Problem 24-29

\[ C_1 = C_2 = C_3 = C_4 = C \]

\[ C_{12} = (\frac{1}{C_1} + \frac{1}{C_2})^{-1} = \frac{C}{2} \]

\[ C_{123} = C_3 + C_{12} = \frac{3}{2}C \]

\[ C_{1234} = \left(\frac{\frac{3}{2}C}{C} + \frac{1}{C}\right)^{-1} = \frac{3}{5}C \]

\[ Q_{\text{tot}} = C_{1234}V = \frac{3}{5}CV \]

\[ Q_4 = \frac{3}{5}CV = CV_4 \Rightarrow V_4 = \frac{3}{5}V \]

\[ Q_{\text{tot}} = \frac{3}{5}CV = \frac{3}{5}CV_{123} \Rightarrow V_{123} = \frac{3}{5}V \]

For capacitors in parallel we have constant voltage so:

\[ V_{12} = V_3 = \frac{3}{5}V \Rightarrow Q_3 = \frac{2}{5}CV \]

\[ V_{12} = \frac{3}{5}V \Rightarrow Q_{12} = \frac{5}{2}V_{12} \Rightarrow C_1 \text{ and } C_2 \text{ are in series so charge is same} \]

\[ Q_{12} = \frac{5}{2}V = Q_1 = Q_2 \]

Since \( C_1 = C_2 \) then we sum it up:

\[ Q_1 = C_1V_1 = \frac{5}{6}CV \]

\[ Q_3 = \frac{2}{5}CV \]

\[ Q_4 = \frac{3}{5}CV \]

\[ Q_2 = \frac{1}{5}CV \]
Problem 24-40

* What is $C_{eq}$ in terms of capacitances of each?

Let's look at voltages first:

For any path through the circuit that starts at a and ends at b, the total potential difference is equal to $V$.

So:

1. $V = V_a + V_1 \quad (1)$
2. $V = V_5 + V_4 \quad (2)$
3. $V = V_3 + V_3 + V_4 \quad (3)$

Now charge conservation: Assume there is a total charge $Q$ that starts at point a. Upon reaching the first junction $Q$ splits into two values $Q_a$ & $Q_5$ (where usually $Q_a + Q_5$) continuity on this:

$$Q = Q_a + Q_5 \implies \text{Put in terms of } C_i V$$

$$C_{eq}V = C_a V_a + C_5 V_5 \quad (4)$$
$$C_{eq}V = C_1 V_1 + C_4 V_4 \quad (5)$$
$$C_2 V_a = C_1 V_1 + C_3 V_3 \quad (6)$$

We have six equations with $V_1 \rightarrow V_5$ as unknowns so we plug equations into each other:

Plug (3) into (6) \implies $C_a V_a = C_1 (V - V_3) + C_3 V_3$

$$V_a (C_a - C_1) = C_3 V_3 + C_1 V$$
Plug (2) into (4) \[ \Rightarrow \] \[ C_{eq} V = C_2 V_2 + C_5 V - C_5 V_4 \]

Going to solve for \( C_{eq} \):

Plug (1) into (5) \[ \Rightarrow \] \[ C_{eq} V = C_1 (V - V_2) + C_4 V_4 \]

We use (3) to rewrite (6) \[ \Rightarrow \] \[ C_3 V_2 = C_1 V_1 + C_3 (V - V_3 - V_4) \]

We then have:

\[ \Rightarrow \left( \left( \frac{1}{C_1} + \frac{1}{C_2 + C_3} \right) \frac{1}{C_3} V_2 + \frac{1}{C_3} V_4 \right) = \left( \frac{1}{C_1} + \frac{1}{C_3} \right) V \]

\[ C_{eq} V = \frac{C_2 V_2 + C_5 V - C_3 V_4}{C_1 + C_3} \]

\[ C_{eq} V = C_4 \left[ \left( \frac{C_2 V_2 + C_5 V - C_3 V_4}{C_1 + C_3} \right) + C_5 C_i V - C_5 V_2 \right] \]

\[ C_5 C_{eq} V = C_4 \left[ \left( \frac{C_2 V_2 + C_5 V - C_3 V_4}{C_1 + C_3} \right) + C_5 C_i V - C_5 V_2 \right] \]

\[ C_5 \left( C_1 + C_3 \right) V = C_4 \left( C_2 + C_5 C_i \right) \]

