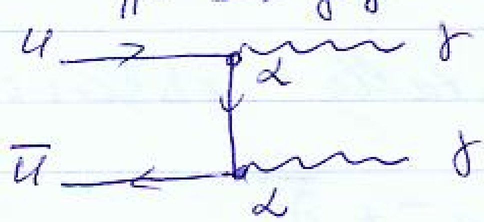
Lecture #3

Electroweak interactions

$\pi^\circ \to \gamma\gamma$



$$|M|^2 \propto \alpha^2$$

$$\Gamma = \frac{\hbar}{\tau} \propto \alpha^2$$

$\pi^\circ \to \gamma\gamma \qquad \tau = 10^{-16}$ sec

$\mu^- \to e^- \bar{\nu}_e \nu_\mu \qquad \tau = 2{,}2 \cdot 10^{-6}$ sec

$\pi^- \to \mu^- \bar{\nu}_\mu \qquad \tau = 2{,}6 \cdot 10^{-8}$ sec

longer time (10 - 8 orders of magnitude) means weaker coupling $\left(\frac{G}{(\hbar c)^3} = 1.16 \cdot 10^{-5} \frac{1}{GeV^2}\right)$

or ....

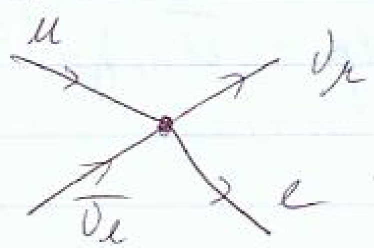
heavy mediator (let's estimate how heavy)

$$G = \frac{g^2}{M_X^2} \qquad \text{let's say } g^2 \approx \alpha$$

$$M_X = \left(\frac{g^2}{G}\right)^{1/2} \cong 25 \text{ GeV} \quad (\text{EW scale})$$

actually 80 GeV $= M_W$ (pretty good)
90 GeV $= M_Z$

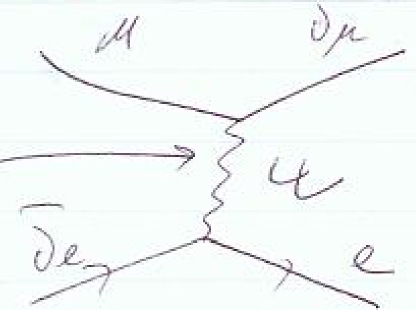
Fermi introduced "contact interactions"



in reality

propagator

$$\frac{g}{M_W^2 - \cancel{q^2}} \quad q^2 \to 0$$



1956 - Lee & Yang suggested P non conservation in weak interactions
1957 - experimentally observed by Wu using

$$^{60}Co \to {}^{60}Ni^{*} + e^{-} + \bar{\nu}_e$$

@ 0.01K

$\uparrow \quad \rightarrow \quad \uparrow \qquad P_0 \uparrow \; \uparrow S_0 \bar{\nu}_e \qquad P_e \downarrow \; \uparrow S_e \; \bar{e}$

$J=5 \qquad J=4$

polarized in magn. field

observe e^- direction

$$I(\theta) = 1 + \alpha \frac{\vec{\sigma}\cdot\vec{p}}{E}$$

pseudo vector! vector

we got a ~~left~~ right handed $\bar{\nu}_R$ current

left handed e_L^-

mathematically

not $\bar{u}_n \gamma^\mu u_p \cdot \bar{u}_{\nu e} \gamma_\mu u_e$

$\bar{u}_\nu \gamma^\mu u_\mu \cdot \bar{u}_e \gamma^\mu u_{\nu e}$, but

$$\bar{u}_n \gamma^\mu \underbrace{\left(\frac{1-\gamma_5}{2}\right)}_{\text{left}} u_p \cdot \bar{u}_{\nu e} \gamma_\mu \underbrace{\frac{(1-\gamma_5)}{2}}_{\text{left}} u_e$$

Reminder about helicities

if $m = 0$, or if a very high energy ($m \ll E$) — helicity is a conserved quantity

