Lecture II  Maxwell's Equations & Electromagnetic Waves

Charging E-field produce Magnetic fields

Before we start let's mention Ampere's Law.

\[ \oint \mathbf{B} \cdot d\mathbf{e} = \mu_0 I \text{enc} \]

I starts \( \oint \mathbf{B} \cdot d\mathbf{e} \) goes around in a circle $\mathcal{C}$ we said that $I$ is the current that passes through the plane (on area) of the loop.

Now we are allowed to extend this area in the loop if we want (as long as we keep the circle the same).

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Ok so now imagine we have that we have the following system $\mathcal{C}$ we put an ampere's loop as such with the area extended out.

\[ I \text{enc} = \mathcal{C} \text{here b/c no current passes through area of loop!} \]
But there is most certainly a B field outside the wire.

We complete Ampere's law by noting our statement above that a changing electric field creates a magnetic field. If an electric field changes, it creates a displacement current (sort of like a changing B creates an induced current)

The electric flux through a given area is \( \Phi_E = \mathbf{E} \cdot \mathbf{A} \)

The displacement current is then: \( I_d = \varepsilon_0 \frac{d\Phi_E}{dt} \)

Ampere's law becomes:

\[
\int \mathbf{B} \cdot d\mathbf{a} = \mu_0 (I_{\text{enc}} + I_d) = \mu_0 I_{\text{enc}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}
\]

In places where there is no current flowing, we can still have a B field create one as long as there is some E field.

**Flux \rightarrow Magnetic**

We know \( \Phi_B = \int \mathbf{B} \cdot d\mathbf{a} \)

And since magnetic lines are always closed, \( \Phi_B = \int \mathbf{B} \cdot d\mathbf{a} = 0 \) always \Rightarrow no existence of magnetic monopole.
We can sum up everything we have done so far in this course by the following equations:

\[ \oint E \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0} \quad \text{Gauss' Law (Electric)} \]

\[ \oint B \cdot d\mathbf{A} = 0 \quad \text{No magnetic monopoles} \]

\[ \oint E \cdot d\mathbf{r} = -\frac{d\Phi_B}{dt} \quad \Rightarrow \quad \text{Faraday's Law} \]

\[ \oint B \cdot d\mathbf{r} = \mu_0 I_{enc} + \mu_0 E_0 \frac{d\Phi_E}{dt} \quad \Rightarrow \quad \text{Ampere's Law} \]

**Electromagnetic Waves**

\( \Rightarrow \) A changing electric field produces a magnetic field.

\( \Rightarrow \) A changing magnetic field will produce an electric field (caused current to flow)

Maxwell envisioned these two fields (electric and magnetic) traveling in space as waves.

\( \Rightarrow \) Imagine that we have the electric field oscillating in space
The velocity of the wave is given by \( \mathbf{E} \times \mathbf{B} \)

* The electric and magnetic fields at any point are perpendicular to each other and direction of travel.

EM waves can come from two of things, one main thing that acceleration electric charge give rise to electromagnetic waves.

(See pages 315-321 for a more detailed description of the follow.)

Using maxwell's Equation (we won't show it here),
we can show that:

\[
\frac{\partial \mathbf{B}}{\partial t} = -\mu_0 \frac{\partial \mathbf{E}}{\partial t} ; \quad \frac{\partial \mathbf{E}}{\partial t} = -\frac{\mathbf{B}}{\mu_0}
\]

Also we assume, that:

\[
\mathbf{E} = E_0 \sin(kx - \omega t) = \mathbf{E}_y
\]

\[
\mathbf{B} = B_0 \sin(kx - \omega t) = \mathbf{B}_z
\]

\[
k = \frac{2\pi}{\lambda} ; \quad \omega = 2\pi f \quad \Rightarrow \quad \omega = \frac{\lambda}{\mu_0}
\]

Using the above equation we get:

\[
\mathbf{w} \cdot \mathbf{B} \cos(kx - \omega t) = k \mathbf{E}_0 \cos(kx - \omega t)
\]

\[
\left( \frac{E_0}{B_0} = k \cdot \text{velocity} \right)
\]
\[ K \beta \cos(kx - \omega t) = \frac{N_0 E_0}{k} \beta \cos(kx - \omega t) \]

\[ \frac{B_0}{E_0} = \frac{\mu_0 \beta \omega}{k} = \frac{N_0 E_0}{k} \beta \]

\[ v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{8.85 \times 10^{-13} \text{ C}^2 / \text{N} \cdot \text{m}^2} \left(4\pi \times 10^{-7} \text{T} \cdot \text{m} / \text{A}\right)} \]

\[ v = 3 \times 10^8 \text{ m/s} = c \]

An EM wave contains energy

The energy in an EM wave is usually given in terms of the Poynting Vector

It is the amount of power per unit area (points in this direction)

\[ S = \frac{1}{A} \frac{dU}{dt} \]

In terms of \( E \) and \( B \):

\[ S = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}) \]