Roughness of a tilted anharmonic string at depinning

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(Received 8 September 2003; revised manuscript received 18 March 2004; published 23 June 2004)

We consider the discretized model of a driven string with an anharmonic elastic energy, in a two-dimensional random potential, as introduced by [Rosso and Krauth, Phys. Rev. Lett. **87**, 187002 (2001)]. Using finite size scaling, we numerically compute the roughness of the string in a uniform applied force at the critical depinning threshold. By considering a string with a net average tilt, we demonstrate that the anharmonic elastic energy crosses the model over to the quenched KPZ universality class, in agreement with recent theoretical predictions.

DOI: 10.1103/PhysRevE.69.062105 PACS number(s): 05.60.Cd, 74.25.Qt, 02.70.Tt

Recently, Rosso and Krauth (RK) reported [1] simulations of roughening at the depinning threshold of a driven one-dimensional string in a two-dimensional (2D) random potential. Introducing higher order anharmonic terms to a quadratic elastic energy for the string, RK found a value for the roughness exponent $\zeta \simeq 0.63$, in contrast to the values $\zeta \sim 1.2$ found in earlier simulations [2], and recently theoretically [3], using a purely quadratic energy. RK noted that the value $\zeta \simeq 0.63$ had previously been found in some cellular automata models for depinning [4].

In a subsequent work [5] RK (with Hartmann) noted that that when an average tilt is applied to the string, the anharmonic terms break the rotational invariance present in the quadratic model, thus suggesting that the anharmonic terms might cross the model into the quenched Kardar-Parisi-Zhang (KPZ) universality class, previously introduced by Kardar [6] to explain the *anisotropic* depinning observed in the automata models. Simulations [7] of a continuum model with the quenched KPZ term found $\zeta \simeq 0.61 \pm 0.06$, consistent with the automata models and with the anharmonic model of RK. Most recently, a functional renormalization group calculation by Le Doussal and Wiese [8] argued that the quenched KPZ term can indeed be generated, not only by the anisotropic disorder considered by Kardar, but also by the anharmonic elastic energy terms introduced by RK.

A key prediction of Kardar's for anisotropic depinning is that the roughness exponent for a *tilted* interface will differ from that of an untilted one; for a tilted string in 2D, he predicted the exact value of $\zeta_{\rm tilt} = 1/2$. In this paper, by computing the $\zeta_{\rm tilt}$ of the RK model for the first time, we offer a direct numerical demonstration that their model does indeed belong in the quenched KPZ universality class.

Our model is the same as that of RK. We take for the energy of the string

$$E[h_i] = \sum_{i=0}^{L-1} \{V(i,h_i) - f h_i + E_{\rm el}(h_{i+1} - h_i)\}, \qquad (1)$$

where h_i is the integer height of the string at position $i = 0, \ldots, L$ on a discretized lattice, $V(i,j)$ is an uncorrelated random Gaussian potential with zero average and unit variance, f is a uniform external driving force, and $E_{\rm el}$ is the elastic energy of deforming the string. $V(i,j)$ is taken periodic on an $L \times L$ system size. In their work, RK used periodic boundary conditions. Here, to model a tilted interface with net slope s, we use boundary conditions $h_L = h_0 + sL$. Defining the height relative to a uniformly tilted line $\delta h_i \equiv h_i - si$, so that $\delta h_L = \delta h_0$, we can rewrite Eq. (1) in terms of the δh_i and recover the same model as RK except that the elastic term now has the form

$$\sum_{i=0}^{L-1} E_{\rm el}(\delta h_{i+1} - \delta h_i + s). \qquad (2)$$

We now carry out simulations of Eq. (1), using the elastic term of Eq. (2). Using the same algorithm [9] as RK, we consider slopes $s=0$ and $s=1$ for one of the specific cases studied in Ref. [1],

$$E_{\rm el}(\Delta) = \Delta^4/16. \qquad (3)$$

We compute the interface roughness W for a system of length L,

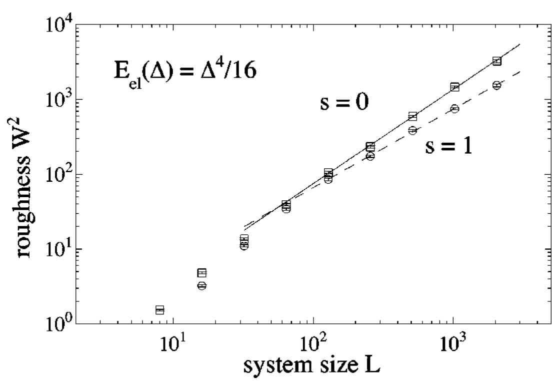

FIG. 1. Roughness W^2 versus system size L, for strings of net slope $s=0$ and $s=1$. The lines are the best fits to $W^2 \sim L^{2\zeta}$ using system sizes $L = 128-2048$.

