Ballistic Transport and Velocity Overshoot in Semiconductors: Part I—Uniform Field Effects

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Abstract—**The relationship between ballistic electron transport and velocity overshoot, in semiconductor materials, is clarified. By considering the behavior of electrons in a uniform electric field, we show that while ballistic transport can coexist with velocity overshoot, it is not necessary for overshoot. Furthermore, we show that ballistic transport will not lead to overshoot unless one of the two classic mechanisms for overshoot is also operative.**

I. Introduction

THE NOTION of ballistic electron transport has been introduced as a means for achieving velocity overshoot in submicrometer semiconductor devices [1]-[4]. Significant controversy has appeared in the literature concerning the time and length scales on which ballistic transport should appear [5]-[10] and whether or not it has been observed experimentally [11]-[14]. In an effort to elucidate the important physics behind electron transport in submicrometer semiconductors, we review the process of velocity overshoot and clarify its relationship to ballistic transport.

In Section II, we consider the case of a uniform electric field. We define precisely what we mean by the terms "velocity overshoot" and "ballistic transport" and we conclude the following.

1) Velocity overshoot is expected to exist in most semiconductor materials due to the presence of one of the following:

a) a momentum relaxation rate which is larger than the energy relaxation rate and which is an increasing function of energy;

or

b) heavier mass satellite conduction valleys at low enough energy to have a significant electron population in the steady state.

2) Ballistic transport can exist in conjunction with velocity overshoot. However, ballistic transport cannot by itself lead to any overshoot, since that can occur only if one of the above mechanisms is also operative.

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Although a uniform field is an idealization of the situation in a real device, we feel it contains much of the essential physics of the effects of phonon collisions on the approach to steady state. Non-uniform field effects will be treated in a second paper. In Section III, we state our conclusions.

II. Velocity Overshoot and Ballistic Transport in Uniform Fields

Consider the problem of an ensemble of electrons with zero initial average velocity released at time $t = 0$ into a uniform electric field. We ask about the evolution of the average velocity of the ensemble v as a function of time. Although in a real device electric fields are non-uniform, the above picture is suggestive [15], [16] of the behavior of a packet of electrons originating in a heavily doped region (low field) suddenly entering a lightly doped region (high field). We also note that indications in the literature suggest that non-uniform fields serve to reduce the velocity overshoot effect [18].

A. Definitions

Ballistic transport, as originally introduced [1], referred to the motion of electrons through the semiconductor material in the complete absence of any collisions. Motion in a uniform field, therefore, would be governed by

$$v = qEt/m. \tag{1}$$

Obviously, this is valid only at short times. "Short" is determined by comparing the average energy acquired by the ballistic electron $\epsilon \simeq \frac{1}{2} mv^2 + \frac{3}{2} k_BT$ with the energy scale at which particular collision mechanisms become important.

At longer times it becomes impossible to ignore collisions: the velocity approaches some steady-state drift value instead of increasing without bound as predicted by (1). The recognition of the importance of collisions led to the introduction [3], [4] of the concept "near-ballistic" transport, which we define as transport in the presence of at most a few collisions. When we wish to distinguish between these two we will refer to the former as true ballistic and the later as near ballistic.

Ballistic transport was introduced as a means of producing velocity overshoot. By velocity overshoot we mean the ability to achieve, on short time scales, velocities exceeding the long-time steady-state drift velocity $v_d(E)$. In Fig. 1, we show some examples of velocity overshoot for Si as determined by the Monte Carlo calculations of Ruch [15]. The classic origins of velocity overshoot, as determined from such Monte Carlo studies [15]-[17], can be separated into two distinct mecha-

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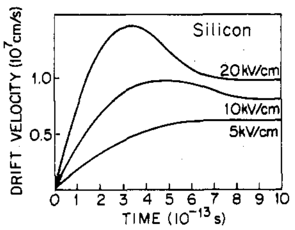

Fig. 1. Average electron velocity as a function of time for Si from Monte Carlo calculations [15].

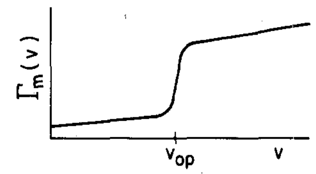

Fig. 2. Typical momentum relaxation rate Γ_m as a function of average electron velocity.

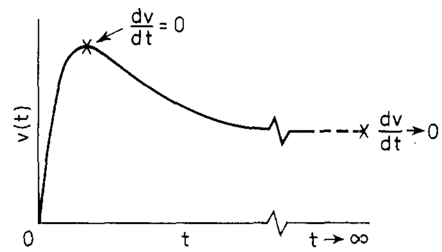

Fig. 3. Average electron velocity v as a function of time for an ensemble displaying velocity overshoot. The points where $dv/dt = 0$ are marked with a cross.

nisms: 1) a difference in momentum and energy relaxation rates; and 2) the presence of higher mass satellite conduction valleys. We review these as follows.

