Lecture #9

Quantum chromodynamics

a bit of history

Nuclei → n, p

$$A = \sum n + \sum p$$

$$Z = \sum p$$

isotops - same Z, different A

neutron existence was established in early 1930's:

→ 1930 - Bothe & Becker α on Be, B, Li's ⇒ penetrating radiation.

not bent by B ⇒ neutral

detect through secondary interactions

first suspected to be γ

→ 1932 - Joliot-Curie & Joliot n + paraffin ⇒ ejects energetic p

→ 1932 Chadwick showed kinem to be not consistent with γ

suggested $\underbrace{m(n) \cong m(p)}_{\text{not a symmetry}}$

"isospin" SU(2) before ~~iso~~ weak isospin

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad s_i = 1/2 \qquad \begin{matrix} \uparrow = p \\ \downarrow = n \end{matrix}$$

nuclear interactions do not distinguish between p & n ⇒ symmetry

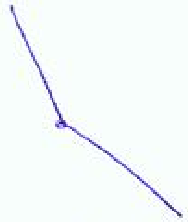
iso(spin up) (iso-spin down)

Powell, Lattes, Occhialini (Bristol)

1947 π was discovered in cosmic rays
"double meson" track in emulsion

$\pi \to \mu + \nu_\mu$

↑ called a meson back then



1948 Lattes, Gardner
Berkley cyclotron $\alpha \to$ C atoms
$\Rightarrow \pi$'s

π^0 seen $\pi^0 \to \gamma\gamma$ in cosmic rays,
then @ Berkley cyclotron

π^+ $\quad$ W^+
π^0 $\sim$ W_3 $\quad$ $S_I = 1$ $\quad$ $\begin{pmatrix} +1 \\ 0 \\ -1 \end{pmatrix}$
π^- $\quad$ W^-

isospin $\quad$ weak isospin
$SU(2)$ $\quad$ $SU(2)_L$

↓
very successful in prediction phenomenology
@ $E \gtrsim 1$ GeV

s-quark was "discovered" through K, Σ^-: $\quad$ $\pi^- p \to K^+ \Sigma^-$

strangeness introduced by Gell-Mann, Nishijima

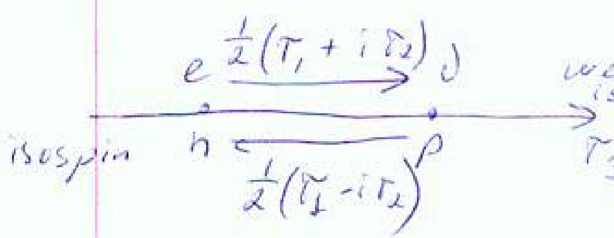
⇓ original idea of quarks

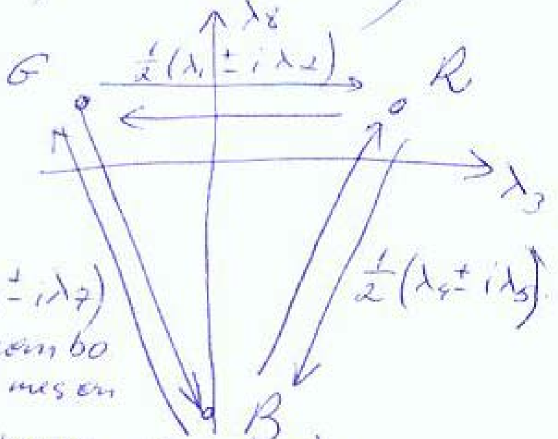
$SU(2)$ $\quad$ u, d
⇓
$SU(3)$ $\quad$ u, d, s $\quad$ - 1961
↓ flavor symmetry

flavor SU(3) is not the same as color, or QCD SU(3), but it prepared the mathematical apparatus

quarks have 3 colors
symmetry between 3 states
SU(3) → red ≠ antired, otherwise SO(3)

$$R = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad G = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$





colorless combo
$i\bar{i}$ - meson
or
ε_{ijk} ← baryon

SU(2)

$2 \otimes 2 = 3 \oplus 1$ ← identity

3: Pauli matrices

SU(3)

