Lecture # 5.

$SU(2)_L \times U(1)$ - EW theory
highly predictive
M_W, $M_{Z boson}$, Widths, asymmetries...
What about the lagrangian?

Remember L_{QED}?

$$L_{QED} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}\, i\gamma^\mu \overbrace{D_\mu}^{\text{long derivative}} \psi - m\bar{\psi}\psi =$$

$$= \underbrace{\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi}_{\text{free electron}} + \underbrace{e\bar{\psi}\gamma^\mu A_\mu \psi}_{\substack{\text{interaction}\\ \text{field}}} \underbrace{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{free } \gamma}$$

remember gauge (phase) invariance
$U(1)$ transformation $e^{i\alpha}$ invariance

$$\left.\begin{array}{l} \psi(x) \to e^{i\alpha(x)}\psi(x) \\ A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha(x) \end{array}\right\} \text{local } \alpha(x)$$

$D_\mu \equiv \partial_\mu - ieA_\mu$

By requiring local (not global) phase invariance we introduced an interaction with a gauge **field**

$U(1)$ invariance $\to$ charge conservation.

N.B. other "charges" = lepton, baryon numbers - these are <u>global</u> $U(1)$'s no interactions ("baryonic photons")!

symmetry under local gauge transformati
"local gauge invariance" is associated with interactions.

NB. mass term $\frac{1}{2} m^2 A_\mu A^\mu$ would break gauge invariance.

Working in analogy with QED we construct EW Lagrangian:

left doublets $Y_L = -1$
right singlets $Y_R = -2$

$$\mathcal{L} = \bar{\chi}_L \gamma^\mu [i\partial_\mu - g\tfrac{1}{2}\tau W_\mu - g'(\tfrac{1}{2}) B_\mu] \chi_L$$
$$+ \bar{e}_R \gamma^\mu [i\partial_\mu - g'(-1) B_\mu] e_R -$$
$$- \tfrac{1}{4} W_{\mu\nu} W^{\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$\sim F_{\mu\nu}$ $\longrightarrow$ $B_{\mu\nu} = \partial_\mu B^\nu - \partial^\nu B_\mu$

$$W_{\mu\nu} = \partial_\mu W^\nu - \partial^\nu W_\mu \overbrace{- g W_\mu \times W_\nu}^{\text{self interaction}}$$

non-abelian character of the group. (τ's do not commute

NB. all interactions are in, but no masses!!!

$$- m_e \bar{e} e = - m_e (\bar{e}_R e_L + \bar{e}_L e_R)$$ breaks gauge invariance

SU(2) doublet

singlet

no $\frac{1}{2} M^2 B_\mu B^\mu$ terms either.

How do we define gauge (phase) transformation?

QED

$$U = e^{i\alpha(x)} = e^{i\alpha(x)Q}$$

EW

$$U = e^{i\alpha_a(x)\,T_a} \qquad a = 1,2,3$$

$$\psi \to \psi' = U\psi$$
$$\bar\psi \to \bar\psi' = \bar\psi\,U^+ \qquad T_a = \tau_a/2$$

infinitesimal gauge transformation

$$\psi' = (1 + i\alpha(x))\psi \qquad \psi' = \left(1 + i\,\alpha_a(x)\frac{\tau_a}{2}\right)\psi(x)$$

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha(x)$$
$$\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu \qquad ?? \qquad D_\mu = \partial_\mu + ig\frac{\tau_a}{2}W_\mu^a$$

plug into L, find that for gauge invariance we need

$$W_\mu^a \to W_\mu^a - \frac{1}{g}\partial_\mu\alpha_a - \underline{\underline{\varepsilon_{abc}\,\alpha_b\,W_\mu^c}}$$

because $[\tau_a\tau_b] = i\varepsilon_{abc}\tau_c$

moreover for the kinetic term $W_{\mu\nu}W^{\mu\nu}$ to be gauge invariant we need

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - g\,W_\mu \times W_\nu$$

or

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - \underline{\underline{g\,\varepsilon_{abc}W_\mu^b W_\nu^c}}$$

we introduced two extra terms for the sake of gauge invariance and we are still not there because of masses!

By the way, introduction of

$W_{\mu\nu} W^{\mu\nu}$ term in the Lagrangian, where $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - g\, \varepsilon_{abc} W_\mu W_\nu$

introduce self interacting terms!

Presents of these diagram is an experimentally observable effect. This was verified by LEP II by measuring $\underset{Z}{W}\,\underset{Z}{W}$ production(s).

Masses for gauge bosons

are generated through "spontaneous symmetry breaking".

