The Boundary Updates

PreSolve
BetweenSolve
PostSolve
EllipticSetBC
Hydro Update

In Hydro Update

$$\frac{\partial(P+\rho v^2)}{\partial y} = -\frac{1}{2}(\rho_0+\rho_1)g$$

$$\kappa T^n \frac{\partial T}{\partial y} = q$$

In the Hydro Update, We solve T0 using:

$$\kappa(\frac{T_0 + T_1}{2})^n \frac{T_0 - T_1}{dy} = q$$

Then we convert the equilibrium equation

$$P_1 + \rho_1 v_1^{\ 2} - P_0 - \rho_0 v_0^{\ 2} = -\frac{1}{2}(\rho_0 + \rho_1) g dy$$

to quadratic equation about density:

$$(-\frac{1}{2} g dy + T_0)\rho_0^{\ 2} + (-\frac{1}{2} g dy \rho_1 - \rho_1 T_1 - \rho_1 v_1^{\ 2})\rho_0 + \rho_1 v_1^{\ 2} = 0$$

The velocity is then solved using mass conservation:

$$\rho_0 v_0 = \rho_1 v_1$$

In Elliptic Solving, we treat the boundary as follows.
We convert the diffusion equation

$$\frac{3}{2}\rho\frac{dT}{dt} = \frac{\partial}{\partial y}\kappa T^n \frac{\partial T}{\partial y}$$

into

$$\frac{3}{2}\rho\frac{dT}{dt} = \frac{1}{2}\frac{\partial^2 z^*}{\partial y^2} + \frac{1}{2}\frac{\partial^2 z}{\partial y^2}$$

where $z = \dfrac{T^{n+1}}{n+1}$

By differentiating the above equation, we have:

$$\frac{3}{2}\rho(T_i^* - T_i) = \frac{\kappa dt}{2dy^2}(z_{i+1}^* + z_{i-1}^* - 2z_i^* + z_i + z_{i-1} - 2z_i)$$

Using Taylor expansion,

$$z^* = z + \frac{dz}{dT}(T^* - T) = z + T^{n+1}(T^* - T)$$

We convert the diffusion equation into:

$$\frac{3}{2}\rho(T_i^* - T_i) = k(\frac{1-n}{1+n}T_{i+1}^{n+1} + \frac{1-n}{1+n}T_{i-1}^{n+1} - 2\frac{1-n}{1+n}T_i^{n+1} + T_{i+1}^n T_{i+1}^* + T_{i-1}^n T_{i-1}^* - 2T_i^n T_i^*)$$

where $k = \frac{\kappa dt}{2dy^2}$

This equation has a form of

$$a_{i+1}T_{i+1}^* + a_i T_i^* + a_{i-1}T_{i-1}^* = a$$

with

$$a_{i+1} = T_{i+1}^n \qquad a_{i-1} = T_{i-1}^n \qquad a_i = -2T_i^n - \frac{3}{2}\rho$$

$$a = -\frac{3}{2}\rho T_i + k\frac{n-1}{n+1}(T_{i+1}^{n+1} + T_{i-1}^{n+1} - 2T_i^{n+1})$$

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For fixed temperature boundary condition, we change coefficients to:

$$a'_{i-1} = 0 \qquad a' = a - a_{i-1}T_{i-1}$$

For fixed flux boundary condition, we start from:

$$\kappa\left(\frac{T_{i-1} + T_i}{2}\right)^n \frac{(T^*_{i-1} - T^*_i)}{dy} = q$$

