Lecture # 4

So far we established

$SU(2)_L \times U(1)_Y$

$\downarrow$

T_1 ⋮ $T_+ = \frac{1}{\sqrt{2}}(T_1 - iT_2) \longrightarrow W^+_\mu$ } couple left currents (massive)

T_2 ⋮ $T_- = \frac{1}{\sqrt{2}}(T_1 + iT_2) \longrightarrow W^-_\mu$

T_3 Y $\longrightarrow W^3_\mu$ & B_μ

$m_\gamma = 0 \longrightarrow A_\mu$; $Z_\mu \longleftarrow$ massive

$Z_\mu \longrightarrow$ couples asym to left & right currents

$A_\mu \longrightarrow$ couples sym to left & right

The experimental facts that lead to this construction are

$\frac{G}{\sqrt{2}}$ → long (compared to EM) lifetime of weak processes

→ parity nonconservation in weak processes

→ charge currents can be described as only left.

c_A, c_V → neutral currents have unequal left & right components, measured in ν scattering experiments.

Construction of a unified EW theory allowed to predict W boson mass

$$\frac{G}{\sqrt{2}} = \frac{e^2}{8\sin^2\theta_W \cdot \underline{M_W^2}}$$

discovered in 1983

prediction for W mass did not involve any mechanism of EW symmetry breaking (next lecture), or in other words a construction that would allow to incorporate masses in a gauge invariant form. Prediction for Z mass did:

$$M_Z = \frac{M_W}{\cos\theta_W}$$

it was confirmed indirectly in NC ν scattering (from propagator $\frac{1}{M_Z^2}$) and most importantly Z was discovered at this mass in 1983.

Yet....

the suggested mechanism of the EW symmetry breaking - so called Higgs mechanism is not considered to be fully established experimentally since it predicts an existance of a scalar boson that is not discovered yet.

At the moment let us concentrate on properties of W & Z bosons.

The probability of two particles interacting is best described by the cross section (σ).
The probability of a decay of a certain particle is described by its lifetime τ, or width
$\Gamma = \hbar/\tau \rightarrow \frac{1}{\tau}$
partial width Γ_i characterized a probability to decay through a process i (one particular reason of death)
total width $\Gamma = \sum_i \Gamma_i$
thus $\tau = \frac{1}{\Gamma} = \frac{1}{\sum_i \Gamma_i}$

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i}$$

(lifetime decreases if there are several possible reasons for death - don't smoke!)

Branching ratio is a relative probability of a certain decay
$\sum_i Br_i = 1$

$$Br_i = \Gamma_i / \Gamma$$

Let us discuss the width of W & Z decays.

$W \to f\bar{f}'$, where $f\,f'$ are members of $SU(2)_L$ doublet.

$$W \to \underbrace{\begin{matrix} e\,\bar{\nu}_e \\ \mu\,\bar{\nu}_\mu \\ \tau\,\bar{\nu}_\tau \end{matrix}}_{3\text{ possibilities}} \quad \underbrace{\begin{matrix} \bar{u}\,d \\ \bar{c}\,s \end{matrix}}_{2\times 3(\text{color})\text{ possibilities}}$$

9 possibilities

all are equally probable in the approximation of $m_f = 0$

$$Br(W \to f\bar{f}') = \frac{1}{9} \text{ if } f = \text{lepton}$$
$$\frac{1}{3} \text{ if } f - \text{quark}$$

$Z \to f\bar{f}$ $\quad f = \begin{matrix} e, & \mu, & \tau \\ \nu_e, & \nu_\mu, & \nu_\tau \end{matrix}$ $\;\Big\|\;$ 6 options