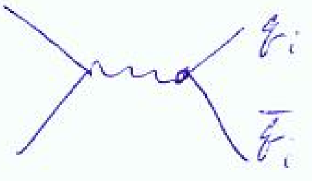
$3 \otimes 3 = 8 \oplus 1$ ← identity

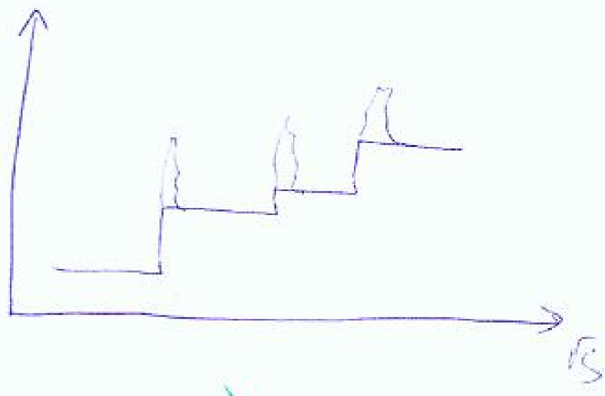
8: λ, or Gell-Mann matrices

Before we go further, let us examine what is the experimental evidence for 3 colors.

9-4

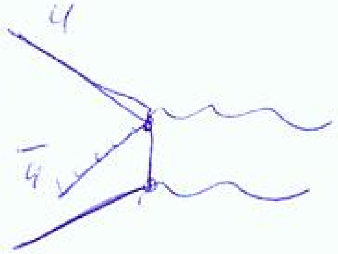
→ $R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$





$\sum_i Q_i^2 \cdot \textcircled{3}$

→ $\Gamma(\pi_0)$ 3 options

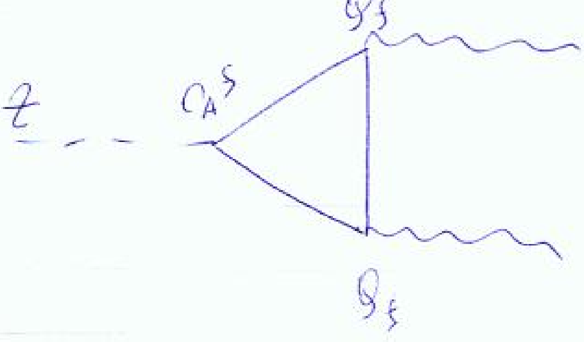


→ $Z \to u\bar{u}$ ~~from~~ from Br to hadrons
$d\bar{d}$
$s\bar{s}$
$c\bar{c}$
$l\bar{l}$

→ $W \to \left.\begin{matrix} u\bar{d} \\ c\bar{s} \end{matrix}\right) 2\times 3$ ← color
$l\nu$) 3

$\frac{1}{9} = Br(W \to e\nu)$
$\frac{6}{9} = Br(W \to \text{hadr})$

→ And we need it to be 3 in order to cancel:



$$\sum_{i=1}^{N} \left(\frac{1}{2}\underset{\nu}{(0)}^2 - \frac{1}{2}\underset{e}{(-1)}^2 + \frac{1}{2}N_c\underset{u}{\left(\frac{2}{3}\right)}^2 - \frac{1}{2}N_c\left(\frac{-1}{3}\right)^2\right) = 0$$

