**Lecture 9**

**Electromagnetic Induction & Faraday's Law**

**Induced EMF**

A changing magnetic field will produce an electric current. This current is called the induced current, which produces current as if there is a "new emf" in the circuit (induced emf).

This phenomenon is called **Electromagnetic Induction**.

**Faraday’s Law of Induction (Lenz’ Law)**

We define the electromagnetic flux as:

\[ \Phi_B = \vec{B} \cdot \vec{A} \quad \text{units of} \quad W(T \cdot m^2) \]

For a differential, we can integrate over the area to get:

\[ \Phi_B = BA \cos \theta \]

We can define the induced emf as:

\[ \varepsilon = -\frac{d\Phi_B}{dt} \quad \text{change in flux per unit time.} \]
Suppose we have a loop of wire:

\[ E_{\text{induced}} = -\frac{N \Delta \Phi_b}{dt} \]

Positive charge

\[ E = \frac{d \Phi_b}{dt} \]

\( E \) induced is negative, so \( I \) is negative to direction. So current in system goes down.

If we place the magnet just in the middle of the loop, \( \Phi_b \) is const. so no induced emf and current.

If the wire had \( N \) loops to it, all the loops would get induced so:

\[ E = -N \frac{d \Phi_b}{dt} \]

More about the minus sign: Experimentally shown that:

* Any current produced by an induced emf will move in a direction so that the magnetic field created by that current opposes the original change in flux (Lenz law)

Other words:

An induced emf is always in a direction that opposes the original change in flux that caused it.
in other words:

An induced emf creates a current that will oppose the change in emf. (wants to keep status quo)

Ex:

This is what we do: We use RTHC to define a "positive direction", we curl our fingers around our "positive" direction and our thumb points up (say our direction is clockwise)

If thumb points in direction of $\mathbb{E}_B$ then $\mathbb{E}_B$ is positive.

Since we are moving magnet up through the wire $\frac{d\mathbb{E}_B}{dt} > 0$, the induced emf is then:

$$\mathcal{E} = -\frac{d\mathbb{E}_B}{dt}$$

so Ind is going to be opposite (neg) to our positive direction. (if count, clockwise)

Note: the choice for "positive" direction does not matter. If we had count, clockwise be positive the $\mathbb{E}_B$ would be negative \( \Rightarrow \mathcal{E} < 0 \) meaning Ind goes counterclockwise.

Note: (emf is increasing \( \mathcal{E} = \frac{d\mathbb{E}_B}{dt} \), so Ind goes against it)

Move Mag. down.

Make clockwise positive direction.

then \( \mathbb{E}_B \) is still positive but \( \frac{d\mathbb{E}_B}{dt} \) is negative \( \Rightarrow \mathcal{E} > 0 \) so Ind is along our count (clockwise positive direction)

(emf is decreasing, Ind goes against it \( \Rightarrow \) decreases)
some examples what direction is \( \mathbf{I} \) ind in the air?

(in all cases treat clockwise as positive)

\[ \nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \]

\[ \mathbf{E} \] is neg. and \( \frac{\partial \mathbf{B}}{\partial t} \) is negative

\[ E_{\text{ind}} < 0 \Rightarrow \text{Ind counterclockwise} \]

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \frac{\partial \mathbf{E}}{\partial t} \]

\[ E > 0 \Rightarrow \text{Ind clockwise} \]

Look at ex. 29-5 on page 764 4th ed.

**EMF induced in moving conductor**

\[ \frac{d\mathbf{B}}{dt} > 0 \text{ b/c area is inc.} \]

The magnitude of induced \( \mathbf{E} \) is:

\[ E = \frac{d\mathbf{B}}{dt} = \frac{B_2 - B_1}{t - t_0} = \frac{B_2 - B_1}{dt} \]

So what if rod has resistance \( R \)? what is power dissipated:

\[ P = I^2R = \frac{E^2}{R} = \frac{B^2v^2}{R} \]
Electric Generators

Easiest generation is \( \text{AC generator} \).

\[
\epsilon = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int B \cdot dA = -\frac{d}{dt}[BA \cos \theta]
\]

\( \theta = \text{angle between } B \& A \)

\( \omega = \frac{d\theta}{dt} \) \( \text{from mechanics} \)

Then \( \theta = \theta_0 + \omega t \) then assuming that \( \theta_0 = 0 \) we have

\[
\epsilon = -BA \frac{d}{dt}(\cos \omega t) = BA \sin \omega t
\]

Notes [In frequency terms \( \omega = 2\pi f \)].

Suppose you have \( N \) turns in our loop then:

\[
\epsilon = \frac{NBA \sin \omega t}{E_0} = \epsilon_0 \sin \omega t
\]

\( \epsilon_0 \) like a sine wave (\( \text{AC current} \)).
DC generator: a little more complicated.

Same basic setup

This rotates (in order to induce current) and every half rev the current rotates (still alternately, but not really) long time before it switches.

Transformers & Transmission of Power

How do they get power (electricity) to your house?

Transformers are used to increase or decrease a voltage. They range in size to cell phones to as large as houses. The transformer consists of two coils: primary & secondary coils.

The idea is to transfer all of magnetic flux from the primary coil to secondary coil. The transfer is 99% efficient.
Okay assume setup as:
iron core that transmits B field.

AC current in primary creates change B in primary coils which looks like change flux in secondary.
The emf (or voltage) in secondary is then

\[ V_s = N_s \frac{dB}{dt} \]

in the primary \[ V_p = N_p \frac{dB}{dt} \]

If we look at a ratio of \( V_s \) to \( V_p \), we get the transformer equation:

\[ \frac{V_s}{V_p} = \frac{N_s}{N_p} \]  
know output is related to primary voltage (assuming we know \( N_s \) & \( N_p \))

If \( N_s > N_p \) \( \Rightarrow \) step up transformer \( V_s \) inc.
If \( N_s < N_p \) \( \Rightarrow \) step down transformer \( V_s \) dec.

By conservation of energy (ignoring heat etc) Power in equals power out.

\[ I_p V_p = I_s V_s \Rightarrow I_s = \frac{N_p}{I_p} \frac{V_s}{N_s} \]

so we have:
Transmission lines: we do use transformers?

Say we are 10 km from our power plant, and we need to send 120 kW of power to a small town. A wire R = 0.4 ohms. Say we transmit at two different V.

1) V = 240 V

\[ I = \frac{P}{V} = \frac{1 \times 10^5 \text{W}}{240 \times 10^3 \text{V}} = 500 \text{A} \]

Line loss: \[ P = I^2 R = 500^2 (0.4) = 100 \text{ kW lost} \]

That's 80% of 120 kW.

2) V = 24,000 V

\[ I = \frac{P}{V} = \frac{1 \times 10^5 \text{W}}{24 \times 10^3 \text{V}} = 5 \text{A} \]

Line loss: \[ P = I^2 R = 5^2 \times 0.4 = 10 \text{ W} \]

1% of 120 kW

So we have a system like this.

Any guesses why power lines are so high? High V can waive air (spark) when close to ground.

I don't know why.
Lastly & Most Importly

A changing Magnetic Flux produces an electric field!

(We sort of knew this in disguise.)

\[ \mathcal{E} = \frac{-d\Phi_B}{dt} = \oint \mathcal{E} \cdot dl \Rightarrow \text{Faraday's Law} \]

This is like a potential diff.

\* Changing mag field produces electric field.

\[ \mathcal{E} \text{ produced by changing } \mathbf{B} \]