B. Velocity Overshoot for One-Valley Semiconductors

For the case where electron transport is mainly confined to one set of equivalent conduction valleys (such as Si or Ge), one can write moment equations for the average electron velocity v and energy ϵ

$$\frac{dv}{dt} = \frac{qE}{m} - v\Gamma_m(\epsilon) \tag{2a}$$

$$\frac{d\epsilon}{dt} = qEv - (\epsilon - \epsilon_0)\,\Gamma_e(\epsilon) \tag{2b}$$

where q is the electron charge, m the effective mass (parabolic dispersion is assumed), and ϵ_0 the equilibrium electron thermal energy. Γ_m and Γ_e are the momentum and energy relaxation rates, which are assumed to depend only on the instantaneous average energy ϵ.

Notice that for times $t \ll \Gamma_m^{-1}$ (2) reduces to the true-ballistic limit (1). However, in this case, the ballistic velocity $v \simeq qEt/m$ will be much less than the average drift velocity $v = qE/m\Gamma_m$. Thus the true-ballistic regime will not produce overshoot. For times $t < \Gamma_m^{-1}$ (2) describe the near-ballistic regime. Overshoot is possible in this regime. However, as we show below, when overshoot is present it is the interplay between Γ_m and Γ_e, and not the time scale $t \simeq \Gamma_m^{-1}$, that is important.

The intuitive argument [16] for the occurrence of velocity overshoot is as follows [19].[1] If $\Gamma_e \ll \Gamma_m$, we expect momentum to relax on time scales which are short compared to that on which the electron distribution heats up. If the average electron energy is initially ϵ_i, then when an electric field E is turned on the momentum will relax to give

$$v_i = \frac{qE}{m\Gamma_m(\epsilon_i)}. \tag{3}$$

At a later time when the electrons have heated up to their final drift energy $\epsilon_f > \epsilon_i$, the momentum relaxes to give

$$v_f = \frac{qE}{m\Gamma_m(\epsilon_f)}. \tag{4}$$

If Γ_m is an increasing function of ϵ, then $v_i > v_f$ and overshoot has occurred. Such is the situation for Si because the dominant collision mechanisms are scattering from acoustic phonons and equivalent intervalley scattering [20]. Scattering is largely isotropic and the relatively large number of backscattering events reduces momentum much more effectively than energy.

Although the peak velocities in the overshoot curves for Si (Fig. 1) occur at roughly one relaxation time after injection into the field, i.e., $t \simeq \Gamma_m^{-1}$, it is important to realize that the overshoot effect is produced by the crucial interplay between energy and momentum relaxation and not just the short time scale involved.

We illustrate this fact clearly by considering the case where the energy relaxation is tied to that of the momentum; then there is no overshoot. In this case, the energy is specified by the instantaneous velocity: $\epsilon(t) = \epsilon_0 + \frac{1}{2}\,mv^2(t)$; and the energy relaxation rate must be twice [due to v^2] the momentum relaxation rate. Then (2b) and (2a) coalesce into

$$\frac{dv}{dt} = \frac{qE}{m} - v\Gamma_m(v) \tag{5}$$

where we have explicitly acknowledged that the relaxation rate depends on the velocity. The near-ballistic region of (5) is again $t < \Gamma_m^{-1}$.

In Fig. 2, we show a typical functional form for the relaxational rate $\Gamma_m(v)$. We have included a step at $v = v_{op}$ corresponding to the onset of an extra collision process as the electron energy increases. The precise form of $\Gamma_m(v)$ will not play an important role in our arguments. We now demonstrate that no velocity overshoot can be obtained in the model (5). For velocity overshoot to exist, a plot of average velocity versus time must resemble that shown in Fig. 3. We see from Fig. 3 that such a solution for $v(t)$ must have two values of v where $dv/dt = 0$: one at the peak velocity and one at the long time limit (these points are marked by a cross in Fig. 3). Using (5a), the condition $dv/dt = 0$ can be rewritten as

$$v\Gamma_m(v) = qE/m. \tag{6}$$

[1] A more quantitative set of restrictions on the rates Γ_e and Γ_m in order to produce velocity overshoot is given in [19].

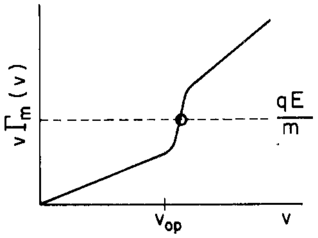

Fig. 4. The quantity $v\Gamma_m(v)$ plotted as a function of v. The intersections with the line qE/m gives all solutions v such that $dv/dt = qE/m - v\Gamma_m(v) = 0$.

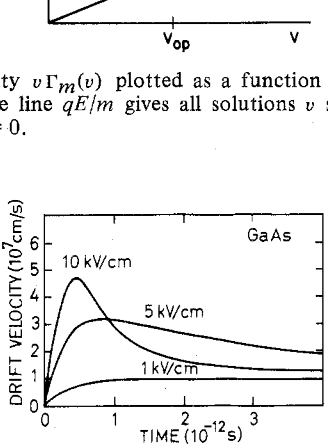

Fig. 5. Average electron velocity as a function of time for GaAs from Monte Carlo calculations [15]. The threshold for negative differential mobility sets in at $E_{th} \simeq 3$ kV/cm. For $E < E_{th}$ no velocity overshoot is seen.