$N_c = 3$

So, $3\otimes 3 = 8 \oplus 1$

$2 \otimes 2 = 3 \oplus 1$

8 Gell-Mann matrices λ_i $i = 1-8$

similar to

3 Pauli matrices τ_i $i = 1, 2, 3$

$SU(2) - 2\times 2$	$SU(3) - 3\times 3$
$Tr(\cancel{\delta}\tau^i) = 0$	$Tr(\lambda^i) = 0$
$Tr(\tau^i \tau^j) = 2\delta^{ij}$	$Tr(\lambda^i \lambda^j) = 2\delta^{ij}$
$[\tau_i, \tau_j] = 2i\varepsilon_{ijk}\tau_k$	$[\lambda^i, \lambda^j] = 2if_{ijk}\lambda_k$
$i; j; k = 1; 2; 3$	$i\, j\, k = 1, 2 \ldots 8$
$\varepsilon_{ijk} = 0$ if $i=j$, or $j=k$, or $i=k$	$f_{ijk} = 0$ if $j=i$, or $j=k$, or $i=k$
$\varepsilon_{123} = 1$	$f_{123} = 1$
$\varepsilon_{ijk} = (-1)^n$	$f_{147} = f_{246} = f_{345} = f_{516} = f_{637} = \frac{1}{2}$
n number of permutations from 1 2 3	$f_{458} = f_{678} = \sqrt{3}/2$
$\varepsilon_{ijk} = -\varepsilon_{jik} = -\varepsilon_{kji} =$ $= -\varepsilon_{ikj}$	$f_{ijk} = -f_{jik} = -f_{kji} =$ $= -f_{ikj}$

$$\lambda_1 = \begin{array}{cc} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{matrix} \bar{R} \\ \bar{B} \\ \bar{G} \end{matrix} \\ \begin{matrix} R & B & G \end{matrix} & \end{array} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$L_{QED} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} i \gamma^\mu D_\mu \psi - m \bar{\psi} \psi$$

$$L_{EW} = -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{\chi}_L i \gamma^\mu D^L_\mu \chi_L +$$

$$+ \bar{e}_R i \gamma^\mu D^R_\mu e_R$$

$$D_\mu(QED) = \partial_\mu + ieQA_\mu$$

$$D^L_\mu(EW) = \partial_\mu + ig \frac{\tau_j}{2} W^j_\mu + ig' \frac{Y}{2} B_\mu \qquad j=1,2,3$$

$$D^R_\mu(EW) = \partial_\mu + ig' \frac{Y}{2} B_\mu$$

$$D_\mu(QCD) = \partial_\mu + i g_s \frac{\lambda_j}{2} B^s_{\mu j} \qquad j=1,..8$$

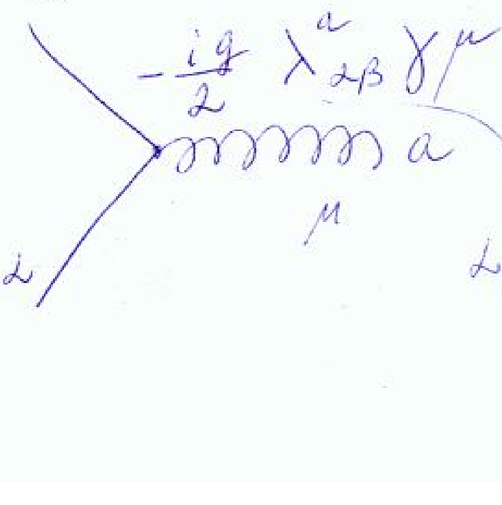
3 W's => 8 B's - gluons
color gauge fields

$$G^l_{\mu\nu} = \partial_\nu B^l_\mu - \partial_\mu B^l_\nu + \underline{g_s f^{jkl} B^j_\mu B^k_\nu}$$

$l = 1, \dots 8$

self interacting fields
non-abelian nature of
the group

$$L_{g-g} = -\frac{g_s}{2} B^a_\mu \bar{\psi} \gamma^\mu \lambda_a \psi$$



$$-\frac{ig}{2} \lambda^a_{\alpha\beta} \gamma^\mu$$

$\alpha, \beta = 1, 2, 3$

$a = 1 \dots 8.$

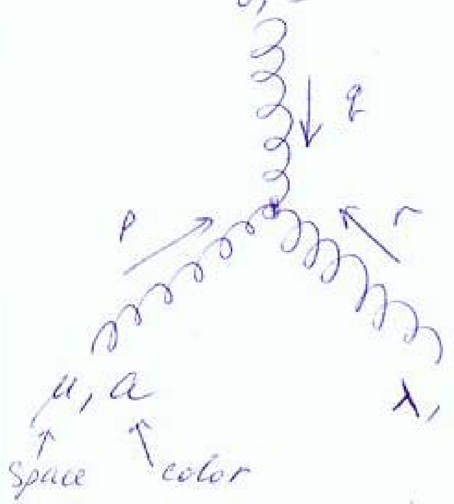
$\alpha, \beta \rightarrow 3 \times 3$ matrix
there are 8 of
them → a

