completeness for particle - anti particle

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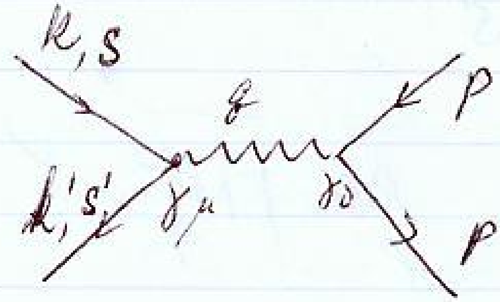
01/20/08

Lecture # 2

$e^+e^- \to \mu^+\mu^-$



Average over initial polarizations, $\frac{1}{2S_A+1}\cdot\frac{1}{2S_B+1}$

Sum over final

$q = k + k'$

$$-iM = \bar u_{k's'}\gamma^\mu u_{ks}\frac{-ig_{\mu\nu}(ie)^2}{q^2}\cdot\bar u(p',s')\gamma^\nu u(p,s) =$$

$$= \bar u_{k's'}\gamma^\mu u_{ks}\frac{+ie^2}{q^2}\bar u(p's'_\mu)\gamma^\mu u(p,s_\mu)$$

average over spins

$$\overline{|M|^2} = M\cdot M^* = \frac{e^4}{q^4}\cdot L_e^{\mu\nu}L^{\mu\nu}_{\mu}$$

$$L_e^{\mu\nu} = \underbrace{\frac{1}{2}\sum_{s,s'}}_{\text{initial spin average}}\bar u_{k's'}\gamma^\mu u_{ks}\Big\{\underbrace{\bar u_{k's'}}_{4\text{-vect}}\underbrace{\gamma^\nu}_{4\times4}\underbrace{u_{ks}}_{4\text{ vect}}\Big\}^*$$

1×1 matrix $\Rightarrow$

$* = +$

$\bar u = u^+\gamma^0$

$\gamma^{\nu+}\gamma^0 = \gamma^0\gamma^\nu$

$$\{\ \}^* = \{\ \}^+ = \{u^+_{k'}\gamma^0\gamma^\nu u_k\}^+ =$$ change order and +

$$= u_k^+\gamma^{\nu+}\gamma^0 u_{k'} = u_k^+\gamma^0\gamma^\nu u_{k'} =$$

$$= \bar u_{ks}\gamma^\nu u_{k's'}$$

use $\sum_s u_{ks}\bar{u}_{ks} = \not{k} + m$

$$(\not{k}'+m)_{\alpha\delta} = (k'^\sigma\gamma^\sigma + m)_{\alpha\delta}$$

$$L_e^{\mu\nu} = \frac{1}{2}\sum_{s,s'} \bar{u}^\alpha_{k's'}\gamma^\mu_{\alpha\beta} u^\beta_{ks}\bar{u}^\gamma_{ks}\gamma^\nu_{\gamma\delta} u^\delta_{k's'} =$$

$$(\not{k}+m)_{\beta\gamma} = (k^\rho\gamma^\rho + m)_{\beta\gamma}$$

$$= \frac{1}{2}(k'^\sigma\gamma^\sigma + m)_{\alpha\delta}\gamma^\mu_{\alpha\beta}(k^\rho\gamma^\rho + m)_{\beta\gamma}\gamma^\nu_{\gamma\delta} =$$

$$= \frac{1}{2} Tr\, k'^\sigma k^\rho \gamma^\sigma\gamma^\mu\gamma^\rho\gamma^\nu + \frac{1}{2}m^2 Tr\,\gamma^\mu\gamma^\nu =$$

N B. $Tr(\text{odd \# of } \gamma) = 0$

$$= \frac{4}{2}k'^\sigma k^\rho(g_{\sigma\mu}g_{\rho\nu} - g_{\sigma\rho}g_{\mu\nu} + g_{\sigma\nu}g_{\rho\mu}) +$$

$$+ \frac{4}{2}m^2 g_{\mu\nu} = 2(k'_\mu k_\nu - (k'k)g_{\mu\nu} + k'_\nu k_\mu) +$$

$$+ 2m^2 g_{\mu\nu} = 2(\underset{1}{k'_\mu k_\nu} + \underset{2}{k'_\nu k_\mu} + \underset{3}{(m^2 - k'k)g_{\mu\nu}})$$

