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Critical Scaling of Compression-Driven Jamming of Frictionless Spheres

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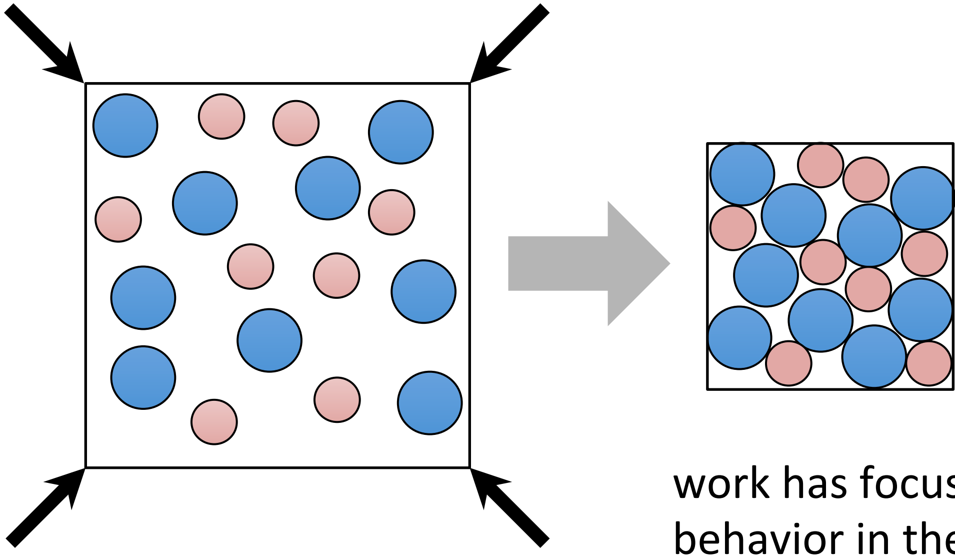
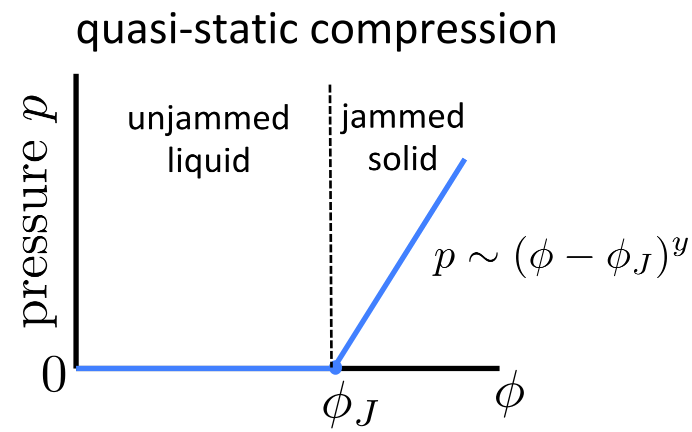
Athermal isotropic compression-driven jamming:





work has focused on quasi-static compression and behavior in the jammed solid phase *above* ϕ_J

We are interested in behavior in the liquid phase *below* ϕ_J, particularly to probe for a diverging time scale as $\phi \to \phi_J$ from below.

Do stress-isotropic (compression-driven) and stress-anisotropic (shear-driven) jamming have the same critical universality?

Ikeda et al, PRL 124, 058001 (2020) computed the relaxation time τ of initial unjammed configurations below ϕ_J. A common divergence of τ for both random isotropic configurations and for configurations in steady-state simple shear, *suggests that compression-driven and shear-driven jamming have a common universality*.

Nishikawa et al, J Stat Phys 182, 37 (2021) have questioned Ikeda et al's results; find $\tau \sim \ln N$ when the system becomes sufficiently large, so τ has no proper thermodynamic limit.