We see that:

\[ V_4 = \frac{1}{C_{eq}} \left( C_2 V_2 + C_5 V - C_3 V_4 \right) \]

Then:

\[ C_5 C_{eq} V = C_4 \left[ \left( \frac{C_2 V_2 + C_5 V - C_3 V_4}{C_1 + C_3} \right) + C_5 C_i V - C_5 V_2 \right] \]

\[ C_5 \left( C_1 + C_3 \right) V = C_4 \left( C_2 + C_5 C_i \right) \]

Group by common voltage:

\[ \left( C_5 C_{eq} + C_4 C_5 - C_4 C_5 - C_5 C_i \right) V = \left( C_4 C_2 - C_5 C_i \right) V_2 \]

\[ C_5 \left( C_1 + C_3 \right) + C_3 \left( C_{eq} - C_3 C_5 \right) V = C_5 \left( C_1 + C_2 + C_3 \right) + C_3 C_2 \]

Set equal to each other:

\[ \left( C_5 C_{eq} + C_4 C_5 - C_4 C_5 - C_5 C_i \right) = \frac{C_4 C_2 - C_5 C_i}{C_5 \left( C_1 + C_2 + C_3 \right) + C_3 C_2} \]

Solve for \( C_{eq} \):
\[ C_{65} = C_1 C_3 C_3 + C_1 C_2 C_4 + C_1 C_2 C_5 + C_1 C_3 C_5 + C_2 C_3 C_5 + C_3 C_4 C_5 + C_3 C_4 C_5 + C_3 C_5 + C_3 C_5 \]
Problem 24-56

For mica $\varepsilon_r = 7$  Area of plates $= (0.08)^2 \text{m}^2$

d$= 1.3 \times 10^{-3} \text{m}$

\[
\text{since } C = \varepsilon_0 C_0
\]

\[
Q = 420 \times 10^{-6} \text{C}
\]

\[
U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2}{\varepsilon_0 C_0} = \frac{1}{2} \frac{Q^2}{\varepsilon_0 \varepsilon_0 A}
\]

\[
U = \frac{1}{2} \frac{Q^2 d}{\varepsilon_0 \varepsilon_0 A} = \frac{(420 \times 10^{-6} \text{C})^2 (1.3 \times 10^{-3} \text{m})}{2 \times (7) \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right) \left(4.4 \times 10^{-4} \text{m}^2\right)}
\]

\[
U = 289.2 \text{ J}
\]
Problem 24-60

We can treat this system as two capacitors in series b/c the voltage across the entire capacitor is not going to const. for $E_{D2}$ and $E_{D3}$ (see problem 24-59 for comparison).

$E_{D2}$ region has capacitance $C_1$,
$E_{D3}$ region has capacitance $C_2$.

The capacitance eq. is then: $C = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}$

Since $C_1 = \frac{E_{D2} A e_0}{d/2}$, $C_2 = \frac{E_{D3} A e_0}{d/2}$

$$C = \left[ \left( \frac{d/2}{A e_0} \left( \frac{1}{E_{D2}} + \frac{1}{E_{D3}} \right) \right)^{-1} = \frac{2 A e_0}{d} \left( \frac{E_{D2} E_{D3}}{E_{D2} + E_{D3}} \right) \right]$$
Problem 24-90

Let's make the four cylinders into groups of two.

Call the one on left $C_1$ and on right $C_2$.

We know the capacitance on cylinders from class. So:

\[
C_1 = \frac{2\pi e_0 l}{\ln(R_a/R_b)} \quad \text{and} \quad C_2 = \frac{2\pi e_0 l}{\ln(R_c/R_d)}
\]

The metal strips indicated in problem mean we can treat $C_1$ & $C_2$ as capacitors in series. Hence:

\[
C_{eq} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = 2\pi e_0 l \left( \frac{\ln(R_a/R_b)\ln(R_c/R_d)}{\ln(R_a/R_b) + \ln(R_c/R_d)} \right)^{-1}
\]

\[
C_{eq} = \frac{2\pi e_0 l}{\ln(R_a R_c / R_b R_d)}
\]

The capacitance per unit length is then:

\[
\frac{C}{l} = \frac{2\pi e_0}{\ln \left( R_a R_c / R_b R_d \right)}
\]
Problem 25-27

we know \( R = \frac{P}{A} \)

since we have a sphere (not a straight line) we need to integrate!

