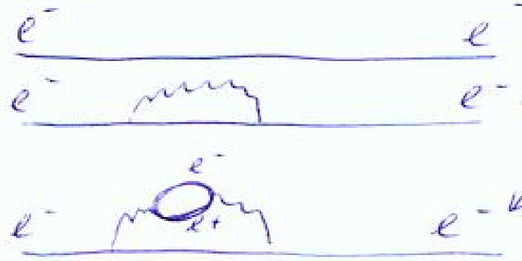
Lecture # 10
QCD, structure functions, fragmentation.

QED:



electron

$\Downarrow$

charge is a function of distance at which it is probed $\Rightarrow$ energy scale - q

$$\alpha_{em}(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{1}{3\pi}\alpha(\mu^2)\ln\frac{q^2}{\mu^2}}$$

μ^2 - ref scale

$\alpha_{em} = \frac{1}{137}$ @ $\mu = 1$ MeV

$\alpha_{em} = 1/129$ @ $q \cong 100$ GeV (M_Z)

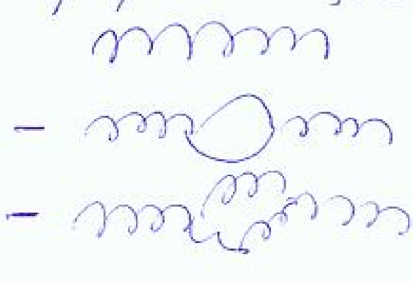
$\alpha_{em} \nearrow$ with q

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{7}{4\pi}\alpha_s(\mu^2)\ln\frac{q^2}{\mu^2}}$$

$\alpha_s \searrow$ with q

asymptotic freedom

Politzer, Gross, Wilczek 1973 - Nobel Prize in 2006



$$\frac{\alpha_s(\mu^2)}{4\pi}\left(-\frac{2}{3}n_f - 5 + 16\right)$$

$n_f = 3$

10-2

asymptotic freedom — The strength of strong interaction goes down for high momentum transfer,

confinement — goes up for low momentum transfer.

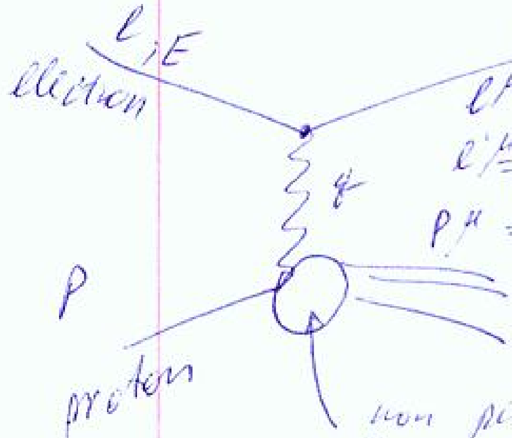
Hard to predict QCD phenomena @ low Q^2 — parametrized models, fit to exp results

structure functions

fragmentation.

form factors

(DIS) = Deep Inelastic Scattering and the Parton Model



l, l' - 4 momentum of e

$q = l - l'$

$l^\mu = (E, 0, 0, E)$

$l'^\mu = (E', E'\sin\theta, 0, E'\cos\theta)$

$P^\mu = (M, 0, 0, 0)$

$Q^2 = -q^2 = -(l-l')^2 = 4EE'\sin^2\theta/2$

$\nu = \frac{q\cdot P}{m} = E - E'$

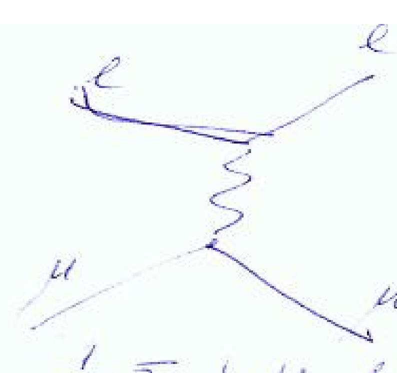
electron beam on target, or ep colliding beams (HERA)

$x = \frac{Q^2}{2q\cdot P} = \frac{Q^2}{2m\nu}$

$y = \frac{\nu}{E} = 1 - \frac{E'}{E}$

dimensionless variables

10-3 recall $e^+e^- \to \mu^+\mu^-$
or $e\mu$ scattering
(same matrix element)



$$\frac{1}{2}\sum_{\text{spins}} |M|^2 = \frac{e^4}{q^4} L_e^{\mu\nu} L^{\mu}_{\mu\nu}$$

for ep scattering - similar structure

same

$$\frac{1}{2}\sum_{\text{spins}} |M|^2 = \frac{e^4}{q^4} L_e^{\mu\nu} W_{\mu\nu}$$

contains our ignorance about low Q^2 QCD

$$L^{\mu\nu} = 2\left(l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu}\, l\cdot l'\right)$$