Athermal isotropic compression at a finite compression rate:

We probe time scales below ϕ_J by compressing at a finite rate $\dot{\epsilon}$
The finite rate introduces a control time scale with which to probe the critical time scale

We measure the bulk viscosity, which we find to diverge at ϕ_J, and compare to the shear viscosity to look for a universality of stress-isotropic and stress-anisotropic jamming

Model:

size-bidisperse, soft core spheres, in non-Brownian suspension

$$N_s = N_b = N/2 \qquad d_b/d_s = 1.4 \qquad \phi = \frac{1}{L^D}\sum_i V_i$$

packing fraction

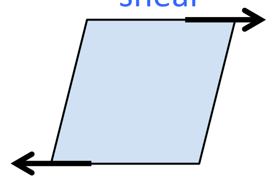
one-sided harmonic elastic contact interaction

$$\mathbf{f}_{ij}^{\rm el} = -\frac{d}{d\mathbf{r}_i}\left[\frac{1}{2}k_e\left(1-\frac{r_{ij}}{d_{ij}}\right)^2\right] \qquad \begin{aligned} r_{ij} &= |\mathbf{r}_i - \mathbf{r}_j| \\ d_{ij} &= (d_i + d_j)/2 \end{aligned}$$

viscous dissipative drag due to host medium

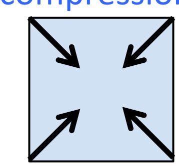
$$\mathbf{f}_i^{\rm dis} = -k_d V_i\left[\mathbf{v}_i - \mathbf{v}_{\rm host}(\mathbf{r})\right]$$

simple shear



$$\mathbf{v}_{\rm host}(\mathbf{r}) = \dot{\gamma} y \hat{\mathbf{x}}$$

isotropic compression



$$\mathbf{v}_{\rm host}(\mathbf{r}) = -\dot{\epsilon}\mathbf{r}$$

dynamics:

$$m_i \ddot{\mathbf{r}}_i = \sum_j \mathbf{f}_{ij}^{\rm el} + \mathbf{f}_i^{\rm dis}$$

$$Q \lesssim 1 \;\Rightarrow\; \text{overdamped}$$

dimensionless parameters:

$$\dot{\epsilon} t_0, \quad t_0 = (D/2) k_d V_s d_s^2 / k_e = 1 \qquad Q = \sqrt{m_s k_e}/k_d V_s d_s$$

