Lecture 6  DC Circuits.

EMF and Terminal Voltage

Up until now we have had the case where our battery in our circuit has a constant potential difference. In real life, our battery only has a constant voltage when no current moves in the circuit (open circuit).

The preconnection voltage is called the electromotive force (emf) \( E \). For real batteries there exists some internal resistance \( r \).

When the battery is connected to a circuit, current flows. The resistance in the battery will create a current that goes against the emf.

\[
V' = Ir
\]

where \( I \) is current.

The potential difference we see \( V_{\text{batt}} \) is then equal to:

\[
V_{\text{batt}} = E - Ir
\]

Resistors in Series

The current in each resistor is the same.

Blc of charge conservation

\( I \rightarrow \text{squeeze but same current} \rightarrow I \rightarrow \text{bottleneck traffic} \)
we know $V = IR$ since $I$ is constant and $R_1 + R_2 + R_3$

we define:

$V_1 = IR_1$
$V_2 = IR_2$
$V_3 = IR_3$

The voltages for resistors in series add:

$V_{\text{circuit}} = V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3) = V_{\text{batt}} = I\cdot R_2$

$R_{\text{eq}} = R_1 + R_2 + R_3$

Resistors in parallel

charge conservation says $I_a$ at point $a$ equals current at $b$.

$I_a$ is like water hitting a fork

current splits at this junction into $I_1, I_2, I_3$ where $I = I_1 + I_2 + I_3$

For resistors connected in parallel the voltage across each is constant.

$V_1 = V_2 = V_3 = V$

$\Rightarrow I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = \frac{V}{R_{\text{eq}}}$

$\Rightarrow R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$
Example:

(a) which bulb burns brighter?

\[
\begin{array}{c}
\text{60W} \\
\text{100W} \\
\text{M} \\
V = 120V
\end{array}
\]

\[\text{V is constant for each }\]
\[\text{voltage of 100W is greater so }\]
\[\text{the 100W bulb is brighter (high P)}\]
\[\text{from } P = \frac{V^2}{R}\]

(b) voltage no longer constant

\[\text{voltage constant,}\]
\[\text{at constant } V\text{ each }\]
\[R_{100} = \frac{V^2}{P} = \frac{120^2}{100W} = \]
\[R_{60} = \frac{120^2}{60W} = \]
\[\Rightarrow R_{100} < R_{60}\]

\[\text{so at constant I power radiated is:}\]
\[P = I^2R \Rightarrow R_{60} > R_{100} \text{ so the 60W bulb is brighter.}\]
Series & Parallel

\[
\text{what current is drawn from battery?}
\]

\[
R_{eq} = \left( \frac{1}{500} + \frac{1}{700} \right)^{-1} = 290 \Omega
\]

in series I is constant.

\[
R_{eq} = 400 + 290 = 690 \Omega
\]

\[
V_{\text{batt}} = I R_{eq} = 690 I = 180 \Rightarrow I = 17 \text{mA}
\]

Kirchhoff's Laws

Two Rules:

1) At any junction point, the sum of all currents entering a junction must equal the sum of all currents leaving junction.

\[
I = I_1 + I_2 + I_3
\]

2) The sum of charges or potential around any closed loop of a circuit must be zero.

Here I is constant so:
Start at a and move clockwise through circuit.

The battery is a positive charge in voltage, the resistor is a negative charge in voltage so we have $V = V_{\text{batt}} - V_1 - V_2$

$$0 = V_{\text{batt}} - IR_1 - IR_2$$

$$\Rightarrow I = \frac{V_{\text{batt}}}{(R_1 + R_2)}$$

adds together in series (we knew this already)

Example:

\begin{align*}
R_1 &= 15\,\Omega \\
R_2 &= 15\,\Omega \\
R_3 &= 15\,\Omega \\
R_4 &= 30\,\Omega \\
R_5 &= 40\,\Omega \\
E_0 &= 45\,\text{V} \\
E_1 &= 8\,\text{V}
\end{align*}

What is $I_1, I_2, I_3$?
we know here that $I_3 = I_1 + I_2$

We do a closed loop around upper half

$$0 = E_0 - I_3 R_3 - I_3 R_2 - I_1 R_1$$

$$-I_3 (15\,\Omega) + 45\,\text{V} - I_1 30\,\Omega$$

\text{Current at } R_4 \text{ is moving opposite to } I_1 \text{ direction so the sign flips (we have an increase potential)}
\[ \Rightarrow 0 = -I_1 \cdot 30 + I_a \cdot 20 + I_0 - 80 \] 

\[ \Rightarrow I_a = \frac{80 + 30I_1}{21} = 3.8 + 1.4I_1 \]

\[ I_a = I_1 - 0.73I_1 \]

\[ I_1 = I_3 - I_2 = 1.1 - 3.8 - (0.73 + 1.4)I_1 \]

\[ \Rightarrow 3.1I_1 = 2.7 \]

\[ I_1 = -0.87A \]

From this \[ I_2 = 2.6A \]

\[ I_3 = 1.7A \]

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**Circuits Containing Resistors and Capacitors**

(RC Circuits)

![Diagram of RC circuit]

When we close the switch that is time \( t = 0 \). After we close \( Q \) will slowly build up on capacitor until potential diff \( = \varepsilon \).

