Magnetic field due to a straight wire

Last time we saw that a wire produces a magnetic field.

- What exactly is this magnetic field? It was discovered empirically that near a long straight wire that:

\[ B \propto \frac{I}{r} \quad \text{(where } u_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A}) \]

\[ B = \frac{u_0 I}{2\pi r} \quad \text{magnitude of } B \text{ at point } r \text{ away from wire} \]

The direction \( B \) is circular around wire.

Suspending a wire with current

Attached

\[ \int 20 \text{ cm} \quad \text{two wires same length} \]

Free to fall

\[ F_g = mg \quad \text{using RHR } F_{\text{wire}} \text{ is up} = I_B l \]

\[ F_{\text{wire}} = I_2 \left( \frac{I_1 u_0}{2\pi d} \right) l = mg \]

\[ I_2 = \frac{3\pi d mg}{I_1 u_0 l} = 15 \text{ A} \quad \text{pretty high amps.} \]
Definition of Ampere & Coulomb

If you ever wondered how we define an amp & coulomb, here it is:

\[ \text{One ampere is defined as the amount of current flowing in two long parallel wires 1 m apart which results in a force of exactly } 2 \times 10^{-7} \text{ N per meter.} \]

\[ \text{One coulomb is defined as being 1 ampere-second.} \quad \text{If } = 14.4 \]

**Ampere's Law** (The Gauss's Law for magnetic fields)

Mr. Ampere found a way to measure the magnetic field for any current configuration (not just a long straight wire).

Let's start w/ a long straight wire:

\[ \begin{align*}
\text{Place of } B \\
\text{We imagine making a path of little segments (each & length &)}}
\end{align*} \]

Ampere noted that \[ \nabla \cdot B = \mu_0 I_{enc} \]

If sum up all the segments \[ \int \nabla \cdot B \parallel \text{ to segment } dL = \mu_0 I_{enc} \]

If we let \[ dL \rightarrow \text{ very small! we get,} \]

\[ \int \nabla \cdot B \cdot dL = \mu_0 I_{enc} \Rightarrow \text{Ampere's Law} \]

\[ \text{dot gets you parallel component above.} \]
So let's do some examples w/ Ampere's Law & we can choose any path.

Circular path around a wire:

Point $P$ is $r$ away:

\[
\oint B \cdot dl = 0 \quad \text{I enc}
\]

Since $dl$ is going around in a circle and $B$ is as well we can assume that $B$ and $dl$ are parallel.

Pull ring $B$ out just like we did w/ $E$ in Gauss' Law.

\[
B \cdot dl = moI
\]

\[
B(\alpha \pi r) = moI \quad \Rightarrow B = \frac{moI}{\alpha \pi r}
\]

direction determined by RHR.

$B$ field inside & outside wire

For $r > R$ make our path a circle of radius $r$.

\[
\oint B \cdot dl = moI \quad \Rightarrow B = \frac{moI}{\alpha \pi r}
\]

For $r < R$ $I$ enc $I$ b/c we don't have whole wire.

How do find $I$ enc? For a given current moving through a cross-sectional area, $A$, $j$ (current density) is constant.

\[
j = \frac{I}{A} = \frac{I}{\pi R^2} \quad \Rightarrow \text{just like charge we say} \quad I \text{enc} = \frac{j}{A \text{enc}}
\]

\[
\Rightarrow I \text{enc} = \frac{\pi R^2 I}{\pi R^2}
\]
\[ \oint \mathbf{B} \cdot d\mathbf{r} = \mu_0 I_{\text{enc}} \]

If \( \mathbf{B} \) and \( d\mathbf{r} \) are parallel.

\[ B(2\pi r) = \mu_0 I \left( \frac{r^2}{R^2} \right) \]

\[ B = \frac{\mu_0 I r}{2\pi R^3} \]

Plot them:

**Magnetic Field of a Solenoid & Toroid**

A long coil of wire with many loops is called a **solenoid** (they are used to make electromagnetics, like the magnets in junkyard).

Using the right-hand rule, we can find magnetic field.

Can also use a different version of RHR when you have loops of current.

Curl your fingers in the direction of current around the loops, your thumb points in the direction of the \( B \) field inside the loops.
Close up view

we make our

circular loop (path)

(Re-rectangular)

we have a rectangular path so:

\[
\oint \vec{B} \cdot d\vec{e} = \int_c \vec{B} \cdot d\vec{e} + \int_b \vec{B} \cdot d\vec{e} + \int_c \vec{B} \cdot d\vec{e} + \int_d \vec{B} \cdot d\vec{e}
\]

We can say outside the solenoid the field is very small (mostly zero) so first term drops.