$\begin{matrix} u & \cancel{d} \\ c & s \end{matrix}$ $\;\Big\|\;$ $4\times 3 = 12$ options

but couplings are different

$$c_V^f = T_f^3 - 2\sin^2\theta_w Q_f$$
$$c_A^f = T_f^3$$

So let us calculate in a general form $\Gamma(X \to f\bar{f}^{(\prime)})$

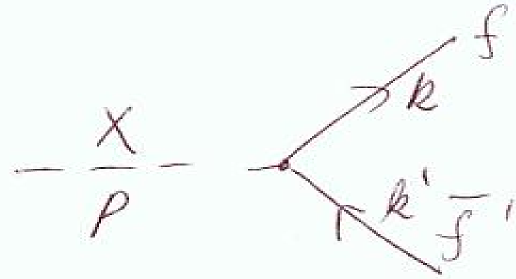
$X = Z, W$

$c_V^w = 1$

$c_A^w = -1$

$$g_X = \begin{cases} g/\sqrt{2} & X = W \\ g/\cos\theta_w & X = Z \end{cases}$$

$$g = e/\sin\theta_w$$



λ - helicity of X_μ
s - helicities of fermions

$$iM = i g_x \gamma^\mu \bar{f}' \tfrac{1}{2}(c_V - c_A \gamma^5) f X_\mu$$

explain!

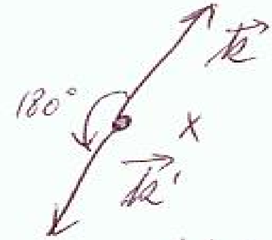
$$\overline{|M|^2} = \frac{g_x^2}{3} \sum_{\lambda, s, s'} \bar{f}' \gamma^\mu \tfrac{1}{2}(c_V - c_A\gamma_5) f X_\mu \cdot$$
$$\cdot X^{*\nu} \left(\bar{f}' \gamma^\nu \tfrac{1}{2}(c_V - c_A \gamma^5) f\right)^* =$$
$$= \frac{g_x^2}{12} \sum_\lambda X_\mu X^{\nu *} \sum_s \bar{f}' \gamma^\mu (c_V - c_A\gamma^5) f \cdot$$
$$\cdot \left(\bar{f}' \gamma^\nu (c_V - c_A \gamma 5) f\right)^* =$$
$$= \frac{g_x^2}{12} \cdot \left(g_{\mu\nu} - \frac{p^\mu p^\nu}{m_x^2}\right) Tr\, \gamma^\mu \not{k} \gamma^\nu \not{k}' \cdot$$
$$\cdot \underbrace{(c_V - c_A \gamma_5)^2}_{c_V^2 - \underbrace{2 c_V c_A \gamma_5}_{\to 0 \text{ trace of odd \# of } \gamma\text{'s}} + c_A^2} =$$

γ now → 0

$$= \frac{g_x^2}{12}(c_V^2 + c_A^2)\left(g_{\mu\nu} - \frac{p^\mu p^\nu}{m_x^2}\right) 4\left(k'^\mu k^\nu - (k'k) g_{\mu\nu} + k'^\nu k^\mu\right) =$$

$$= \frac{g_x^2}{3}\left(c_V^2 + c_A^2\right)\left(2(k'k) - 4(k'k)\right) =$$

$$= \frac{2g_x^2}{3}\left(c_V^2 + c_A^2\right)\cdot\left(k'k\right)$$



$$m_f = 0 = m_{f'}$$

$$|k| = E = \frac{M_X}{2}$$

$$k'k = E^2 - E^2 \cdot \cos\theta =$$

$$= 2E^2 = \frac{M_X^2}{2}$$

hence, $\overline{|M|}^2 = \frac{g_x^2}{3}\left(c_V^2 + c_A^2\right) M_X^2$

$$\Gamma = \int \frac{1}{2E_X}\overline{|M|}^2\, d\Phi = \int \frac{1}{2M_X}|M|^2\, d\Phi$$

$$d\Phi = LIPS = (2\pi)^4\,\delta^4\left(\Sigma p_i - \Sigma p_f\right)\frac{d^3k}{(2\pi)^3 2E_f}\cdot\frac{d^3k}{(2\pi)^3 2E_{f'}}$$