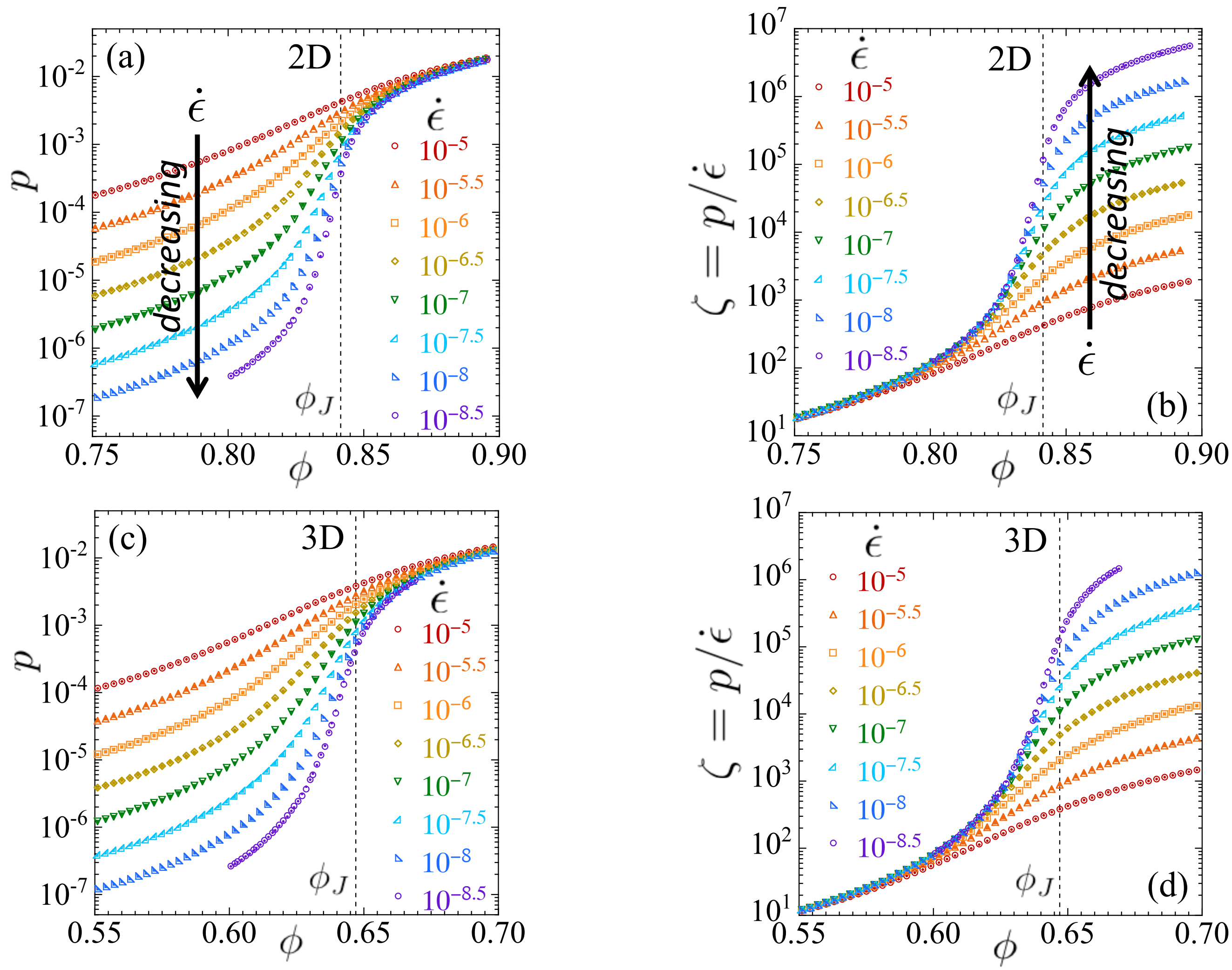
N=16,384
32,768

Results:

pressure p | bulk viscosity $\zeta = p/\dot{\epsilon}$

2D

3D



$\phi < \phi_J : \quad p \to 0 \text{ as } \dot{\epsilon} \to 0$

$\phi > \phi_J : \quad p \to \text{ constant as } \dot{\epsilon} \to 0$

$\phi < \phi_J : \quad \zeta \to \text{ constant as } \dot{\epsilon} \to 0$

$\phi > \phi_J : \quad \zeta \to \infty \text{ as } \dot{\epsilon} \to 0$

Critical scaling ansatz:

$$p = \dot{\epsilon}^q \, f\left(\frac{\phi - \phi_J}{\dot{\epsilon}^{1/z\nu}}\right)$$

$\phi > \phi_J$: since $p \to$ constant as $\dot{\epsilon} \to 0$,
then $f(x \to +\infty) \to |x|^{qz\nu}$