$$L_{QCD} = -\frac{1}{4} G^{l}_{\mu\nu} G^{\mu\nu}_{l} + \bar{\psi} i \gamma^{\mu} \partial_{\mu} \psi -$$

$$- \bar{\psi} \frac{g_s}{2} B^{l}_{\mu} \cancel{\bar{\psi}} \gamma^{\mu} \lambda_l \psi - m \bar{\psi} \psi$$

note, no problem here, L-R symmetric

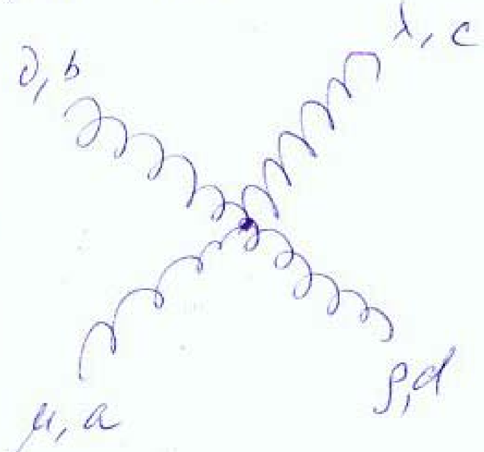
Also have vertices



$$-g_s f^{abc} [(p-q)_{\lambda} g_{\mu\nu} + (q-r)_{\mu} g_{\nu\lambda} + (r-p)_{\nu} g_{\lambda\mu}]$$

$$p+q+r=0$$

$$a, b, c = 1, \dots 8$$



$$-i g_s^2 [f^{abe} f^{cde} (g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}) + f^{ace} f^{dbe} (g_{\mu\rho} g_{\nu\lambda} - g_{\mu\nu} g_{\lambda\rho}) + f^{ade} f^{bce} (g_{\mu\nu} g_{\lambda\rho} - g_{\mu\lambda} g_{\nu\rho})]$$

$$a, b, c, d, e = 1, \dots 8$$

no problem with masses

$m(g) = 0$

$m \bar{\psi} \psi$ term is SU(3) invariant

$= \bar{\psi}_i \psi_i + \bar{\psi}_j \psi_j + \psi_k \psi_k \quad i, j, k = 1, 2, 3$

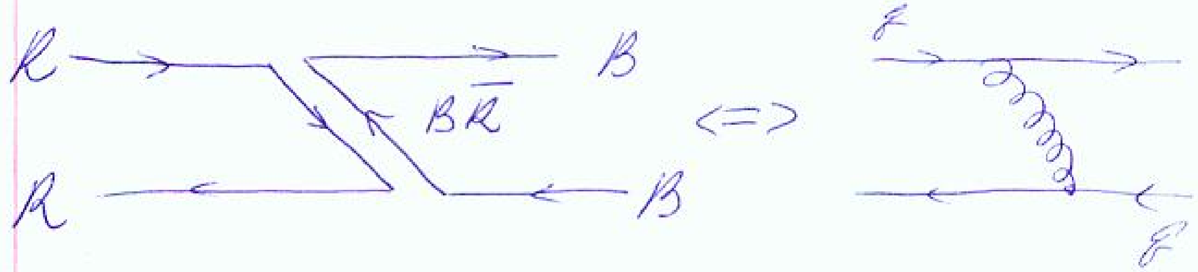
9 - 8

$3 \otimes 3 = 8 \oplus 1$

8 gluons - color octet can be presented as:

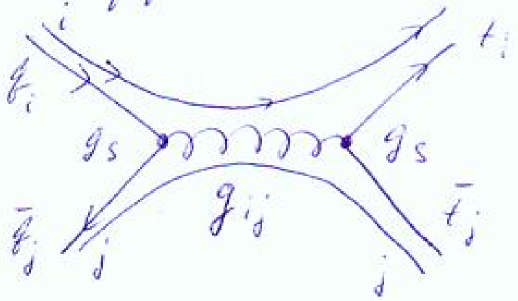
$R\bar{G}, R\bar{B}, G\bar{R}, G\bar{B}, B\bar{R}, B\bar{G}, \sqrt{\frac{1}{2}}(R\bar{R}-G\bar{G}),$
$\sqrt{\frac{1}{6}}(R\bar{R}+G\bar{G}-2B\bar{B})$