We postulate the existence of a scalar complex $SU(2)$ doublet

$$\phi = \begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

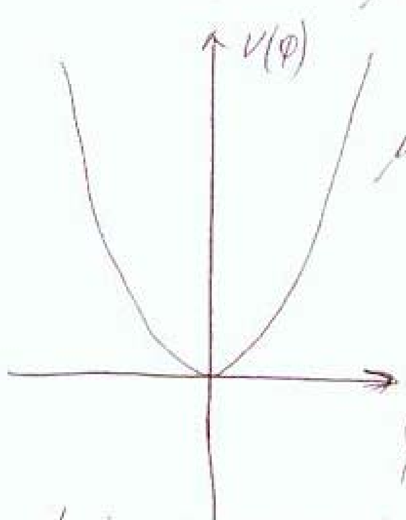
it has a Lagrangian:

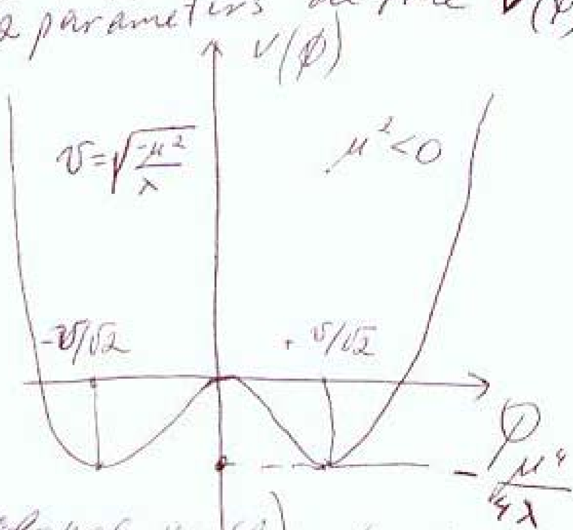
$$L = (\partial_\mu \varphi)^+ (\partial^\mu \varphi) - \mu^2 \varphi^+ \varphi - \lambda (\varphi^+ \varphi)^2$$

or in other words it has a potential

$$V(\varphi) = \mu^2 \varphi^+ \varphi + \lambda (\varphi^+ \varphi)^2$$

2 parameters define $V(\varphi)$





L is invariant under global $SU(2)$ phase transformation

impose local gauge invariance

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ig \frac{\tau_a}{2} W^a$$

$$\varphi(x) \rightarrow \varphi'(x) = (1 + i\alpha(x)\tau/2)\,\varphi(x)$$

$$W_\mu \rightarrow W_\mu - \frac{1}{g}\partial_\mu \alpha - \alpha \times W_\mu$$

gauge symmetric Lagrangian!

$$\mathcal{L} = \left(\partial_\mu \varphi + ig\frac{\tau}{2} W_\mu \varphi\right)^+ \left(\partial_\mu \varphi + ig \frac{\tau}{2} W^\mu \varphi\right) - V(\varphi) - \frac{1}{4} W_{\mu\nu} W^{\mu\nu}$$

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - g\, W_\mu \times W_\nu$$

5-6

Potential has a minimum @ $\varphi^+\varphi = -\frac{\mu^2}{2\lambda}$

$$\varphi^+\varphi = \frac{1}{2}\left(\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2\right) = -\frac{\mu^2}{2\lambda}$$

We'll see why this is a good choice

(We can choose a particular minimum (Nature also made this choice!)

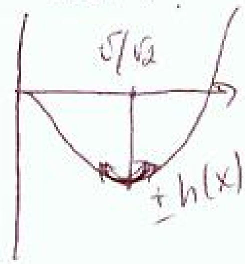
$$\varphi_1 = \varphi_2 = \varphi_4 = 0 \qquad \varphi_3^2 = -\frac{\mu^2}{\lambda} = v^2$$

$$\varphi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$T = 1/2$
$T_3 = -\frac{1}{2}$
$Y = 2(Q - T_3) = 1$
$Q = 0!$

Let us expand around this min:

$$\varphi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$



invariant under $U_{em}(1)$ transformation

$$\left(ig\tfrac{1}{2}\tau W_\mu \varphi\right)^+ \left(ig\tfrac{\tau}{2} W^\mu \varphi\right) =$$

$$= \frac{g^2}{8}\left|\begin{pmatrix} W_3^\mu & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW^2 & -W_\mu^3 \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2$$

$$= \frac{g^2 v^2}{8}\left(\left(W_\mu^1 - iW_\mu^2\right), -W_\mu^3\right)\begin{pmatrix} W_\mu^1 + iW_\mu^2 \\ -W_\mu^3 \end{pmatrix} =$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\ \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix};\ \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \frac{g^2 v^2}{8}\left((W_\mu^1)^2 + (W_\mu^2)^2 + (W_\mu^3)^2\right)$$

mass term for vector bosons

$\Longrightarrow \quad M_W = \frac{1}{2} g v \Longrightarrow v = 246$ GeV

NB. min @ $v/\sqrt{2} = 174$ GeV!