$$L_m^{\mu\nu} = 2(\overset{4}{p'_\mu p_\nu} + \overset{5}{p'_\nu p_\mu} + \overset{6}{(M^2 - p'p)g_{\mu\nu}})$$

$$\overline{|M|}^2 = \frac{4e^4}{q^4}\Big(2\underset{\substack{1\\2}}{(k'\cdot p')}\underset{\substack{4\\5}}{(kp)} + 2\underset{\substack{1\\2}}{(kp')}\underset{\substack{5\\4}}{(k'p)} +$$

$$+2\underset{\substack{1\\2}}{(k'k)}\underset{\substack{6\\6}}{(M^2 - p'p)} + 2\underset{\substack{3\\3}}{pp'}\underset{\substack{4\\5}}{(m^2 - k'k)} + 4\overset{g_{\mu\nu}\cdot g^{\mu\nu}}{\underset{3-6}{(m^2 - k'k)(M^2 - p'p)}}\Big) =$$

$$= \frac{8e^4}{q^4}\Big((k'p')(kp) + (kp')(k'p) \mp M^2(k'k) \mp m^2(p'p) +$$

$$+ 2m^2M^2\Big) \qquad OK!$$

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consider ultrarelativistic situation

$m = 0 = M$

center of mass system

$k = (E; 0; 0; k_z)$

$k' = (E; 0; 0; -k_z)$

$q = 2E = \sqrt{s}$

$q^4 = 16E^4$

$p = (E; E \cdot \sin\theta \cos\varphi; E \cdot \sin\theta \sin\varphi; E \cdot \cos\theta)$

$p' = (E; -E \sin\theta \cos\varphi; -E \cdot \sin\theta \sin\varphi; -E \cdot \cos\theta)$

$$\overline{|M|^2} = \frac{e^4}{2E^4}\left((k'p')(kp) + (kp')(k'p)\right)$$

$$k'p' = E^2 - (-E)\cdot(-E\cos\theta) = E^2(1-\cos\theta)$$
$$kp = E^2 - E \cdot E \cdot \cos\theta = E^2(1-\cos\theta)$$
$$kp' = E^2 - E\cdot(-E\cos\theta) = E^2(1+\cos\theta)$$
$$k'p = E^2 - (-E)\cdot(E\cdot\cos\theta) = E^2(1+\cos\theta)$$

$$|M|^2 = \frac{e^4}{2}\left((1-\cos\theta)(1-\cos\theta) + (1+\cos\theta)(1+\cos\theta)\right) =$$
$$= \frac{e^4}{2}\left(1 - \cancel{2\cos\theta} + \cos^2\theta + 1 + \cancel{2\cos\theta} + \cos^2\theta\right) =$$
$$= e^4(1+\cos^2\theta)$$

now need σ.

$AB \to CD$ Fermi golden rule

$$d\sigma = \frac{|M|^2}{F} d\Phi$$

$$d\Phi = LIPS = (2\pi)^4 \delta^4(p_C + p_D - p_A - p_B) \frac{d^3 p_C}{(2\pi)^3 2E_C} \frac{d^3 p_D}{(2\pi)^3 2E_D}$$

Lorentz Invariant phase space

F - incoming flux $= |v_A| \, 2E_A 2E_B$ for $\underset{\rightarrow}{A} \; \underset{\circ}{B}$

$|v_A - v_B| \, 2E_A 2E_B$ $\underset{\rightarrow}{A} \; \underset{\leftarrow}{B}$

$$F = 4\left((p_A \cdot p_B)^2 - m_A^2 m_B^2\right)^{1/2} \text{ - prove}$$

$\frac{1}{2E}$ comes from wave function normalization $\frac{1}{\sqrt{2E}}$

δ^4 - energy momentum conservation from e^{ipx} integration (see L1)

$d^3p \, d^3x$ - phase space

$(2\pi\hbar)^3$ - min volume in phase space per particle

thus $\frac{d^3p \, d^3x}{(2\pi\hbar)^3}$ - # of final states.