$$W^{\mu\nu} = -W_1\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right] + \frac{W_2}{m^2}\left[p_\mu - \frac{p\cdot q}{q^2} q_\mu\right] \times \left[p_\nu - \frac{p\cdot q}{q^2}\, q_\nu\right]$$

must have a Lorentz invariant form
must be a tensor
current conservation $\Rightarrow q^\mu W_{\mu\nu} = 0 = q^\nu W_{\mu\nu}$
W_1, W_2 - form factors
Lorentz invariant scalars
depend on x; Q^2

$$\left.\begin{matrix} W_1(x, Q^2) \\ W_2(x, Q^2) \end{matrix}\right\} \text{ - measured experimentally}$$

$$\frac{d\sigma}{dE'\,d\Omega'} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2}\Big[2W_1(x, Q^2)\sin^2\theta/2 + W_2(x, Q^2)\cos^2\theta/2\Big]$$

electron's final energy

electron's scat angle

Structure functions

other way of representing $d\sigma$ is in terms of x & y

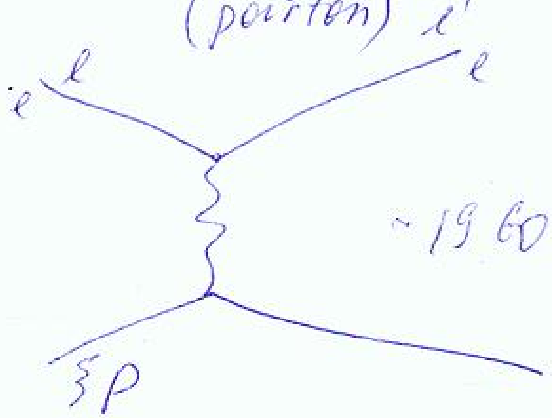
$\left(\begin{array}{l} F_1 = M W_1 \\ F_2 = \nu W_2 \end{array} \right\|$ $F_{1,2}$ depend on x & Q^2, but the dependence on Q^2 is very weak $\underset{Q^2 \gg m_p^2}{F_{1,2}(x)}$ – Bjorken scaling

$$\frac{d\sigma}{dx\,dy} = \frac{4\pi\alpha^2}{Q^2 x y}\left[\frac{1+(1-y)^2}{2} 2x F_1 + (1-y)\left[F_2 - 2xF_1\right] - \frac{M^2}{Q^2} x^2 y^2 F_2\right]$$

neglecting proton mass we get and changing from y to Q^2

$$\frac{d\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left(\left(1 + (1-y^2)\right)F_1 + \frac{1-y}{x}\left[F_2 - 2x F_1\right]\right)$$

comparing this to ~~an~~ expression of e scattering off a point-like object, we conclude that



$$\xi = \frac{p_{parton}}{p_{proton}} = x$$

~1960s – Bjorken scaling suggest point-like constituents of proton = quarks gluons

By scattering "point-like" object like electron or neutrino off proton we can probe (and parametrize) its substructure.
Bjorken scaling tells us that these parametrizations can be used in other ~~scaling~~ scattering experiments

$ep \Rightarrow$ structure $\Rightarrow$ $\bar{p}p, pp$
(hera) functions predict x sections

$$p = \underbrace{uud}_{\text{valence}} + \underbrace{\Sigma q\bar{q}}_{\text{quark sea}} + \underbrace{\Sigma g's}_{\text{gluon sea}}$$

$u(x)$ - probability to find u-type quark with a fraction of total momentum x inside proton.

$u(x) = u_v(x) + \xi_u(x)$ $\qquad \bar{u}(x) = \xi_u(x)$
$d(x) = d_v(x) + \xi_d(x)$ $\qquad \bar{d}(x) = \xi_d(x)$
$s(x) = \xi_s(x)$ $\qquad \bar{s}(x) = \xi_s(x)$
$c(x) = \xi_c(x)$ $\qquad \bar{c}(x) = \xi_c(x)$

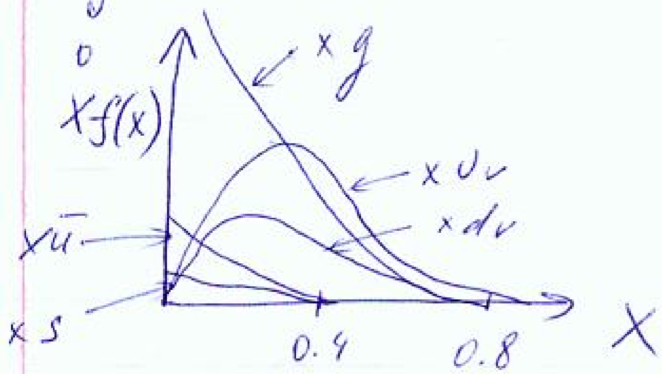
$$\int_0^1 u_v(x)dx = \underbrace{2}_{2u \text{ quarks}} \qquad \int_0^1 d_v(x)dx = \underbrace{1}_{1d \text{ quark}}$$

