Helicity Modulus and Meissner Effect in a Fluctuating Type-II Superconductor

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The helicity modulus for a fluctuating type-II superconductor is computed within the elastic medium approximation, as a probe of superconducting phase coherence and the Meissner effect in the mixed state. We argue that at the vortex line lattice melting transition, there remains superconducting coherence parallel to the applied magnetic field, provided the vortex line liquid retains a finite shear modulus at finite wave vector.

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In the high T_c superconductors, fluctuation effects are important over a wide region of the H-T phase diagram [1–4], resulting in the melting of the vortex line lattice well below the mean field H_{c2} line. It is generally believed that the resulting vortex line liquid is not truly superconducting, and has only a smooth crossover to the normal state [1,2]. Recently, however, Feigel'man and co-workers [5], using the 2D boson analogy, have argued that for large magnetic penetration length λ this line liquid can retain superconducting coherence in the direction parallel to the applied magnetic field. This conclusion has been supported by recent numerical simulations [6] in the special $\lambda \to \infty$ limit. Experimental evidence for such a possibility is suggested in new studies of MoGe/Ge multilayers [7]; below a well defined "decoupling temperature" T_D, resistivity perpendicular to H remains linear as in a vortex line liquid, while resistivity parallel to H shows the onset of strong nonlinearities indicating coherence between planes. To understand such experiments, it is therefore vital that one establish the properties of the thermodynamic states which may exist in the fluctuating vortex line system.

In the present work we present a simple, physically appealing, analytic demonstration that, for *any* λ, superconducting phase coherence parallel to the magnetic field does indeed exist in a *hexatic* vortex line liquid [8]; or more generally in any vortex line liquid state which retains a finite shear modulus at finite wave vector. Working within the elastic medium approximation we compute the helicity modulus [9], which is equivalent to the linear response coefficient between a perturbation in applied magnetic field and the resulting supercurrent. Hence it directly probes one of the most characteristic properties of a superconductor, the ability to screen out magnetic fields. Parallel to H, we find a total screening as in the Meissner effect, which we show persists into the hexatic line liquid state. Our calculation also helps clarify recent controversy [3,4,10–12] as to whether or not fluctuations destroy superconducting coherence even in the vortex line lattice state. For simplicity we carry out our calculation for an isotropic superconductor; the extension to the uniaxial case is straightforward. We work within the London approximation which is valid provided one is not too close to H_{c2}.

The Landau-Ginzburg Helmholtz free energy [13] for an isotropic uniform superconductor, within the London approximation of constant wave function amplitude, can be written as

$$\mathcal{H} = \frac{1}{2} J_0 \int d^3r \left\{ |\nabla\theta - \mathbf{A}|^2 + \lambda^2 |\nabla \times \mathbf{A}|^2 \right\}, \qquad (1)$$

where θ is the phase of the superconducting wave function, λ is the "bare" magnetic penetration length, $J_0 = \phi_0^2/16\pi^3\lambda^2$ with ϕ_0 the flux quantum, and $(\phi_0/2\pi)\mathbf{A}$ is the magnetic vector potential. $\nabla \times \mathbf{A} = 2\pi \mathbf{f}$, where $\mathbf{f}(\mathbf{r}) \equiv \mathbf{B}(\mathbf{r})/\phi_0$ is the local density of magnetic flux quanta. The partition function Z is computed averaging over *independently* [14] fluctuating θ and $\mathbf{A}$, subject to the constraint that $\langle \mathbf{f}(\mathbf{r}) \rangle = f\hat{\mathbf{z}}$ for a uniform average magnetic induction $B\hat{\mathbf{z}}$. In evaluating Eq. (1), the integration is to be cut off at the core of a vortex line, so that the energy stays finite.