LIPS in center of mass & ultrarelativistic limit

$m = 0 = M$

$p_C = (E;\ E\sin\theta\cos\varphi;\ E\sin\theta\sin\varphi;\ E\cos\theta)$

$p_D = (E;\ -E\sin\theta\cos\varphi;\ -E\sin\theta\sin\varphi;\ -E\cos\theta)$

$$d\Phi = \int (2\pi)^4\, \delta^{4 = 3+1}(p_A + p_B - p_C - p_D)\, \frac{d^3 p_C}{(2\pi)^3 2E_C}\cdot\frac{d^3 p_D}{(2\pi)^3 2E_D} =$$

$$= \int \frac{1}{(2\pi)^2}\, \delta^{1 = 4-3}(\underbrace{E_A + E_B}_{2E_i}\ \underbrace{- E_C - E_D}_{-2E_F})\, \frac{p_C^2\, dp_C\, d\Omega}{2E_C 2E_D} =$$

$E^2 = p^2 + M^2$

$E\,dE = p\,dp$

$p_C = p_F$

$E_C = E_D = E_F$

$$= \frac{1}{(2\pi)^2} \int \delta(2E_i - 2E_F)\, \frac{p_F\, \cancel{E_F}\, d(E_F \cdot 2)}{2\cancel{E_F}\ \underbrace{(2E_F)}_{\sqrt{s}}\ 2}\, d\Omega =$$

$$= \frac{1}{(2\pi)^2}\, \frac{p_F\, d\Omega}{4\cdot\sqrt{s}} = \frac{p_F\, d\Omega}{16\pi^2\sqrt{s}}$$

$$F = \frac{\overbrace{2p_i}^{= v_A - v_B}}{\cancel{E_A}} \cdot 2\cancel{E_A}\ \underbrace{(2E_B)}_{\sqrt{s}} = 2\cdot 2 p_i \sqrt{s}$$

$$\frac{d\Phi}{F} = \frac{p_F\, d\Omega}{16\pi^2\sqrt{s}\cdot 2\cdot 2 p_i\sqrt{s}} = \frac{p_F\, d\Omega}{64\pi^2 p_i\, s}$$

$\alpha = \frac{e^2}{4\pi}$ 2-6

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$\alpha^2 = \frac{e^4}{16\pi^2}$

$$\frac{d\sigma}{d\Omega} = \frac{\overline{|M|^2}}{F} \, dQ = \frac{e^4 \, (1+\cos^2\theta) \, p_F}{64\pi^2 p_i \; 4E^2} \, d\Omega =$$

$$= \frac{\alpha^2}{4\cdot 4E^2} \frac{p_F}{p_i} (1+\cos^2\theta) \, d\Omega = \frac{\alpha^2}{4\cdot 4E^2} (1+\cos^2\theta) \, d\Omega =$$

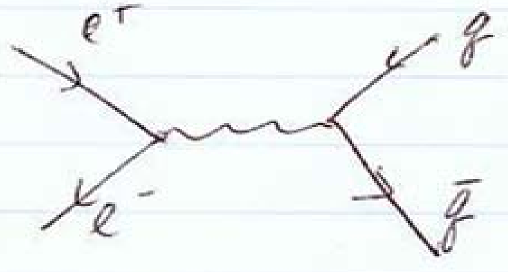
$$= \frac{\alpha^2}{4s} (1+\cos^2\theta) \, d\Omega$$

Total σ

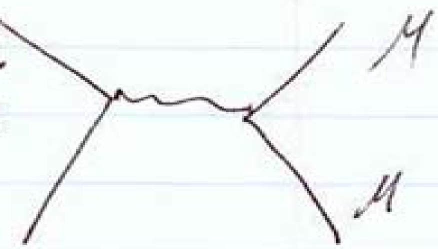
$$\int \frac{\alpha^2}{4s} (1+\cos^2\theta) \, d\varphi \cdot d\cos\theta =$$

$$= \frac{2\pi\alpha^2}{4s} \left(\int_{-1}^{1} 1 \, d\cos\theta + \int_{-1}^{1} \cos^2\theta \, d\cos\theta \right) =$$

$$= \frac{2\pi\alpha^2}{4s} \left(2 + \frac{2}{3} \right) = \frac{4\pi}{3} \frac{\alpha^2}{s}$$



$= e_q^2$



$$R = \frac{ee \to \text{hadrons}}{ee \to \mu\mu}$$