Lecture # 8

Neutrinos: interactions, flavors, mass, mixing.

$q(\nu) = 0$ $m(\nu) \to 0$ $\overset{\text{lepton}}{q_s}(\nu) = 0$

hard to detect - participate only in weak interactions

→ first suggest by Pauli in 1930 to explain energy/momentum/angular momentum non conservation in β-decays.

$n \to p + e^- + \nu_e$ need $s(\nu) = 1/2$

→ the name was suggested by Fermi

→ 1942 K.C. Wang proposed to use beta-capture to detect ν's

→ 1956 Cowan, Reines, Harrison, Kruse & McGuire published observation article - Nobel prize in 1995 (65 years after the idea was suggested)

$\nu_e + p \to n + e^+$

→ 1962 Lederman, Schwartz, Steinberger showed that there is more than one type of ν: $\nu_\mu + N \to \mu + N'$

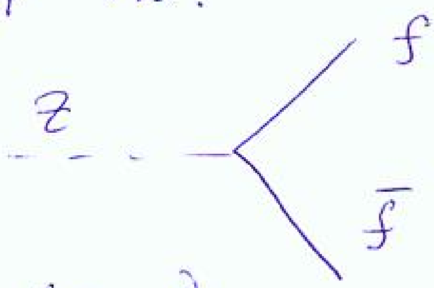
→ 1975 τ-lepton discovered

~2000 DoNut confirmed ν_τ

in SM:

$$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L \quad e_R;\ \mu_R;\ \tau_R$$

from Z line shape we know that
there 3 ν flavors.
at least the ones that are lighter
than Z/2 and couple to Z.
Does this rule out the existence
of ν_R?



$$-i\frac{g}{\cos\theta_W}\frac{\gamma^\mu}{2}\left(c_V^f - c_A^f\gamma^5\right)$$

$f = \nu_L$

$T_3 = +\frac{1}{2}$

$Q = 0$

$c_V = \frac{1}{2}$

$c_A = \frac{1}{2}$

$c_V^f = T_f^3 - 2\sin^2\theta_W Q_f$

$c_A^f = T_f^3$

$f = \nu_R$ $\quad T_3 = 0 \quad c_V = 0$

$Q = 0 \quad c_A = 0$

No, it does not rule out ν_R.
ν_R does not couple to Z.
ν_R does not couple to W either.

→ Dirac particles $\underset{\overset{\uparrow}{\text{left}}}{\overset{q=0}{\nu_e}}$; $\underset{\overset{\uparrow}{\text{right}}}{\overset{q=0}{\bar{\nu}_e}}$ $\quad s = 1/2$

just like $\underset{\substack{\text{particle}\\ q=-1}}{e^-}$ $\quad \underset{\substack{\text{antiparticle}\\ q=1}}{e^+}$

→ Majorana $\nu \equiv \bar{\nu}$, but
ν - left helicity
$\bar{\nu}$ - right helicity

How to test of ν is Dirac or Majorana particle?

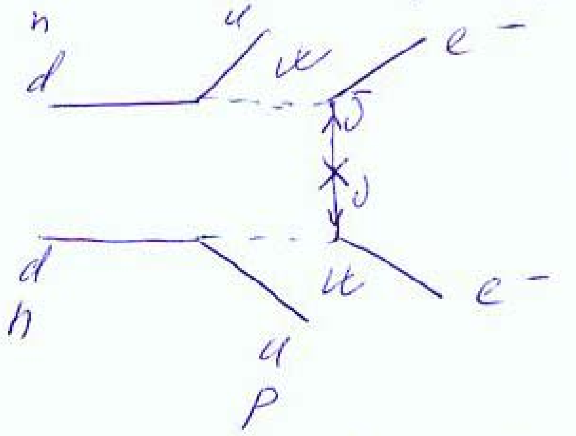
double beta decay, e.g. ${}^{82}Se \to {}^{82}Kr$

$2n \to 2p + 2e^- + 2\bar{\nu}_{e_R}$ //

if $\nu \equiv \bar{\nu}$ we can have

$n \to p + e^- + \bar{\nu}_{e_R}$

$\bar{\nu}_{e_R} (\equiv \nu_{e_R}) + n \to p + e^-$



neutrinoless double beta decay = $0\nu_{\beta\beta}$

Suppressed by helicity flip

$$1 - \frac{v}{c} = 1 - \frac{p}{E_0} = 1 - \sqrt{\frac{E^2 - m^2}{E^2}} =$$

$$= 1 - \sqrt{1 - \frac{m^2}{E^2}} = 1 - \left(1 - \frac{m^2}{2E^2}\right) = \frac{m_\nu^2}{2E_0^2}$$

Thus, limits on the rate of $0\nu_{\beta\beta}$ sets limits on m_ν^2, but only <u>for Majorana type of ν's.</u> $\overset{m_\nu}{<}$ 1 eV

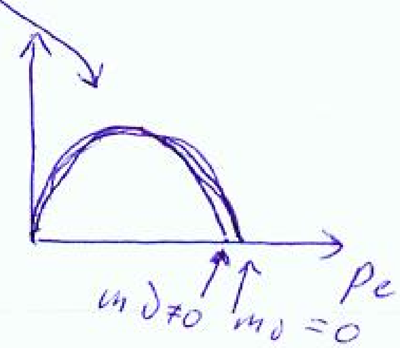
also limit heavy W' of $V+A$ nature.