In terms of the supervelocity $\mathbf{v} \equiv \nabla\theta - \mathbf{A}$, and its Fourier transform $\mathbf{v}_q \equiv \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}} \mathbf{v}(\mathbf{r})$, the helicity modulus is defined [9] as the linear response coefficient between induced supercurrent $J_0\mathbf{v}$ and an applied twist in phase. If we take $\mathbf{v}_q \to \mathbf{v}_q + \delta v_q \hat{\boldsymbol{\mu}}$ in Eq. (1), then the helicity modulus in direction $\hat{\boldsymbol{\mu}}$ is

$$\Upsilon_\mu(\mathbf{q}) \equiv \left. \frac{\partial^2 \mathcal{F}}{\partial(\delta v_q)^2} \right|_{\delta v_q = 0} = J_0 \left\{ 1 - \frac{J_0}{VT} \langle v_{q\mu} v_{-q\mu} \rangle \right\}, \quad (2)$$

where $\mathcal{F} = -T \ln Z$ is the total free energy, and $V = L_z L_\perp^2$ is the system volume. Because of the symmetry with which $\nabla\theta$ and $\mathbf{A}$ enter $\mathbf{v}$, Υ_μ equivalently gives the induced supercurrent that flows in response to an applied perturbation in magnetic field, given by the vector potential $\delta\mathbf{A}_q = \delta v_q \hat{\boldsymbol{\mu}}$. In evaluating $\Upsilon_\mu(\mathbf{q})$, the physically relevant case is the limit $q_\mu \to 0$. This follows from the convention of Baym [15], where to describe a system with a current flowing in the direction $\hat{\boldsymbol{\mu}}$, the appropriate thermodynamic limit is to take the system size $L_\mu \to \infty$ first, followed by $L_\nu \to \infty$ for the directions $\hat{\boldsymbol{\nu}} \perp \hat{\boldsymbol{\mu}}$ (equivalently, $\mathbf{q} \cdot \delta\mathbf{A}_q = 0$ in the London gauge).

Defining the vortex line density $\mathbf{n}$ by $\nabla \times \nabla\theta = 2\pi\mathbf{n}$, one can write an arbitrary configuration $\mathbf{v}_q$ in gauge invariant form as

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$$\mathbf{v}_q = 2\pi i \left\{ \mathbf{q}\chi_q + \frac{\mathbf{q}\times(\mathbf{n}_q-\mathbf{f}_q)}{q^2} \right\}. \qquad (3)$$

χ is a smooth function which gives the longitudinal part of $\mathbf{v}_q$, while the transverse part of $\mathbf{v}_q$ is determined by $\nabla\times\mathbf{v} = 2\pi(\mathbf{n}-\mathbf{f})$. Substituting Eq. (3) into the Hamiltonian (1), and decoupling the $\mathbf{n}_q$ and $\mathbf{f}_q$ degrees of freedom by completing the square in $\mathbf{f}_q$, we get

$$\mathcal{H} = \frac{2\pi^2 J_0}{VT}\sum_q \left\{ q^2\chi_q\chi_{-q} + \frac{1}{q^2}(1+\lambda^2q^2)\delta\mathbf{f}_q\cdot\delta\mathbf{f}_{-q} + \frac{\lambda^2}{1+\lambda^2q^2}\mathbf{n}_q\cdot\mathbf{n}_{-q}\right\}, \qquad (4)$$

where $\delta\mathbf{f}_q \equiv \mathbf{f}_q - \mathbf{f}_q^0$ is the fluctuation of the magnetic flux density away from the value $\mathbf{f}_q^0 = \mathbf{n}_q/(1+\lambda^2q^2)$, which minimizes the Hamiltonian for a given vortex configuration $\mathbf{n}_q$. The partition sum is now an average over all smooth functions χ, all $\delta\mathbf{f}_q$ such that $\mathbf{q}\cdot\delta\mathbf{f}_q = 0$ (so that $\nabla\cdot\mathbf{B}=0$), and all vortex configurations $\mathbf{n}$. The vortex line interaction in Eq. (4) is just the familiar London result [13,16].

The helicity modulus is evaluated by substituting Eq. (3) into Eq. (2), and using the Hamiltonian (4) to evaluate the averages over χ_q and $\delta\mathbf{f}_q$. Taking the limit $q_\mu \to 0$ one gets

$$\lim_{q_\mu\to 0}\Upsilon_\mu(\mathbf{q}) = \lim_{q_\mu\to 0}\frac{J_0\lambda^2q^2}{1+\lambda^2q^2}\left\{1 - \frac{4\pi^2J_0\lambda^2}{VT}\frac{(\hat{\boldsymbol{\mu}}\times\hat{\mathbf{q}})_\alpha(\hat{\boldsymbol{\mu}}\times\hat{\mathbf{q}})_\beta\langle n_{q\alpha}n_{-q\beta}\rangle}{1+\lambda^2q^2}\right\}. \qquad (5)$$