$$\Gamma = \frac{g_x^2}{3\cdot 2}\left(c_V^2 + c_A^2\right) M_X \int d\Phi$$

$$\int d\Phi = \frac{1}{4\cdot(2\pi)^2}\int \delta\left(M_X - E_1 - E_1\right)\frac{k^2\, dk\cdot d\Omega}{E_1^2} =$$

$$= \frac{1}{4(2\pi)^2}\int \delta\left(M_X - \underline{\underline{2}}\,E\right) dE\, d\Omega =$$

$$= \frac{1}{8(2\pi)^2}\int d\Omega = \frac{4\pi}{8\cdot 4\cdot \pi^2} = \frac{1}{8\pi}$$

$g_w = g/\sqrt{2}$ 4-7
$g_Z = g/\cos\theta_w$

$$\Gamma\left(X \to f\bar{f}^{(\prime)}\right) = \frac{g_X^2}{48\pi}\left(C_V^2 + C_A^2\right) M_X$$

$$\Gamma_1\left(W \to f\bar{f}'\right) = \frac{g^2}{96\pi}\left(1+1\right) M_W = \frac{g^2}{48\pi} M_W =$$

$$= \frac{e^2}{\sin^2\theta_w\, 48\pi} \cdot M_W = \frac{\alpha}{12\sin^2\theta_w} \cdot M_W = 0.225\,\text{GeV}$$

$\sin^2\theta_w = 0.234$

$\alpha(M_W) = \frac{1}{128}$

$M_W = 81\,\text{GeV}$

$\Gamma_{total} = \Gamma_1 \cdot 9 = 2.03\,\text{GeV}$.
in PDG $\Gamma = 2.141 \pm 0.041\,\text{GeV}$
we ignored masses & higher order corrections.

Now Z-decays.

$C_V^e = -\frac{1}{2} + 2\sin^2\theta_w = -0.03$

$C_A^e = -1/2 = -0.5$

$g_Z^2 = \frac{g^2}{\cos^2\theta_w} = \frac{e^2}{\sin^2\theta_w(1-\sin^2\theta_w)}$

$$\Gamma_{\ell\bar{\ell}} = \frac{g^2}{48\pi\cos^2\theta_w}\left({C_V^e}^2 + {C_A^e}^2\right) M_Z =$$

$$= \frac{\alpha\, M_Z}{12\,\cancel{\pi}\,\sin^2\theta_w(1-\sin^2\theta_w)}\overbrace{\left(0.03^2 + 0.5^2\right)}^{0.2509} = 0.083\,\text{GeV}$$

$\times 3$ for lepton flavor

4-8

ν: $c_V = c_A = \frac{1}{2}$

$$\Gamma(Z \to \nu\nu) = \frac{g^2}{48\pi \cos^2\theta_W} \cdot \left(\frac{1}{4} + \frac{1}{4}\right) M_Z =$$

$$= \frac{g^2}{96\pi \cos^2\theta_W} M_Z = \frac{\alpha\, M_Z}{24 \sin^2(1-\sin^2)} = 0.167\ \text{GeV}$$

$$\times 3 = 0.502\ \text{GeV} =$$

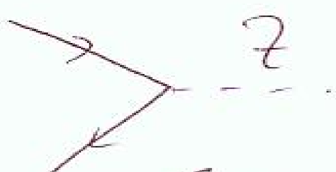
= "invisible" width

Note! that $\nu\bar{\nu}$ width $>$ $l\bar{l}$ ($\times 2$)
important for Higgs searches.