For the segments b->c and d->a we see that \( \vec{B} \cdot d\vec{e} \) means \( \vec{B} \cdot d\vec{e} = 0 \) so we are left with:

\[
\oint \vec{B} \cdot d\vec{e} = N \text{enc}
\]

\[ B = \mu_0 I \text{enc} ; \text{enc} = NI \]

\[ B = \frac{NI}{l} \text{ we make } l \text{ length of solenoid.} \]

\[ B = \mu_0 n I \quad \text{ } \rightarrow n = \frac{N}{l} \text{ # of loops per unit length.} \]
Toroid
A toroid is a circular solenoid.

What is $B$ inside and out of toroid.

For $r<R$, $\oint B \cdot dl = \text{ad become } 0$

\[ B = 0 \]

For $r>R$, we have equal # of current pass up through our plane and down through our plane so $\oint B \cdot dl = 0$

\[ \oint B \cdot dl = 0 \]

\[ r = R \text{ inside toroid:} \]

only getting all down (or up)

\[ \oint B \cdot dl = \mu_0 n I \implies B = \mu_0 n I \frac{2\pi r}{\sigma} \]

Biot-Savart Law
When Ampere's law can't be easily applied, (when we don't have nice straight wires or perfect loops) we use a differential form of $E$

(much like we have $dE = k \int ds$)

According to Biot and Savart, for any point $P$.

\[ dB = \frac{\mu_0 I}{4\pi} \frac{dl \times \hat{r}}{r^2} \]

\[ dB = \frac{\mu_0 I}{4\pi} \frac{dl \times \hat{r}}{r^2} \]

\[ \text{where } r = \frac{r_2}{1} \]

\[ \text{non-straight wire} \]
To find the entire B field due to a non-straight wire we integrate over the length of the wire.

\[
\hat{B} = \int \frac{\mu_0 I \, \hat{d}x \hat{r}}{4\pi \, r^2}
\]

The magnitude of \( \hat{B} \) is then:

\[
B = \frac{\mu_0 I}{4\pi} \int \frac{\sin \theta}{r^2}
\]

For rest of lecture will go over some examples of finding B field.

\[\hat{B} \text{ due to a straight wire: we know } B = \frac{\mu_0 I}{2\pi R} \text{ a distance } R \text{ away.}\]

\[
\int_I \quad \text{d}^2 \text{ inward} \quad \text{blc} \quad \text{d} \hat{r} \quad \text{we know direction}
\]

\[
R \quad \text{magnitude of } \hat{B}
\]

\[
\hat{B} = \frac{\mu_0 I}{4\pi} \int \frac{\sin \theta}{r^2} \quad ; \quad r^2 = y^2 + R^2
\]

\[
\sin \theta = \frac{R}{\sqrt{y^2 + R^2}}
\]

\[
\text{let } \text{d}e = \text{d}y
\]

\[
B = \frac{\mu_0 I}{4\pi} \int \frac{dy \sin \theta}{(y^2 + R^2)}
\]

\[
\text{we want to go to } \theta \quad \text{let } y = -R \quad \text{dy} = +R \csc^2 \theta \text{d}\theta
\]

\[
\frac{R^2}{r^2} \quad \text{dy} = \frac{R^2}{r^2} \text{d}e
\]

\[
B = \frac{\mu_0 I}{4\pi} \int \frac{\sin \theta}{R \, r^2}
\]

\[
dy = \frac{r^2}{R} \text{d}e
\]
\[
\begin{align*}
B &= \frac{\mu_0 I}{4\pi R} \int_0^\pi \sin \theta \, d\theta \, d\phi \\
\left[ B = \frac{\mu_0 I}{2\pi R} \right] \quad \text{as expected}
\end{align*}
\]

Current Loop (redrawn)

From symmetry \( dB_y = 0 \)

\( dB_y \) in \(+x\) direction

Magnitude

\[ B_{\parallel} = B_{\parallel} = \int \frac{\mu_0 I \, dl \cos \phi}{4\pi r^2} \]

\[ B = \frac{\mu_0 I}{4\pi} \int \frac{dl}{r^2} \left( \frac{R}{(x^2 + R^2)^{3/2}} \right) \]

\[ B = \frac{\mu_0 I}{4\pi} \left[ \frac{R}{(x^2 + R^2)^{3/2}} \right] \int_0^{2\pi} \frac{d\theta}{a (R^2 + x^2)^{3/2}} \]

what happens when \( x = 0 \)?

\[ B = \frac{\mu_0 I}{2R} \]
Curved wire segment:

A curved wire segment with radius $R$. What is $B$ at center? (just circular part)

d$e$, $r$ are $1$

$B$ is into page.

\[
B = \frac{\mu_0 I}{4\pi R}
\]

\[
\left[ \frac{R^2}{4\pi R^3} \right] = \frac{\mu_0 I}{8R}
\]

A little more on magnetic materials:

A magnetic fields comes from two things

1) magnetic materials (permanent magnets)
2) electric curronts

Some materials (such as iron) can be made into strong magnets \(\Rightarrow\) they are called ferromagnets.

we will get to why magnets attract metals in a few lectures.

Remember north pole attracts south pole of magnetic.