NEMO3 (France)
under analysis

→ Neutrino mass - "direct" measurement
study the kinematics of decay products

$^3H \to {}^3He + e^- \bar{\nu}_e$ $m_{\nu_e} < 2$ eV
Katrin down to $\underline{0.35\,\text{eV}}$

$\pi \to \mu + \nu_\mu$ $m_{\nu_\mu} < 0.17$ MeV
$\tau \to \nu_\tau + 5\pi$ $m_{\nu_\tau} < 18$ MeV



study end spectrum, very sensitive measurement

So, do ν have zero mass? for a while we thought so. But now there evidence that this is not the case.

→ The evidence comes from the fact that $\underline{\nu \text{ mix}}$ = change flavor in the course of their life

weak eigenstates $\nu_e; \nu_\mu; \nu_\tau$
mass eigenstates $\nu_1; \nu_2; \nu_3$

not so surprising
recall CKM.

Let us review the experimental evidence for mixing.

ν mass, mixing and flavor change

8-5

Experimental evidence

Solar ν: nuclear fusion — solar neutrinos

$$^1H + {}^1H \to \boxed{^2H} + e^+ + \boxed{\nu_e}$$

no ν_μ, ν_τ

$$^1H + {}^2H \to {}^3He + \boxed{\gamma}$$

This is the light that we see

$$^3He + {}^3He \to \underbrace{^4He}_{\text{heavier element creation}} + {}^1H + {}^1H$$

The reaction is well understood (fission = hydrogen bomb)

The amount of H, He can be checked from spectral lines

N.B. He was discovered in solar spectral analysis

Total energy (N_γ) is measured.

So, the ν_e flux is well predicted.

Yet, for decades solar ν experiments report ~a factor of 2 deficit in the number of ν_e observed. $\nu_e + {}^{71}Ga \to {}^{71}Ge + e^-$

Atmospheric ν:

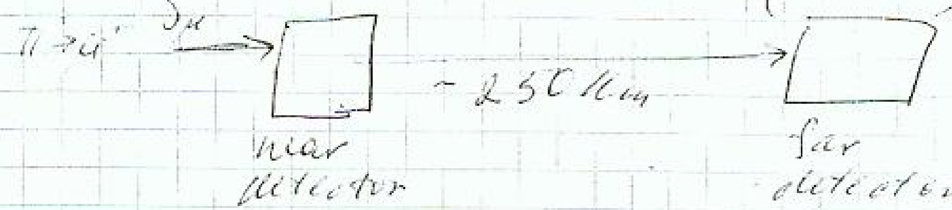
ν_{atm} produced in the Earth atmosphere by cosmic rays. Expect isotropic flux at E ≳ few GeV

Yet Super Kamiokande underground detector sees ~ factor of two difference in up coming ν_μ's compared to down coming ν's

$N(\nu_\mu \text{ up}) = 1/2 \, N(\nu_\mu \text{ down})$.

Accelerator ν's:

KEK to Kamioka (K2K)



flux near — compare — flux far

expect

$155.9 \pm 0.3 \, ^{+13.6}_{-15.6}$

observe

112 events

distorted spectrum.

Possible explanation:

$\boxed{\nu \text{ change flavor}}$

e.g. $\nu_e \to \nu_\mu$ That would mean the ν have masses

mass eigenstates $\nu_1; \nu_2; \nu_3$ $- \nu_i$

flavor eigenstates $\nu_e; \nu_\mu; \nu_\tau$ $- \nu_\alpha$

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$$

U - unitary leptonic mixing matrix most likely this is 3×3 matrix

$Z \to \nu_\alpha \bar{\nu}_\alpha$ $\alpha = 1; 2; 3$ - invisible Z-decays. But the possibility of sterile ν (does not couple to Z, W) is not excluded.

~~11~~ in ν_i frame: from Schrödinger's eq:

$$|\nu_i(\tau_i)\rangle = e^{-i m_i \tau_i} |\nu_i(0)\rangle$$

m_i – ν_i mass

τ_i – time in ν frame.