Weyl representation

$$(\vec{\alpha}\vec{p} + \cancel{\beta m})\, u = E u \qquad E = \pm p, \quad m \to 0$$

$$E^2 = p^2 \qquad E = \pm p$$

$$\boxed{\vec{\alpha} = \begin{pmatrix} -\vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}}$$

$$\vec{\alpha}\vec{p}\, u = p u$$

$$\vec{\alpha}\vec{p}\, u = -p u$$

$$H = \frac{\vec{\sigma}\vec{p}}{|p|} = \begin{cases} +1 \to \text{right} \to E < 0 \ \text{antiparticle} \\ -1 \to \text{left} \to E > 0 \ \text{particle} \end{cases}$$

$$\gamma_5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

Let us define

$$P_R = \frac{1}{2}(1 + \gamma^5) \qquad P_L = \frac{1}{2}(1 - \gamma_5)$$

$$\begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} u_A \\ u_B \end{pmatrix} \qquad \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

"pulls" out u_B antiparticle

"pulls" out u_A particle

left current $\quad \bar{u}\gamma_\mu \frac{1-\gamma_5}{2} u \quad$ "V − A"

right current $\quad \bar{u}\gamma_\mu \frac{1+\gamma_5}{2} u \quad$ "V + A"

γ_μ – vector

$\gamma_\mu \gamma_5$ – axial vector

What have we established so far?

→ There are weak interactions that are (probably) mediated by massive bosons.

$\frac{G}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$

→ Only left particles or right antiparticles participate in charged currents

Neutral currents

→ 1973 ν scattering experiments established neutral currents by studying angular correlations of recoiling nuclei it was established that they are neither left symmetric nor left-right symmetric (!!)

$\nu q \to \nu q$

currents $\frac{1}{2} \gamma^\mu \left(c_V^q + c_A \gamma_5^q \right)$

from experimental data

$c_V^q = \pm 0.7 \pm ??$

$c_A^q = \mp 0.4 \pm ??$

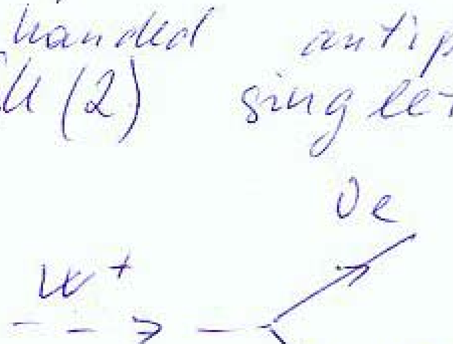
|| average over q, hard to interprete! better data $e\nu$ scattering

→ We know that EM interaction is left-right symmetric, or in other words $c_A = 0$; $c_V = 1$ only vector current

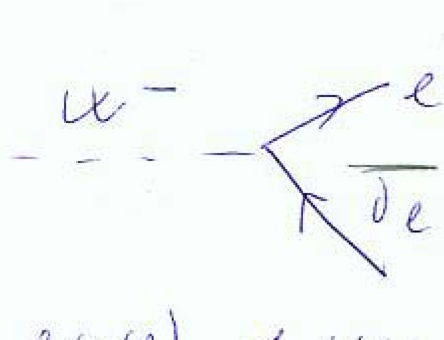
With this experimental evidence Let us build a theory & hopefully make observable predictions.

SU(2) of isospin was already developed by then (people thought that there were 3 quarks u, d, s) Let's recycle it and build $SU(2)_L$ of weak isospin