\[ dR = \frac{P \, dl}{4 \pi r^2} \quad r_i < r < r_2 \quad ; \quad P = \frac{1}{6} \quad ; \quad dl = dr \]

\[ \Rightarrow R = \frac{1}{4 \pi 6} \int_{r_i}^{r_2} \frac{dr}{r^2} \]

\[ R = \frac{1}{4 \pi 6} \left( -\frac{1}{r} \right)_{r_i}^{r_2} = \frac{1}{4 \pi 6} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]
Problem 25-30


As done in 25-27 \( R = \frac{L}{2\pi e} \left( \ln \left( \frac{r_2}{r_1} \right) \right) = \frac{L}{2\pi e} \ln \left( \frac{r_2}{r_1} \right) \)

\[ R = \frac{L}{2\pi e} \left( \ln \left( \frac{1.8 \times 10^{-3} m}{1 \times 10^{-3} m} \right) \right) = \frac{L}{2\pi e} \ln \left( \frac{1.8 \times 10^{-3} m}{1 \times 10^{-3} m} \right) \]

b) \( \rho_{\text{copper}} = 1.5 \times 10^{-5} \text{ R} / \text{m} \)

\( L = 2.4 \times 10^{-3} \text{ m} \)

\( r_1 = 1 \times 10^{-3} \text{ m}, \ r_2 = 1.8 \times 10^{-3} \text{ m} \)

Plug in to get:

\[ R = 5.8 \times 10^{-4} \text{ R} \]

c) Here \( A = \pi (r_2^2 - r_1^2) \) \( \Rightarrow \) no need to integrate.

\[ R = \frac{\frac{L}{\pi (r_2^2 - r_1^2)}}{0.51 \text{ R}} \]
Problem 25-40

\[ P = 25 \text{ W} \quad \text{cost: } \frac{\$0.095}{\text{kWh}} \]

Night & day for 1 year

\[
\text{Total cost} = (25 \times 10^{-3} \text{ kW}) \left( \frac{\$0.095}{\text{kWh}} \times \frac{365 \text{ days}}{1 \text{ day}} \right) \left( \frac{24 \text{ h}}{1 \text{ day}} \right) = \$81
\]

\[ \Rightarrow \text{pretty cheap!} \]
Problem 25-56

\[ V_{\text{rms}} = \left( \frac{1}{T} \int_0^T V^2 \, dt \right)^{\frac{1}{2}} \]

a) \( V(t) = V_0 \sin(\omega t / \sqrt{T}) \)

\[ V_{\text{rms}} = \left[ \frac{V_0^2}{T} \int_0^T \sin^2(\omega t / \sqrt{T}) \, dt \right]^{\frac{1}{2}} = \left[ \frac{V_0^2}{T} \left( \frac{T}{2\pi} \right) \right]^{\frac{1}{2}} \int_0^{\frac{3\pi}{2}} \sin^2(u) \, du \]

Note:
\[
\begin{align*}
\int_0^L \sin^2(u) \, du &= \frac{L}{2} \\
\int_0^L \cos^2(u) \, du &= \frac{L}{2}
\end{align*}
\]

Try it!

So:

\[ V_{\text{rms}} = \left[ \frac{V_0^2}{\omega \sqrt{2}} \right]^{\frac{1}{2}} = \frac{V_0}{\sqrt{2}} \]

b) \( V(t) = \left\{ \begin{array}{ll} V_0 & 0 \leq t \leq T/2 \\ \ 0 & T/2 \leq t \leq T \end{array} \right. \)

\[ V_{\text{rms}} = \left[ \frac{V_0^2}{T} \int_0^{T/2} \, dt \right]^{\frac{1}{2}} = V_0 \left[ \frac{1}{T} (T/2) \right]^{\frac{1}{2}} \]

\[ V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \]
Problem 25-91

This is very much like 25-30 part c.

\[ A = \pi \left( (0.035)^2 - (0.015)^2 \right) \text{m}^2 \]

\[ l = 0.1 \text{m} \]

\[ \rho = 1.68 \times 10^8 \text{ S.m} \]

Plug in

\[ R = \frac{\rho l}{A} = \frac{1.34 \times 10^{-9}}{\pi} \text{ S} \]
Problem 26-19

* all Resistors have resistance R.