$$e^{-i m_i \tau_i} = e^{-i(E_i t - p_i L)}$$

in ultrarelativistic case $t \approx L$

$$E_i - p_i \approx \frac{m_i^2}{2p}$$

$$e^{-i\frac{m_i^2}{2p}L} \approx e^{-i\frac{m_i^2}{2E}L}$$

$$|\nu_\alpha(L)\rangle \approx \sum_i U^*_{\alpha i}\, e^{-i\frac{m_i^2}{2E}L} |\nu_i\rangle =$$

$$= \sum_\beta \left[\sum_i U^*_{\alpha i}\, e^{i\frac{m_i^2}{2E}L}\, U_{\beta i} \right] |\nu_\beta\rangle$$

$$P(\nu_\alpha \to \nu_\beta)(L) = |\langle \nu_\beta | \nu_\alpha(L)\rangle|^2 =$$

$$= \left|\langle \nu_\beta | \sum_i U^*_{\alpha i}\, e^{-i\frac{m_i}{2E}L}\, U_{\beta i} | \nu_\beta\rangle\right|^2 =$$

$$= \sum_i U^*_{\alpha i}\, e^{-i\frac{m_i^2}{2E}L}\, U_{\beta i} \sum_j U_{\alpha j}\, e^{i\frac{m_j^2}{2E}L}\, U^*_{\beta j} =$$

$$= \sum_{ij} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\, e^{i\frac{m_j^2 - m_i^2}{2E}L} =$$

$$= \sum_{ij} \underbrace{M_{\alpha\beta ij}}_{\left(\mathrm{Re}\, M_{\alpha\beta ij} + i\, \mathrm{Im}\, M_{\alpha\beta ij}\right)} \left(\underbrace{\cos\frac{\Delta m_{ij}^2}{2E}L}_{1 - 2\sin^2\frac{\Delta m_{ij}^2}{4E}L} + i \sin\frac{\Delta m_{ij}^2}{2E}L \right)$$

$$\frac{\Delta m_{ij}^2 L}{4E} \to \frac{\Delta m^2 c^4 L}{4E\hbar c} = \underline{\underline{1.27}}\, \Delta m^2(\mathrm{eV}^2)\frac{L(\mathrm{km})}{E(\mathrm{GeV})}$$

Probability that ν born of flavor α will dye as ν of flavor β

$$\underline{\underline{P(\nu_\alpha \to \nu_\beta)}} = \delta_{\alpha\beta} - 4 \sum_{i>j} \mathrm{Re}\left(M_{\alpha\beta ij}\right) \times$$

$$\times \sin^2\left[1.27 \frac{\Delta m_{ij}^2 L}{E}\right] \quad +$$

$$+ 2 \sum_{i>j} \mathrm{Im}\left(M_{\alpha\beta ij}\right) \sin\left[2.54 \frac{\Delta m_{ij}^2 L}{E}\right]$$

$$\alpha; \beta; i; j = 1; 2; 3$$

In the approximation of two neutrino oscillation

$$U = \begin{array}{c|cc} & \nu_1 & \nu_2 \\ \hline \nu_\alpha & \cos\theta & \sin\theta \\ \nu_\beta & -\sin\theta & \cos\theta \end{array}$$

$$\underset{\alpha \neq \beta}{P(\nu_\alpha \to \nu_\beta)} = \sin^2 2\theta \sin^2\left[1.27 \frac{\Delta m^2 L}{E}\right]$$

survival probability

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \left[1.27 \frac{\Delta m^2 L}{E}\right]$$

two parameters θ (or $\tan^2\theta$)

and Δm (or Δm^2)

solar $\theta_\odot$; $\Delta m^2_\odot$

atmospheric θ_{atm} ; Δm^2_{atm}

SNO

1. $\nu + d \to e^- + p + p$ — only ν_e (solar ν_e)
2. $\nu + d \to \nu + p + n$ — $\nu_e ; \nu_\mu ; \nu_\tau$ equal σ
3. $\nu + e^- \to \nu + e^-$ — $\frac{\sigma(\nu_{\mu;\tau}\, e \to \nu_{\mu\tau}\, e)}{\sigma(\nu_e\, e \to \nu_e\, e)} = \frac{1}{6.5}$

$$① \to \varphi(\nu_e)$$
$$② \to \varphi(\nu_e) + \varphi(\nu_{\mu;\tau})$$
$$③ \to \varphi(\nu_e) + \varphi(\nu_{\mu;\tau}) / 6.5.$$

$$\frac{\varphi(\nu_e)}{\varphi(\nu_e) + \underbrace{\varphi(\nu_{\mu;\tau})}_{\text{not zero}}} = 0.340 \pm 0.023 \, {}^{+0.029}_{-0.031}$$

more over

$\varphi(\nu_e) + \varphi(\nu_{\mu;\tau}) \simeq$ agrees with the predicted flux from the Sun based on Standard Solar Model.

Atmospherie ν

$\pi^+ \to \mu^+ + \enclose{circle}{\nu_\mu}$ $\mu^+ \to e^+ \nu_e \enclose{circle}{\bar\nu_\mu}$

$\pi^+ \to \mu^- \enclose{circle}{\bar\nu_\mu}$ $\mu^- \to e^- \bar\nu_e \enclose{circle}{\nu_\mu}$

$N(\nu_\mu) \cong 2\, N(\nu_e)$ actually 2.1

depends on energy

for $E \gtrsim 5$ GeV not all μ decay by sea level (we catch them with CMS), no ν_e produced, more ν_μ at higher E