to get mass for Z we need to add B_μ

$$\left|\left(-ig\frac{\tau}{2}W_\mu - i\frac{g'}{2}B_\mu\right)\varphi\right|^2 =$$

$$= \frac{1}{8}\left|\begin{pmatrix} gW^3 + g'B & g(W^1 - iW^2) \\ g(W^1 + iW^2) & -gW^3 + g'B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2 =$$

$$= \frac{1}{8}v^2 g^2\left[W_1^2 + W_2^2\right] + \frac{1}{8}v^2\left(g'B_\mu - gW_\mu^3\right)\times$$

$$\times\left(g'B^\mu - gW^{3,\mu}\right) =$$

$$= \left(\frac{vg}{2}\right)^2 W_\mu^+ W_\mu^- + \frac{1}{8}v^2\left(W^3, B^3\right)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\times$$

$$\times\begin{pmatrix} W^3 \\ B \end{pmatrix}$$

$$\tfrac{1}{8} v^2 \left(g^2 (W^3)^2 - 2 g g' W^3 B + g'^2 B^2 \right] =$$

$$= \tfrac{1}{8} v^2 \underbrace{[g W^3 - g' B]}_{Z}{}^2 +$$

$$+ 0 \, \underbrace{[g' W^3 + g B]}_{\gamma}{}^2$$

$M_\gamma = 0$, not a coincidence our choice of $\varphi_{min} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ guaranteed it!

Higgs field in this particular min (!) does not interact with γ; light thus remains massless

$$\frac{g'}{g} = \tan \theta_W$$

$$\Longrightarrow M_Z = \frac{v}{2} \sqrt{g^2 + g'^2} \qquad \frac{M_W}{M_Z} = \cos \theta_W$$

This is a prediction that turns out to be true; $M_Z = 91$ GeV. $M_\gamma = 0$, but this is by design.

$$A_\mu = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}} \; ; \; Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}}$$

$$A_\mu = \cos \theta_W B_\mu + \sin \theta_W W_\mu^3 \; ; \; Z_\mu = - \sin \theta_W B_\mu + \cos \theta_W W_\mu^3$$

So we got ourselves massive vector bosons. But at what cost?

We introduced quite a few new terms into the Lagrangian $\Rightarrow$ new interactions:

$$\mathcal{L} = \left| \left(i\partial_\mu - g\frac{\tau}{2}W_\mu - g'\frac{Y}{2}B_\mu \right) \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \right|^2 -$$

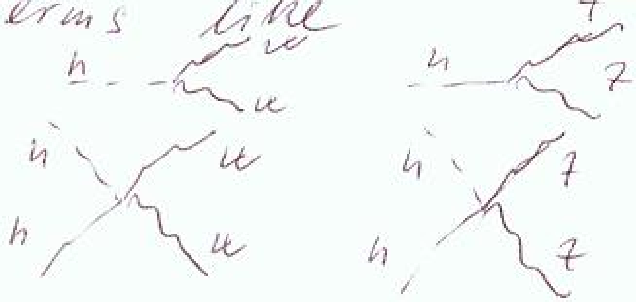
$$- \mu^2 \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}^2 - \lambda \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}^4$$

v - constant; h - field

We will get terms like

Higgs-vector boson

$\left(\begin{array}{l} hW^2;\ hB^2 \\ h^2W^2;\ h^2B^2 \end{array} \right.$



Higgs self interaction $\rightarrow h^3;\ h^4$

and finally Higgs mass term

$-\lambda v^2 h^2$ $\qquad M_H = \sqrt{\lambda} \cdot v$

not constrained! $\qquad$ constrained by W mass

the theory does not predict Higgs mass.

Two parameters $\mu;\ \lambda$ or

M_H ; $v = 246$ GeV

?

5-10

One last thing - fermion masses.

Add terms ad hoc:

$$g_f \left[\left(\bar{f}_L \phi \right) f_R + h.c. \right]$$

OK

SU(2) doublets — SU(2) singlet

SU(2) invariant,

not OK

$$m_f \bar{f}_L f_R$$

SU(2) doublet — SU(2) singlet

not invariant.

plug in $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

$$\underbrace{\frac{1}{\sqrt{2}} g_f v}\left(\bar{f}_L f_R \right)$$

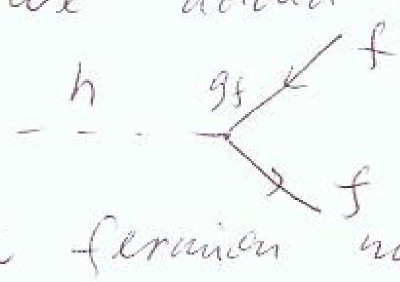
$$m_f = g_f \frac{v}{\sqrt{2}} = g_f \cdot \frac{246}{\sqrt{2}} = g_f \cdot 174 \text{ GeV}$$

$$g_f = \frac{m_f}{174}$$

we did not predict anything, the number of free parameters is the same $m_f \longleftrightarrow g_f$

as a result we added terms like

$g_f \bar{f} h f$



Higgs interaction with fermions $\propto$ fermion mass $\frac{m_f}{v/\sqrt{2}}$