Before we go on let’s redraw this into something more familiar:

Let’s number them

\[ R_1, R_2 \text{ are in series } \Rightarrow R_{12} = 2R \]

\[ R_{12} \text{ and } R_3 \text{ are in parallel } \Rightarrow R_{123} = \left( \frac{1}{2R} + \frac{1}{R} \right)^{-1} = \frac{2}{3}R \]

\[ R_{123} \text{ and } R_4 \text{ are in series } \Rightarrow R_{1234} = \frac{5}{3}R \]

\[ R_{1234} \text{ and } R_5 \text{ are in parallel } \Rightarrow R_{12345} = \left( \frac{3}{5R} + \frac{1}{R} \right)^{-1} = \frac{5}{8}R \]

\[ R_{12345} \text{ and } R_6 \text{ are in series } \Rightarrow R_{123456} = \frac{13}{8}R \]

So

\[ R_{eq} = \frac{13}{8}R \]
Problem 26-31

From the diagram we can write:

\[ I_3 = I_1 + I_2 \]

Using Kirchhoff's Rules we then have:

Loop 1 ⇒ around upper half:

\[
45 \text{ V} = I_3 + 47I_3 + 34I_1
\]

\[ I_3 = \frac{45 - 34I_1}{48} \]

Loop 2 ⇒ around outside of circuit:

\[
75 \text{ V} = I_2 + 18I_2 - 34I_1
\]

\[ I_2 = \frac{75 + 34I_1}{19} \]

Then \[ I_1 + I_2 - I_3 = 0 \] Plugging in from above:

\[
I_1 + \frac{75 + 34I_1}{19} - \frac{45}{48} + \frac{34I_1}{48} = 0
\]

\[ I_1 (1 + \frac{34}{19} + \frac{34}{48}) = - \left( \frac{75 - 45}{19} \right) \]

\[ I_1 = -6.861 \text{ A} \]

\[ I_2 = 2.41 \text{ A} \]

\[ I_3 = 1.55 \text{ A} \]

Then, to find voltage across a & d we can treat between a & d as a "battery:"

\[ V' = I_3 \times \frac{34R}{\text{Wt}} \]

\[ V' = (-0.861)(34) = 29.27 \text{ Volts} \]
b) The terminal voltage is given as the voltage across:

\[ V_{\text{terminal}} = \varepsilon_2 - I_3 r = 45 - 1.55 = 43V \]

\[ V_{\text{terminal}} = \varepsilon_1 - I_2 r = 75 - 2.41 = 73V \]
Problem 26-44

\[ R = 15 \times 10^3 \, \Omega \]
\[ \tau = 24 \times 10^6 \, s \]
\[ E = 24 \, V \]

a) \[ C = \frac{R \tau}{R + \tau} = \frac{24 \times 10^6}{15 \times 10^3} = 1.6 \times 10^9 \, F \]

b) For an RC circuit: \[ V_c = E \left( 1 - e^{-t/\tau} \right) \]
\[ \text{If } V_R = 16 \, V \Rightarrow 24 = 16 + V_c \Rightarrow V_c = 8 \, V \]
\[ \Rightarrow -\frac{V_c}{E} + 1 = e^{-t/\tau} \]
\[ \Rightarrow \ln \left( 1 - \frac{V_c}{E} \right) = -\frac{t}{\tau} \]
\[ t = -\tau \ln \left( 1 - \frac{V_c}{E} \right) = -24 \times 10^6 \, s \ln \left( 1 - \frac{8}{24} \right) = 9.73 \times 10^6 \, s \]
Problem 26-49

\[ \begin{align*}
\text{we know that } I_1 &= I_2 + I_3 \\
\end{align*} \]

a) at \( t=0 \) the capacitor is uncharged, so there is no voltage across it \( (V_c = 0) \) we can redraw it as:

\[ \begin{align*}
\Rightarrow I_1 &= \frac{E}{R_{eq}} = \frac{E}{\frac{3}{2}R} \\
I_2 &= I_3 = \frac{3}{8} I_1 = \frac{E}{3R} \\
\end{align*} \]

b) at \( t=\infty \) the capacitor is charged to \( E \Rightarrow \) hence \( I_3 = 0 \) b/c current out of capacitor goes against \( I_3 \) so we can write:

\[ \begin{align*}
\Rightarrow I_1 &= \frac{E}{R_{eq}} = \frac{E}{2R} \\
I_3 &= 0 \\
\end{align*} \]

c) at \( t=\infty \) since \( I_3 = 0 \) no current will flow through the resistor. So we consider only the \( I_2 \) current.

\[ V_c = I_2 R = \frac{E}{2R} R = \frac{1}{2} E \]