$1 = \sqrt{\frac{1}{3}}(R\bar{R}+G\bar{G}+B\bar{B})$ - color singlet does not ~~de~~ mediate between color charges

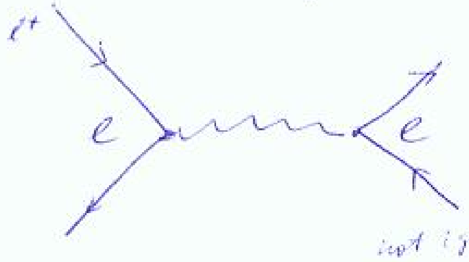


For example

$q\bar{q} \to t\bar{t}$:



$e^+e^- \to \mu^+\mu^-$



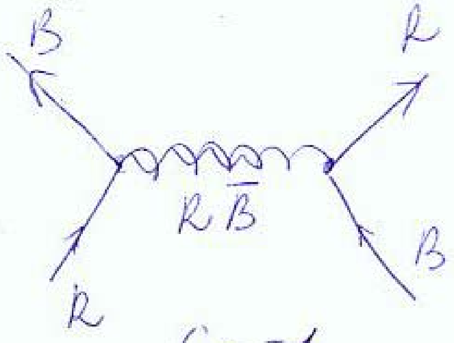
$i = 1, 2, 3$ } averaging over ~~spins~~ color in
$j = 1, 2, 3$ } initial state 1/9

not ignoring mass

$$\sigma_{QED} = \frac{4\pi\alpha^2}{3S}\frac{\beta(3-\beta^2)}{2}$$

$$\sigma = \frac{4\pi\alpha_s^2}{3\hat{S}}\frac{\beta(3-\beta^2)}{2}\cdot\boxed{\frac{2}{9}}$$

~~Sum over final = 3~~

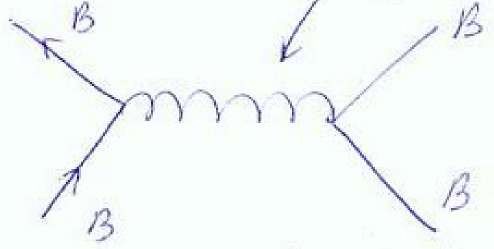


6 × 1 - different color combinations

$C_F = 1$

$\frac{1}{\sqrt{6}}(R\bar{R} + G\bar{G} - 2B\bar{B})$

produce in color singlet state



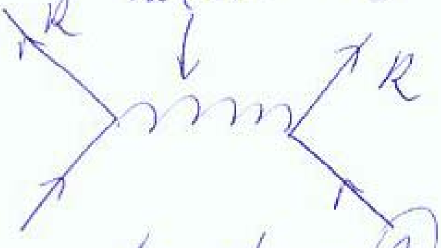
$3 \times \frac{2}{3}$ - same color combinations.

$$C_F = \left(-\frac{2}{\sqrt{6}}\right) \cdot \left(-\frac{2}{\sqrt{6}}\right) = \boxed{\frac{2}{3}}$$

or $\frac{1}{\sqrt{6}}(R\bar{R} + G\bar{G} + 2B\bar{B})$

+

$\frac{1}{\sqrt{2}}(R\bar{R} - G\bar{G})$



$$\frac{1}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \boxed{\frac{2}{3}}$$

$$\underbrace{\frac{2}{3}}_{\text{"fraction of gluon"}} \times \underbrace{3}_{\text{sum over final}} \times \underbrace{\frac{1}{3} \times \frac{1}{3}}_{\text{average over initial}} = \frac{2}{9}$$

9-10

Main problem with strong interaction is that it is strong

$\alpha_s \cong 0.2 \longleftrightarrow \alpha_{QED} = 1/137$

perturbation theory bare works

different approach - Lattice QCD

$S = T\exp\left(-i\int L_I(x)\,d^4x\right)$

do not expand, but evaluate numerically

$M_{if} = \langle i | S | f \rangle$.