"L" means that only left handed particles & right handed antiparticles will participate in it (in other words are SU(2) doublets) right handed particles & left handed antiparticles will be SU(2) singlets



$$J_\mu = \bar{u}_\nu \gamma_\mu \frac{1}{2}(1-\gamma_5) u_e = \bar{\nu}_L \gamma_\mu e_L$$



$$J^+_\mu = \bar{e}_L \gamma_\mu \nu_L$$

SU(2) doublet: $\chi_L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L$

step up operator $\tau_+ = \frac{1}{2}(\tau_1 + i\tau_2)$

step down $\tau_- = \frac{1}{2}(\tau_1 - i\tau_2)$

τ_i – Pauli spin matrices $= \sigma_i$

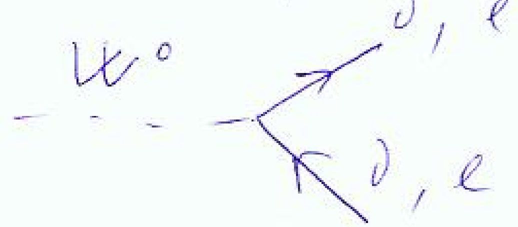
$J^+_\mu(x) = \overline{\chi}_L \gamma_\mu \tau_+ \chi_L$ $\quad$ $J^{1,2}_\mu(x) =$

$J^-_\mu(x) = \overline{\chi}_L \gamma_\mu \tau_- \chi_L$ $\quad$ $= \overline{\chi}_L \gamma_\mu \frac{\tau_{1,2}}{2} \chi_L$

We immediately get $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$J_3 = \overline{\chi}_L \gamma_\mu \frac{1}{2} \tau_3 \chi_L =$

$= \frac{1}{2} \bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma_\mu e_L$



neutral current?

But, oh-no, it is only left!!

we need $c_V + c_A$

Combine with EM left-right.

How?

charges of $SU(2)$

$T^i = \int J^i_0(x) d^3x$

$[T^i, T^j] = i\varepsilon_{ijk} T^k$ ← like angular momenta in QM

For completeness let us introduce EM charge operator Q

$$j_\mu = e\bar{\Psi}\gamma_\mu Q\Psi \qquad U(1)_{em}$$

$$Q\Psi = -1\Psi \quad \text{for } e^-$$
$$Q\Psi = +1\Psi \quad \text{for } e^+$$

By analogy let us define another $U(1)_Y$ with a "hypercharge" Y

$$j_\mu^Y = \bar{\Psi}\gamma_\mu Y\Psi$$

$$Q = T^3 + Y/2$$

our EM charge is a linear combination of T^3 ("z" of T) & hypercharge!

$$j_\mu^{em} = J_\mu^3 + \frac{1}{2} j_\mu^Y$$

must be the same for SU(2) doublet members

Leptons

	T	T^3	Q	Y
ν_e	1/2	1/2 (isospin up)	0	−1
e_L^-	1/2	−1/2 (isospin down)	−1	−1
e_R^-	0			−2

HW - derive for quarks, antiparticles.

Fields that mediate these interactions
→ W_i couples to T_i with strength g
→ B couples to Y with strength $g'/2$

tradition

$$-ig\,(J^i)_\mu W_\mu^i - i\,\frac{g'}{2}\,(j^Y)_\mu B_\mu$$

$$W_\mu^\pm = \sqrt{\tfrac{1}{2}}\left(W_\mu^1 \mp i W_\mu^2\right) -$$

massive charged vector bosons W^+ W^-
neutral fields W_μ^3, B_μ mix(!!)
to form γ & Z

$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W \quad (m_\gamma = 0)$$
$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W \quad (m_Z \neq 0)$$

$$-ig\,(J^3)_\mu (W^3)^\mu - i\,\frac{g'}{2}\,(j^Y)_\mu B^\mu =$$
$$= -ig\,(J_\mu^3)\left(A_\mu \sin\theta_W + Z_\mu \cos\theta_W\right) -$$
$$- i\,\frac{g'}{2}\,(j^Y)_\mu \left(A_\mu \cos\theta_W - Z_\mu \sin\theta_W\right) =$$
$$= -i\left(g \sin\theta_W \cdot J_\mu^3 + \frac{g'}{2}\cos\theta_W\, j_\mu^Y\right)A_\mu -$$
$$- i\left(g \cos\theta_W \cdot J_\mu^3 - \frac{g'}{2}\sin\theta_W\, j_\mu^Y\right)Z_\mu$$

em current

neutral weak current

Since $Q = T^3 + Y/2$

$$e\, j^{em}_\mu = e\left(J^3_\mu + \frac{1}{2} j^Y_\mu\right)$$

then $$\boxed{g \cdot \sin\theta_W = g' \cos\theta_W = e}$$

indeed g is at the order of e (but not equal to e)

remember $$\frac{G}{\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{e^2}{8 \sin^2\theta_W M_W^2}$$

measured in experiment e.g. μ-decay

can we constraint this?