Close \( S \) \( \Rightarrow \) as \( \varepsilon \) \( = \frac{Q}{C} + IR \)

Current steps

We show this

Close \( S \) \( \Rightarrow \) as \( \varepsilon \)

Current steps

We know \( I = \frac{dQ}{dt} \)

\[ \varepsilon = \frac{+dQ}{R} + \frac{Q}{C} \]

Solve for \( dQ \)

\[ \Rightarrow \frac{dQ}{(CE-Q)} = \frac{dt}{RC} \]

We integrate from \( 0 \to Q \) on left and \( 0 \to t \) on right.

Time when \( Q \) is stored on capacitor.
\[ \int_0^\infty \frac{d\theta}{(C\varepsilon - \theta)} = \int_0^\infty \frac{dt}{R\varepsilon} \]

\[-\ln(C\varepsilon - \theta) \bigg|_0^\theta = \frac{t}{RC} \]

Multiply by minus:
\[+\ln(C\varepsilon - \theta) - \ln(C\varepsilon) = \frac{-t}{RC} \]

\[\exp \left( \ln \left( 1 - \frac{\theta}{C\varepsilon} \right) \right) = e^{-t/RC} \]

\[1 - \frac{\theta}{C\varepsilon} = e^{-t/RC} \]

\[RC = \tau \text{ the time constant} \]

In terms of \( Q \):
\[ Q = C\varepsilon \left( 1 - e^{-t/RC} \right) \]

Since \( V_c = \frac{Q}{\varepsilon} \)
\[ V_c = \varepsilon \left( 1 - e^{-t/RC} \right) \]

\[ V_c \rightarrow \varepsilon \text{ when } V_c \rightarrow 0 \]

\[ I(t) = \frac{dQ}{dt} = \frac{\varepsilon}{R} e^{-t/\tau} I \]

\[ I \rightarrow 0 \text{ as } t \rightarrow \infty \]

\[ I = -IR + V_c \]

\[ \varepsilon = -IR + \theta \Rightarrow I = 0 \]
The circuit we had was charging a capacitor using an emf. Now let's look at discharge.

Suppose capacitor has some $V_c$ on it, it now acts like a battery!!

When $S$ is closed $t=0$

$$V_c = \frac{-e}{-\frac{e}{V_c}}$$

Using Kirchhoff

$$I_R = \frac{\Delta V}{R} = \frac{V_c}{R}$$

$$I \propto \frac{dQ}{dt} =$$ leaves at negative direction

it came in. Remember we had leaving + going to minus now

it is reversed.

$$I = \frac{Q}{C}$$

$$\Rightarrow \frac{-dQ}{d+} = \frac{Q}{C} = 0$$

we integrate from $Q_0$ to capacitor having some

and we integrate $0$ to $+\infty$

$$\ln \left( \frac{Q}{Q_0} \right) = -\frac{t}{T}$$

$$Q = Q_0 e^{-t/RC}$$

$$\Rightarrow V_c = V_0 e^{-t/RC}$$

$$\Rightarrow I = \frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC} = I(0) e^{-t/T}$$

$V_c$ vs. $t$:

decreases w/ time.
Let's put together both:

\[ 20V = \mathcal{E} + \frac{V}{C} \]

**In position 1** charges capacitor up to \( \mathcal{E} \):

\[ Q_0 = C \mathcal{E} \quad \text{then we move to position 2 \Rightarrow discharge capacitor}! \]

\[ Q_0 = C \mathcal{E} \quad \text{given } C = 1.0 \mu F \]

\[ I = I_0 e^{-t/RC} \quad \mathcal{E} = 20V \]

I suppose current drops \( \frac{1}{2} \) of its initial value in 40ms:

\[ \Rightarrow \frac{1}{2} I_0 = I_0 e^{-40 ms/RC} \quad \text{what is } Q_0 \text{ at } t = 0 \]

\[ e^{-\ln(2)} = \frac{40 ms}{RC} \]

\[ R = \frac{40 ms}{\ln(2)} = 575 \Omega \]

**c) what is \( Q \) at 60ms**

\[ Q = Q_0 e^{-t/RC} = (0.4) e^{-60ms/(87)(1.02 \mu F)} = 7.3 \mu C \]
Electric kill \[ P=I^2R \]

\[ \Rightarrow \text{most people a current of } 1 \text{mA , A few mA is not known to cause damage in a healthy person.} \]

\[ >10 \text{mA causes severe contraction of muscles (can't let go)} \]
\[ \text{Death can occur from paralysis of respiratory system} \]

between 80-100mA the heart will contract irregularly & blood will improperly pump.

Larger currents will stop heart, but after remove current the heart will pump regularly again by defibrillators.

Usually warnings are in voltages. \[ V=IR \text{ the resistance of skin is } 10^4 \Omega \text{ (wet is } 10^3 \Omega) \]

Say you touch 120V bare foot wet.

\[ I=\frac{120}{10^3}=120 \text{mA} \]

Power radiated (say heat) \[ =I^2R=(120\text{mA})^2(10^3)=\underline{\text{Ouch!}} \]

Look at Example 26-8 on page 683 of 4ed.