The issue of this method is that we are solving linear equation instead nonlinear.
From this equation we get:

$$T^*_{i-1} = T^*_i + dT$$

where

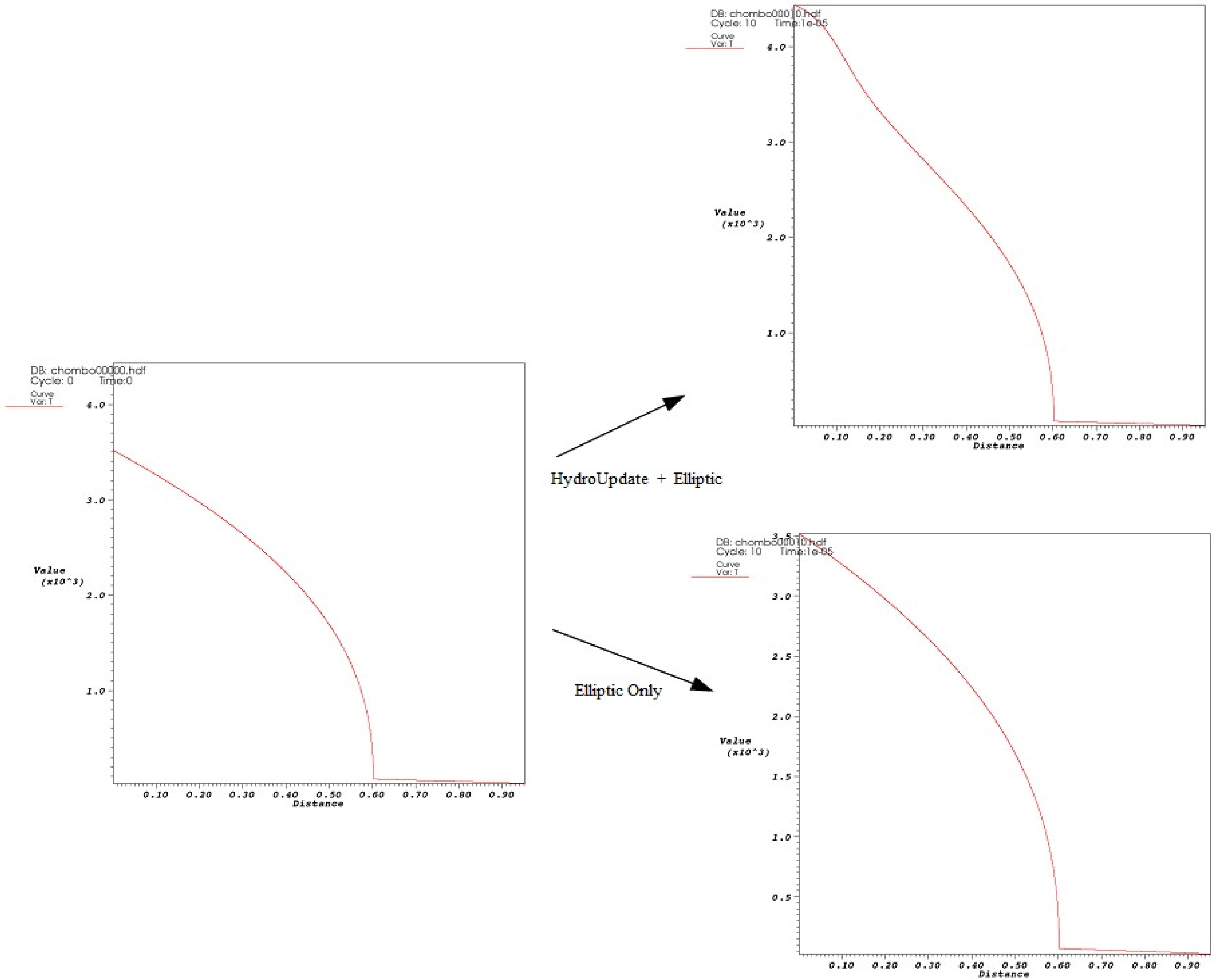
$$dT = \frac{q\,dy}{\kappa}\left(\frac{2}{T_{i-1} + T_i}\right)^n$$

We change the coefficients to:

$$a_{i-1}' = 0$$

$$a_i' = a_i + a_{i+1}$$

$$a' = a - a_{i-1}dT$$



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HydroUpdate + Elliptic
Elliptic Only
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