$q_v(x) \sim x^{-1/2}$ $\qquad \xi(x) \sim x^{-1}$

$$\sum_{q,\bar{q}} \int_0^1 dx\, x\, q(x) \cong 0.5$$

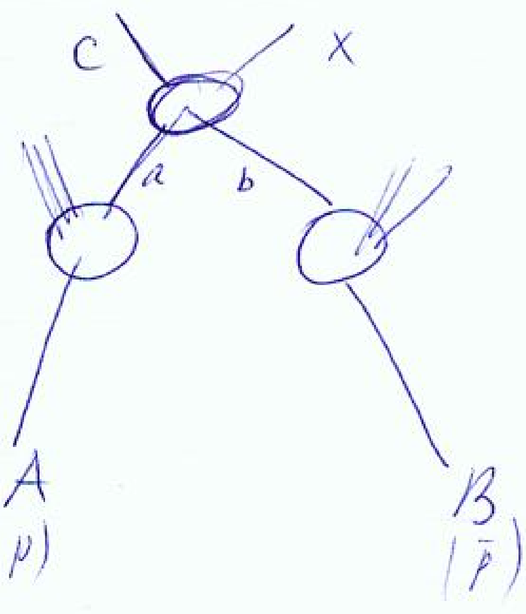
only 50% of proton momentum is carried by quark! The rest is gluons

$$\int_0^1 dx\, x\, G(x) \cong 0.5$$



libraries of structure functions
MCNLO,
CTEQ
MRS ~~00~~

$\Longrightarrow$ hadron-hadron collisions



σ: $A + B \to c\, X$

$\hat{\sigma}$; $a + b \to c\, X$

$$\sigma(AB \to cX) = \sum_{a,b} C_{ab} \int dx_a\, dx_b\, f_{a/A}(x_a) \cdot f_{b/B}(x_b)\, \hat{\sigma}(ab \to cX)$$

C_{ab} - color averaging

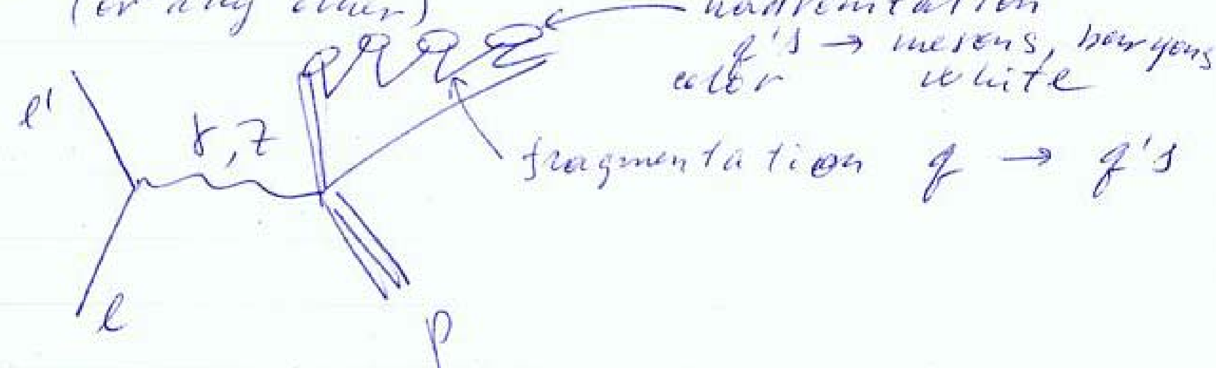
$$C_{qq} = C_{q\bar{q}} = \frac{1}{9}$$

$$C_{qg} = \frac{1}{24} \quad ; \quad C_{gg} = \frac{1}{64}$$

$\hat{s} = x_a x_b s$ — center of mass energy squared in ab frame of ref.

That was "pre model" treatment

⟹ What happens to quarks produced in $p\bar{p}$ collisions? (or any other)



⟹ Feynman scaling

$$z = \frac{p_{hL}}{p_{kL}}$$

h — hadron h; L - longitudinal; k — parton k

Probability to find hadron h in the range z to $z+dz$ $D_k^h(z)\,dz$ is called k-to-h fragmentation function. Depends only on z — Feynman scaling

$$\frac{d\sigma}{dE_h}\left(AB \to hX\right) = \sum_k \frac{d\sigma}{dE_k}\left(AB \to kX\right) \times$$

$$\times\, D_k^h\left(E_h/E_k\right)\frac{dE_k}{E_k}$$

since $dz = E_k^{-1}\, dE_h$

a simple parametrization

$$D(z) = f\left((1-z)^n\right)/z$$

Peterson parametrization of the fragmentation functions
Bowler,
others

Generally the heavier the ~~quark~~ meson, the larger fraction of the momentum of initial parton it will get.