For an uncharged superfluid or spin model [9] phase coherence is indicated by a nonvanishing Υ_μ in the limit $\mathbf{q}\to 0$. For the superconductor, however, the gauge field $\mathbf{A}$ is free to adjust itself to screen out the applied phase twist (or perturbation $\delta\mathbf{A}$), and so even in the superconducting state $\Upsilon_\mu(\mathbf{q}\to 0) \sim q^2$, as seen in Eq. (5) above [17]. In fact, if no vortex lines are present ($\mathbf{n}_q = 0$), Eq. (5) just gives the familiar total screening response of the Meissner state [15]. With the presence of vortex lines in the mixed state, we can generalize the form of the Meissner response, by defining a renormalized coupling $(J\lambda^2)_R$ and penetration length λ_R such that

$$\lim_{q\to 0}\Upsilon_\mu \equiv \frac{(J\lambda^2)_R q^2}{1+\lambda_R^2q^2}, \qquad (6)$$

where λ_R and $(J\lambda)_R$ may depend on the direction $\hat{\mathbf{q}}$. Thus to examine superconductivity it is necessary to consider the form of Υ_μ at small but finite $\mathbf{q}$.

At high T, one can make a hydrodynamic approximation [18] and average over $\mathbf{n}(\mathbf{r})$ as if it was a continuous function, subject to the constraint that vorticity is conserved $\mathbf{q}\cdot\mathbf{n}_q = 0$. Using the Hamiltonian (4) one gets $\Upsilon_\mu(\mathbf{q}) = 0$ as expected. At low T, one can evaluate the vortex line correlations using the elastic approximation for small fluctuations about a vortex line lattice. If $\mathbf{u}_i(z)$ is the transverse displacement of vortex line i at height z from its position $\mathbf{R}_i$ in the vortex line lattice, then $\mathbf{n}(\mathbf{r}_\perp, z) = \sum_i \delta(\mathbf{r}_\perp - \mathbf{R}_i - \mathbf{u}_i(z))(\hat{\mathbf{z}} + d\mathbf{u}_i/dz)$. To evaluate Eq. (5) to lowest order in T, it is only necessary to consider the expansion of $\mathbf{n}_q$ to linear order in the strain, $\mathbf{q}\cdot\mathbf{u}_q$. For small $q > 0$ we have

$$\mathbf{n}_q = if(\mathbf{q}\cdot\mathbf{u}_q\hat{\mathbf{z}} - q_z\mathbf{u}_q), \qquad (7)$$

where $\mathbf{r}_\perp$, $\mathbf{R}_i$, and $\mathbf{u}_i$ lie in the x-y plane, q_z and $\mathbf{q}_\perp$ are the components of $\mathbf{q}$ parallel and perpendicular to $\hat{\mathbf{z}}$, and $\mathbf{u}_q \equiv f\sum_i\int dz\, e^{i(q_z z + \mathbf{q}_\perp\cdot\mathbf{R}_i)}\mathbf{u}_i(z)$. Correlations of $\mathbf{u}_q$ may be evaluated using the elastic Hamiltonian, as derived by Brandt [16],

$$\mathcal{H}_{\rm el} = \frac{1}{2V}\sum_q\left\{(c_{44}q_z^2 + c_{11}q_\perp^2)u_{qL}u_{-qL} + (c_{44}q_z^2 + c_{66}q_\perp^2)u_{qT}u_{-qT}\right\}, \qquad (8)$$

where u_{qL} and u_{qT} are the components of $\mathbf{u}_q$ parallel and transverse to $\mathbf{q}_\perp$, and $c_{44}(\mathbf{q})$, $c_{11}(\mathbf{q})$, and $c_{66}(\mathbf{q})$ are the tilt, compression, and shear moduli, respectively.