$$\lim_{\dot{\epsilon} \to 0} p \sim (\phi - \phi_J)^y$$

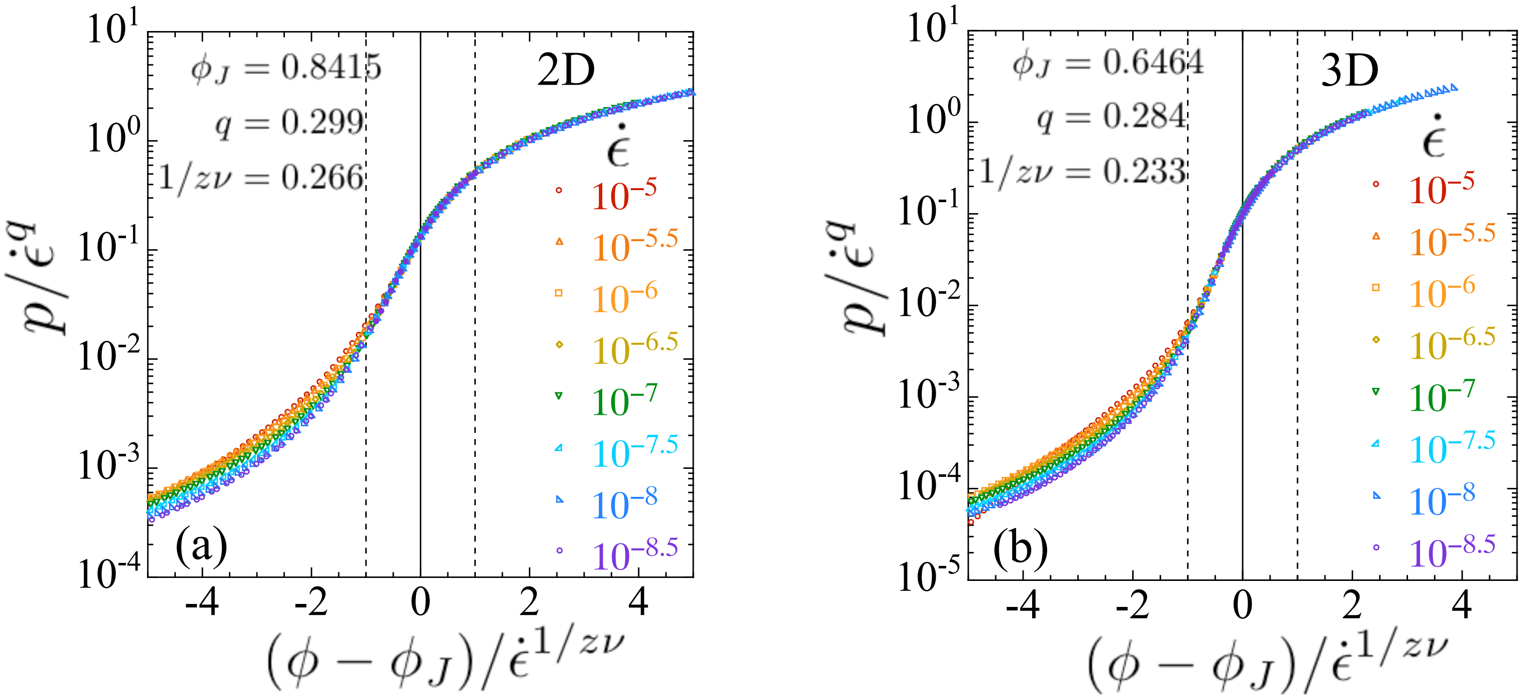
$$y = qz\nu$$

$\phi < \phi_J$: since $\zeta \equiv p/\dot{\epsilon} \to$ constant as $\dot{\epsilon} \to 0$,
then $f(x \to -\infty) \to |x|^{-(1-q)z\nu}$

