We consider the discretized model of a driven string with an anharmonic elastic energy, in a two-dimensional random potential, as introduced by [Rosso and Krauth, Phys. Rev. Lett. 87, 187002 (2001)]. Using finite size scaling, we numerically compute the roughness of the string in a uniform applied force at the critical depinning threshold. By considering a string with a net average tilt, we demonstrate that the anharmonic elastic energy crosses the model over to the quenched KPZ universality class, in agreement with recent theoretical predictions.

DOI: 10.1103/PhysRevE.69.062105

PACS number(s): 05.60.Cd, 74.25.Qt, 02.70.Tt

Roughness of a tilted anharmonic string at depinning

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(Received 8 September 2003; revised manuscript received 18 March 2004; published 23 June 2004)

Recently, Rosso and Krauth (RK) reported [1] simulations of roughening at the depinning threshold of a driven one-dimensional string in a two-dimensional (2D) random potential. Introducing higher order anharmonic terms to a quadratic elastic energy for the string, RK found a value for the roughness exponent \( \xi = 0.63 \), in contrast to the values \( \xi \sim 1.2 \) found in earlier simulations [2], and recently theoretically [3], using a purely quadratic energy. RK noted that the value \( \xi = 0.63 \) had previously been found in some cellular automata models for depinning [4].

In a subsequent work [5] RK (with Hartmann) noted that that when an average tilt is applied to the string, the anharmonic terms break the rotational invariance present in the quadratic model, thus suggesting that the anharmonic terms might cross the model into the quenched Kardar-Parisi-Zhang (KPZ) universality class, previously introduced by Kardar [6] to explain the anisotropic depinning observed in the automata models. Simulations [7] of a continuum model with the quenched KPZ term found \( \xi = 0.61 \pm 0.06 \), consistent with the automata models and with the anharmonic model of RK. Most recently, a functional renormalization group calculation by Le Doussal and Wiese [8] argued that the quenched KPZ term can indeed be generated, not only by the anisotropic disorder considered by Kardar, but also by the anharmonic elastic energy terms introduced by RK.

A key prediction of Kardar’s for anisotropic depinning is that the roughness exponent for a tilted interface will differ from that of an untitled one; for a tilted string in 2D, he predicted the exact value of \( \zeta_{\text{anh}} = 1/2 \). In this paper, by computing the \( \zeta_{\text{anh}} \) of the RK model for the first time, we offer a direct numerical demonstration that their model does indeed belong in the quenched KPZ universality class.

Our model is the same as that of RK. We take for the energy of the string

\[
E[h_i] = \sum_{i=0}^{L-1} \{V(i,h_i) - fh_i + E_{\text{el}}(h_{i+1}-h_i)\}, \tag{1}
\]

where \( h_i \) is the integer height of the string at position \( i = 0, \ldots, L \) on a discretized lattice, \( V(i,j) \) is an uncorrelated random Gaussian potential with zero average and unit variance, \( f \) is a uniform external driving force, and \( E_{\text{el}} \) is the elastic energy of deforming the string. \( V(i,j) \) is taken periodic on an \( L \times L \) system size. In their work, RK used periodic boundary conditions. Here, to model a tilted interface with net slope \( s \), we use boundary conditions \( h_L = h_0 + sl \). Defining the height relative to a uniformly tilted line \( \delta h_i = h_i - si \), so that \( \delta h_{i+1} = \delta h_i + s \), we can rewrite Eq. (1) in terms of the \( \delta h_i \) and recover the same model as RK except that the elastic term now has the form

\[
\sum_{i=0}^{L-1} E_{\text{el}}(\delta h_{i+1} - \delta h_i + s). \tag{2}
\]

We now carry out simulations of Eq. (1), using the elastic term of Eq. (2). Using the same algorithm [9] as RK, we consider slopes \( s = 0 \) and \( s = 1 \) for one of the specific cases studied in Ref. [1].

\[
E_{\text{el}}(\Delta) = \Delta^4/16. \tag{3}
\]

We compute the interface roughness \( W \) for a system of length \( L \).

![Fig. 1. Roughness \( W^2 \) versus system size \( L \), for strings of net slope \( s = 0 \) and \( s = 1 \). The lines are the best fits to \( W^2 \sim L^{2.1} \) using system sizes \( L = 128 – 2048 \).](image)
We also compute the disorder average of the critical force, $f_c$, where $f_{c} = 0$ and $s = 1$. The values agree with those in Ref. [1], however, we find the value $\zeta = 0.52 \pm 0.01$. We use this value to plot the dashed line in Fig. 1. Given that $\zeta$ for $s = 1$ still shows a small systematic decreases as $L_{\text{min}}$ increases, we believe our value is in excellent agreement with the exact value of 1/2 predicted by Kardar, and thus verifies that the anharmonic model of RK is in the quenched KPZ universality class.

Finally, in Fig. 3 we plot the critical force $f_c$ as a function of system size $L$ for the tilted and untilted strings. We see clearly that the critical forces approach different values as $L$ increases, another signature of anisotropic depinning.

The authors thank T. Nattermann and Y. Shapir for helpful discussions. This work has been supported by DOE Grant No. DE-FG02-89ER14017 and NSF grant PHY-0242483.