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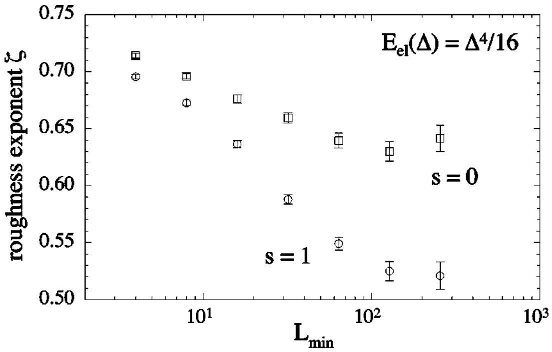

FIG. 2. Roughness exponents ζ for the tilted $(s=1)$ and untilted $(s=0)$ strings, as obtained by fitting the data of Fig. 1 to $W^2 \sim L^{2\zeta}$ using data for sizes $L_{\min}$ to 2048.

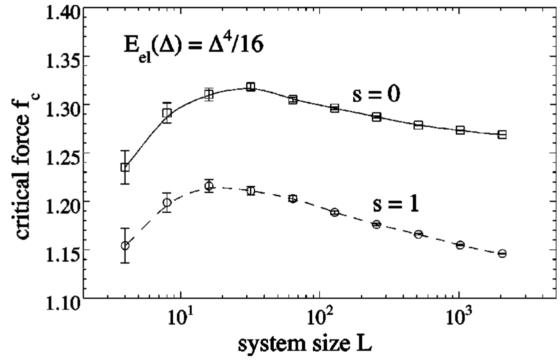

FIG. 3. Critical force f_c versus system size L, for strings of net slope $s=0$ and $s=1$. The lines are guides to the eye.

$$W^2 \equiv \frac{1}{L}\sum_{i=0}^{L-1} [(\delta h_i^c - \overline{\delta h^c})^2] \sim L^{2\zeta}, \tag{4}$$

where δh_i^c is the relative height at site i of the critical string at depinning, $\overline{\delta h^c}$ is the average relative height of the critical string, $[\cdots]$ represents an average over many realizations of the random potential $V(i,j)$, and ζ is the roughness exponent. We also compute the disorder average of the critical force, f_c.

Our results for string roughness W^2 versus L, averaged over 500 disorder samples (for $L=2048$ we use only 200 samples), are plotted in Fig. 1. For $s=0$, our numerical values agree with those in Ref. [1]. The straight lines on the log-log plot indicate the power law relation $W^2 \sim L^{2\zeta}$ and the difference in slopes indicate clearly different roughness exponents for the tilted $(s=1)$ and untilted $(s=0)$ strings.

To determine the values of the exponent ζ, we fit the data in Fig. 1 to $W^2 \sim L^{2\zeta}$, using system sizes from $L_{\min}$ to $L=2048$. We plot the resulting values of ζ versus $L_{\min}$ in Fig. 2, for $L_{\min}=4$ to 256. We see that as $L_{\min}$ increases, the values of ζ decrease and saturate to a constant value characterizing the roughness in the asymptotic large L limit. Using the results from fitting with $L_{\min}=128$ we find for $s=0$ the value $\zeta \simeq 0.63 \pm 0.01$. This value is used to plot the solid straight line in Fig. 1, and agrees with the value found by RK. For $s=1$, however, we find the value $\zeta = 0.52 \pm 0.01$. We use this value to plot the dashed line in Fig. 1. Given that ζ for $s=1$ still shows a small systematic decreases as $L_{\min}$ increases, we believe our value is in excellent agreement with the exact value of $1/2$ predicted by Kardar, and thus verifies that the anharmonic model of RK is in the quenched KPZ universality class.

Finally, in Fig. 3 we plot the critical force f_c as a function of system size L for the tilted and untilted strings. We see clearly that the critical forces approach different values as L increases, another signature of anisotropic depinning.

The authors thank T. Nattermann and Y. Shapir for helpful discussions. This work has been supported by DOE Grant No. DE-FG02-89ER14017 and NSF grant PHY-0242483.

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