$$\lim_{\dot{\epsilon} \to 0} \zeta \sim (\phi_J - \phi)^{-\beta}$$

$$\beta = (1-q)z\nu$$

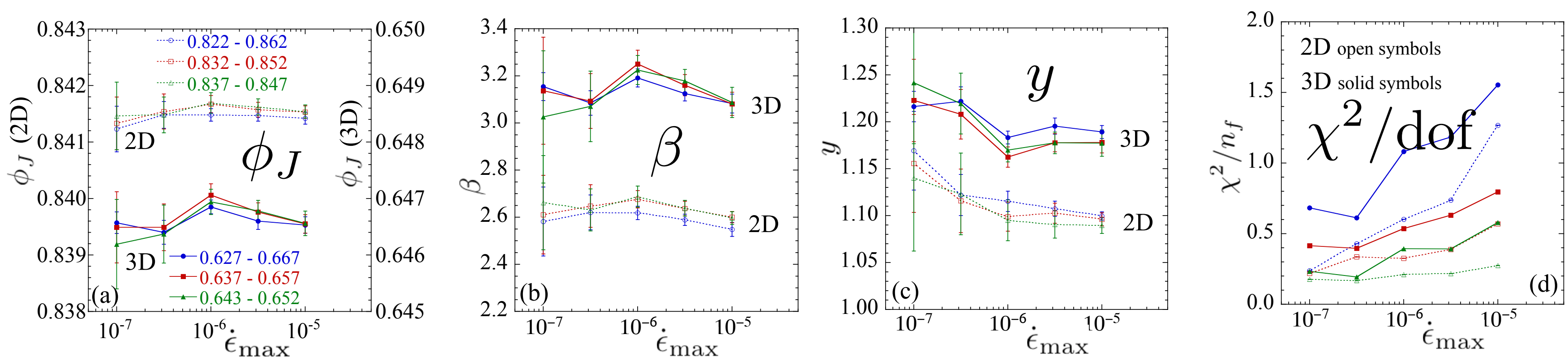
plot $p/\dot{\epsilon}^q$ vs $(\phi - \phi_J)/\dot{\epsilon}^{1/z\nu}$ – data for different $\dot{\epsilon}$
should collapse to a common curve.



$$p = \dot\epsilon^q \, f\left(\frac{\phi - \phi_J}{\dot\epsilon^{1/z\nu}}\right) \qquad \lim_{\dot\epsilon \to 0} p \sim (\phi - \phi_J)^y \qquad y = qz\nu$$

$$\zeta = p/\dot\epsilon \qquad \lim_{\dot\epsilon \to 0} \zeta \sim (\phi_J - \phi)^{-\beta} \qquad \beta = (1-q)z\nu$$

test of sensitivity of fitted parameters to size of data window used in fit



Conclusions:

	compression	simple shearing (numerical results)	theory: marginal stability
2D	$\beta = 2.63 \pm 0.09$ $y = 1.12 \pm 0.04$	$\beta = 2.77 \pm 0.20$ [1], $\beta = 2.58 \pm 0.10$ [2] $y = 1.08 \pm 0.03$ [1], $y = 1.09 \pm 0.01$ [2]	$\beta = 2.83$ [4]
3D	$\beta = 3.07 \pm 0.15$ $y = 1.22 \pm 0.03$	$\beta = 2.56$ [3], $\beta = 2.8$ [4], $\beta = 2.94$ [5], $\beta = 3.8 \pm 0.1$ $y = 1.16 \pm 0.01$ [6]	$\beta = 2.83$ [4]

compression:

bulk viscosity:

$$\phi < \phi_J \quad \lim_{\dot{\epsilon}\to 0} p/\dot{\epsilon} \sim (\phi - \phi_J)^{-\beta}$$

pressure:

$$\phi > \phi_J \quad \lim_{\dot{\epsilon}\to 0} p \sim (\phi - \phi_J)^{y}$$

shearing:

pressure analog of shear viscosity:

$$\phi < \phi_J \quad \lim_{\dot{\gamma}\to 0} p/\dot{\gamma} \sim (\phi - \phi_J)^{-\beta}$$

pressure on yield stress line:

$$\phi > \phi_J \quad \lim_{\dot{\gamma}\to 0} p_Y \sim (\phi - \phi_J)^{y}$$

Our results are consistent with a common universality for stress-isotropic compression-driven jamming vs stress-anisotropic shear-driven jamming (though 3D is tentative)

[1] Olsson and Teitel, PRE 83, 020201(R) (2011)
[2] Olsson and Teitel, PRL 109, 108001 (2012)
[3] Kawasaki et al, PRE 91, 012203 (2015)
[4] DeGiuli et al, PRE 91, 062206 (2015)
[5] Lerner et al, PNAS 109, 4798 (2012)
[6] Olsson, PRL 122, 108003 (2019)