$\Gamma(Z \to \text{hadrons}) = 1.744$ GeV

$\Gamma_{total} = \underline{2.49 \text{ GeV}}$

$e^+e^- \to Z$ inverted diagram for decay



$J=1$ (spin Z)

$$\sigma = \frac{4\pi(2J+1)}{2 \cdot 2\, M_Z^2} \cdot \frac{\Gamma_i\, \Gamma_f}{(E - m_Z)^2 + \Gamma^2/4}$$

Breit-Wigner resonance shape

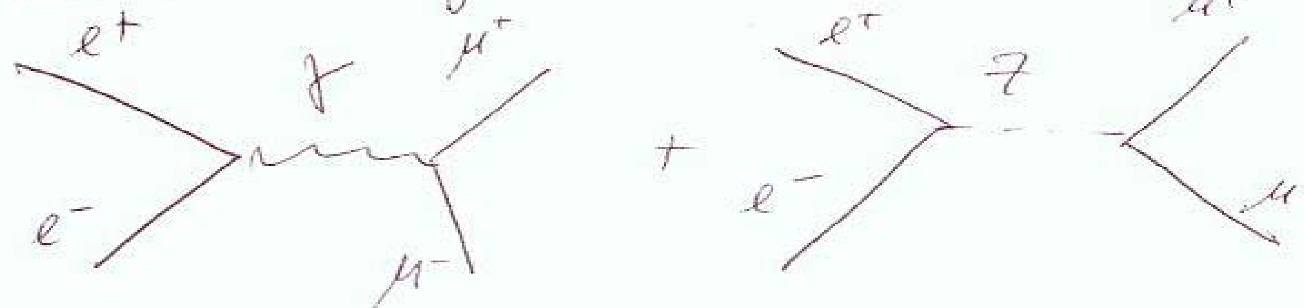
$\Gamma_i = \Gamma(Z \to ee)$

$\Gamma_f = \Gamma(Z \to \text{final})$

$\Gamma = \Gamma_{total}$

Asymmetries @ Z

consider $\gamma - Z$ interference



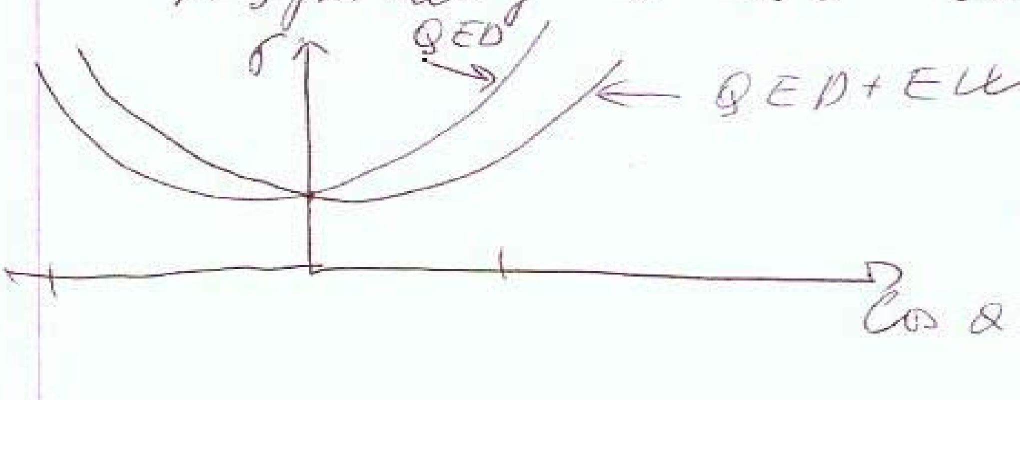
$$\frac{d\sigma}{d\Omega}\left(e_L^- e_R^+ \to \mu_L^- \mu_R^+\right) = \frac{\alpha^2}{4s}(1+\cos\theta)^2 \left|1 + r c_L^e c_L^\mu\right|^2$$

$$\frac{d\sigma}{d\Omega}\left(e_L^- e_R^+ \to \mu_R^- \mu_L^+\right) = \frac{\alpha^2}{4s}(1-\cos\theta)^2 \left|1 + r c_R^\mu c_L^e\right|^2$$

$$r = \frac{\sqrt{2}\, G M_Z^2}{s - M_Z^2 + i M_Z \Gamma_Z}\left(\frac{s}{e^2}\right)$$

$c_R = c_V - c_A$
$c_L = c_V + c_A$

Asymmetry in $\cos\theta$ distribution



$$A = \frac{F - B}{F + B}$$