need to predict this

Let us now turn to NC

$$e j^{em} = g \sin\theta_W J^3 + \frac{g'}{2} \cos\theta_W j^Y$$

$$\frac{g'}{2} j^Y = \frac{e}{\cos\theta_W} j^{em} - g \frac{\sin\theta_W}{\cos\theta_W} J^3$$

$$\mathcal{L}_{NC}\; J^{NC} = g \cos\theta_W J^3 - \frac{\sin\theta_W}{\cos\theta_W} e j^{em} + g \frac{\sin^2\theta_W}{\cos\theta_W} J^3 =$$

$$= \frac{g}{\cos\theta_W}\left(J^3 - \frac{e \sin\theta_W}{g} j^{em}\right)$$

$\sin^2\theta_W$

3-10

relate to c_V, c_A measured in ν scattering

$$\overbrace{V-A}\qquad\qquad \checkmark$$

$$J_\mu^{NC} = \overbrace{J_\mu^3} - \sin^2\theta_W\, \overbrace{j_\mu^{em}}$$

weak NC interaction

$$-i\frac{g}{\cos\theta_W} J_\mu^{NC} \cdot Z^\mu =$$

$$= -i\frac{g}{\cos\theta_W}\bar{\psi}\gamma^\mu\left[\frac{1}{2}(1-\gamma^5)T^3 -\right.$$

$$\left.-\sin^2\theta_W Q\right]\psi Z_\mu =$$

$$= -\frac{ig}{\cos\theta_W}\bar{\psi}\frac{\gamma^\mu}{2}\left[c_V^f - c_A^f\gamma^5\right]\psi Z_\mu$$

$$c_V^f = T_f^3 - 2\sin^2\theta_W Q_f$$

$$c_A^f = T_f^3$$

HW - derive c_V, c_A for quarks & leptons

$$c_A^e = -0.52 \pm 0.06$$

$$c_V^e = 0.06 \pm 0.08$$

from $e\nu$ scattering

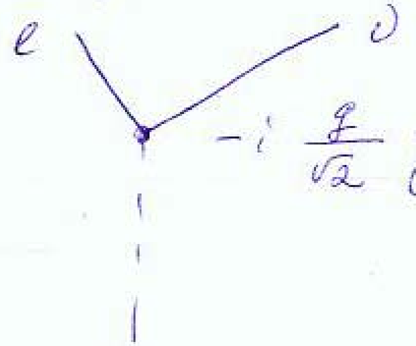
$\sin^2\theta_W$ (large uncertainty)

more careful analysis of νN scattering

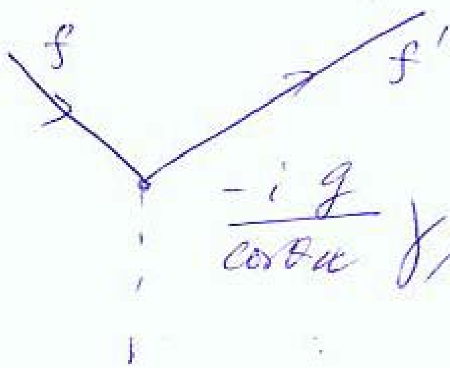
1981: $\sin^2\theta_W = 0.234 \pm 0.013$

now we can predict W mass!

Feynman rules



$$-i\frac{g}{\sqrt{2}}\gamma^\mu\frac{1}{2}\left(1-\gamma_5\right)$$



$$\frac{-ig}{\cos\theta_W}\gamma^\mu\frac{1}{2}\left(c_V^f-c_A^f\gamma^5\right)$$

$$\frac{-i\left(g_{\mu\nu}-p_\mu p_\nu/M^2\right)}{p^2-M^2}$$