Thus for overshoot to exist there must be two values of v that solve equation (6).

In Fig. 4, we plot the function $v\Gamma_m(v)$ versus v. The intersection of this curve with the line qE/m gives all possible solutions to (6). As is seen clearly, only one solution exists. The result holds generally, provided only that $v\Gamma_m(v)$ is an increasing function of v.

Thus in a one-valley model, no overshoot can exist either in the true-ballistic or near-ballistic regimes without the introduction of separate energy and momentum relaxation rates. We also note that it is possible, at least in theory, to make the peak velocity occur at several times Γ_m^{-1} away from injection ($t = 0$) by taking certain choices of Γ_m and Γ_e in (2) [19]. Whether such cases exist for real materials, however, remains to be seen.

C. Velocity Overshoot for Semiconductors with Satellite Conduction Valleys

For materials where significant conduction takes place in higher mass satellite valleys (such as GaAs or InP), (2) is inadequate. One must introduce separate equations for each valley with appropriate transfer rates coupling them [21].[2] Such complications introduce a second mechanism for achieving velocity overshoot.

In Fig. 5 we show curves of velocity overshoot for GaAs from the work of Ruch [15]. Monte Carlo simulations [15]-[17] of such materials show no overshoot until the electric field exceeds the threshold value E_{th} for negative differential mobility. As negative differential mobility is due to the transfer of electrons to the higher mass valleys, these calculations indicate that transfer is the key mechanism producing overshoot. At short times, the electrons remain in the central valley with light mass and, hence, achieve high velocities. However, once they achieve energies comparable to that of the satellite valley, transfer to the satellite valley takes place accompanied by a loss in velocity. At longer times, the heavy mass of the satellite valley prevents transferred electrons from reaching velocities comparable to those achieved in the central valley. Thus overshoot occurs.

An additional mechanism for overshoot of the central-valley electrons arises from the effect of successive scatterings in and out of the satellite valleys. This scattering process produces: 1) a rapid increase in the momentum relaxation at energies ($\sim$0.3 eV) appropriate to the in-and-out scattering and; 2) the condition $\Gamma_m \gg \Gamma_e$. Accordingly, velocity overshoot is then possible by the mechanism discussed in Section II-B. However, as shown in the work of Grubin *et al.* [22], this effect is small compared to that disussed first. Thus the major reason for the overshoot of the average electron velocity is the finite time required for the transfer of electrons to the satellite valleys.

In particular, contrary to comments made in the paper by Hess [5], we do not expect velocity overshoot to be associated with central-valley electrons at onset energy ($\sim$0.04 eV) for the emission of polar optical phonons, an energy which is far below the threshold energy ($\sim$0.3 eV) for transfer into satellite valleys. Since polar-optical-phonon scattering is peaked in the forward direction [20], the energy and momentum scattering rates are comparable. Thus by the arguments in Section II-B, no overshoot is expected, and this is confirmed by the Monte Carlo result [15]-[17] that overshoot is absent for electric fields $E < E_{th}$. In summary, all velocity overshoot effects in materials such as GaAs are a direct consequence of the presence of heavy mass-satellite valleys.

These observations may be relevant for devices. For the case of submicrometer GaAs devices, it should be possible to apply sufficiently low voltages to effectively confine the electrons to the high-mobility central valley while in the same device creating large electric fields $E > E_{th}$. Under such conditions, it should be possible to produce average velocities exceeding the maximum steady-state drift velocity $v_d(E_{th})$. Such an effect is observed in the calculations of Bosch and Thim [23]. In a future paper, we will treat the effect of a non-uniform field in a submicrometer device on velocity overshoot.

III. CONCLUSIONS

The origins of velocity overshoot and their relation to ballistic electron transport in uniform electric fields can be characterized by the following observations. Ballistic transport at short times does not by itself lead to large velocities. In the true-ballistic regime ($t \ll \Gamma_m^{-1}$), even though collisions will have a negligible effect on the motion of the electron, the times are so short that the electron will not be accelerated up to velocities comparable to the steady-state value. In the near-ballistic regime ($t < \Gamma_m^{-1}$), large velocities can be achieved through the mechanisms of velocity overshoot. However, these are not so much a manifestation of the rarity of collisions as of the subtle way collisions act in the approach to steady state. For semiconductors where conduction takes place in a single set of equivalent valleys, velocity overshoot is not possible unless

[2] Satellite valleys are treated by the addition of an energy dependent mass $m(\epsilon)$ in (2), in the paper by M. S. Shur [21].

momentum can relax faster than energy. For semiconductors, with heavier mass satellite valleys, larger than steady-state drift velocities can occur before the electrons have transferred to satellite valleys. For submicrometer devices it should be possible to apply low enough voltages such that the electrons will be unable to transfer to satellite valleys and still create large enough electric fields to produce large velocities.

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