Warm up discussion problem:

Counting problems are a traditional issue in computer science because the number of elements in a data set directly relate to the time required for an algorithm to examine them.

So, a standard chess board is a square 8x8 checkered grid. How many squares, of all sizes, are there in this structure. Can you come up with a mathematical expression, or sequence, to describe the total number of squares? Given such an expression or sequence, could you write a function that takes the length of the side of the board and returns the total number of squares? This is a good example of something that may be hard to arrive at a closed form mathematical solution for, but is easy to write program to solve.

Workshop:
Simulation and Design

1. Craps is a dice game played at many casinos. A player rolls a pair of normal six-sided dice. If the initial roll is 2, 3, or 12, the player loses. If the roll is 7 or 11, the player wins. Any other initial roll causes the player to roll for point. That is, the player keeps rolling the dice until either rolling a 7 or re-rolling the value of the initial roll. If the player re-rolls the initial value before rolling a 7, it’s a win. Rolling a 7 first is a loss.
Write a program to simulate multiple games of craps and estimate the probability that the player wins. For example, if the player wins 249 out of 500 games, then the estimated probability of winning is \( \frac{249}{500} = 0.498 \).

2. Blackjack (twenty-one) is a casino game played with cards. The goal of the game is to draw cards that total as close to 21 points as possible without going over. All face cards count as 10 points, aces count as 1 or 11, and all other cards count their numeric value. The game is played against a dealer. The player tries to get closer to 21 (without going over) than the dealer. If the dealer busts (goes over 21), the player automatically wins (provided the player had not already busted). The dealer must always take cards according to a fixed set of rules. The dealer takes cards until he or she achieves a total of at least 17. If the dealer’s hand contains an ace, it will be counted as 11 when that results in a total between 17 and 21 inclusive; otherwise, the ace is counted as 1.

Write a program that simulates multiple games of blackjack and estimates the probability that the dealer will bust. Hints: treat the deck of cards as infinite (casinos use a “shoe” containing many decks). You do not need to keep track of the cards in the hand, just the total so far (treating an ace as 1) and a boolean variable that tells whether or not the hand contains an ace. A hand containing an ace should have 10 points added to the total exactly when doing so would produce a stopping total (something between 17 and 21 inclusive.)

3. A blackjack dealer always starts with one card showing. It would be useful for a player to know the dealer’s bust probability (see previous problem) for each possible starting value. Write a simulation program that runs multiple hands of blackjack for each possible starting value (Ace, 2, 3, . . . 10) and estimates the probability that the dealer busts for each starting value.