Substituting Eq. (7) into Eq. (5), and evaluating the displacement correlations using $\mathcal{H}_{\rm el}$, we find for perpendicular and parallel responses

$$\lim_{q_x\to 0}\Upsilon_x(\mathbf{q}) = \lim_{q_x\to 0}\frac{J_0\lambda^2q^2}{1+\lambda^2q^2}\left\{1 - \frac{B^2}{4\pi(1+\lambda^2q^2)}\frac{q^2}{c_{44}q_z^2 + c_{11}q_\perp^2}\right\}, \qquad (9)$$

$$\lim_{q_z\to 0}\Upsilon_z(\mathbf{q}) = \lim_{q_z\to 0}\frac{J_0\lambda^2q^2}{1+\lambda^2q^2}\left\{1 - \frac{B^2}{4\pi(1+\lambda^2q^2)}\frac{q_z^2}{c_{44}q_z^2 + c_{66}q_\perp^2}\right\}. \qquad (10)$$

For the transverse response $\Upsilon_x(\mathbf{q})$ there are two cases to consider: (i) $\mathbf{q} = q\hat{\mathbf{z}}$, and (ii) $\mathbf{q} = q\hat{\mathbf{y}}$. In (i) the perturbation $\delta\mathbf{A}_q$ gives a magnetic induction along $\hat{\mathbf{y}}$, oscillating in the $\hat{\mathbf{z}}$ direction. It is thus a tilt modulation of the original induction $B\hat{\mathbf{z}}$. In (ii), the perturbation gives a magnetic induction along $\hat{\mathbf{z}}$, which oscillates along $\hat{\mathbf{y}}$; it is thus a compression modulation of $B\hat{\mathbf{z}}$. Accordingly, Eq. (9) shows that in (i) Υ_x depends on c_{44}, while in (ii) Υ_x depends on c_{11}. We consider in detail case (i). A comparison of Eq. (9) with Eq. (6) shows that $(J\lambda^2)_R$ is determined by $c_{44}(\mathbf{q}=0)$, while λ_R is determined by $dc_{44}(0)/dq^2$. Using the result of Brandt [16],

$$c_{44} = \frac{B^2}{4\pi}\left[\frac{1}{1+\lambda^2q^2} + \left(\frac{dH_\perp}{dB_\perp} - 1\right)\right], \qquad (11)$$

we find

$$\frac{(J\lambda^2)_R}{J_0\lambda^2} = 1 - \frac{dB_\perp}{dH_\perp}, \qquad \frac{\lambda_R^2}{\lambda^2} = 1 - \frac{dB_\perp}{dH_\perp}. \qquad (12)$$

For an isotropic system, the factor $dH_\perp/dB_\perp$ in c_{44} above, where the derivative is evaluated at the average magnetic induction $B\hat{\mathbf{z}}$, is equal to the more familiar H/B [19]. The renormalization factor for the coupling $(J\lambda^2)_R$ has a simple physical interpretation. Since the induced magnetic induction is determined from Maxwell's equations as $\mathbf{A}_{\text{ind}} = -[\Upsilon_x(\mathbf{q})/J_0\lambda^2q^2]\delta\mathbf{A}$, Eq. (12) results in a fraction $dB_\perp/dH_\perp$ of the perturbation $\delta H\hat{\mathbf{y}} = \nabla \times \delta\mathbf{A}$ penetrating the superconductor, while the remainder is screened out as in the Meissner effect.

We now consider the more interesting parallel response Υ_z of Eq. (10). As long as the shear modulus c_{66} is finite as $q_z \to 0$, the term in Eq. (10) due to vortex line fluctuations vanishes, and one has total screening of the perturbation as in the Meissner state. Such total screening we take as the signature of superconducting coherence along the direction of the magnetic field. If, however, c_{66} is identically zero, $\Upsilon_z(\mathbf{q}_\perp)$ has exactly the same form as $\Upsilon_x(q\hat{\mathbf{z}})$, with the greatly reduced coupling $1 - dB_\perp/H_\perp$, resulting in the loss of total screening. A related analysis of order parameter correlations [12] has led Ikeda *et al.* to conclude that vortex line lattice melting [where $c_{66}(0) \to 0$] results in the loss of superconducting correlations along $\mathbf{H}$. However, a careful analysis of Eq. (10) shows that Υ_z continues to be unaffected by vortex line fluctuations provided that $c_{66}(q_z, \mathbf{q}_\perp)$ does not vanish as fast (or faster than) q_z^2, as $q_z \to 0$ for *finite* $\mathbf{q}_\perp$.

We now use this result to extend our analysis into the vortex line liquid state. For *long-wavelength* behavior (small q), Nelson has shown [20] that the average vortex line density, even in a liquid, may still be expressed in terms of a well defined strain field, and that the free energy of the line liquid can be expressed in terms of this strain field and effective elastic moduli, as in Eq. (8) (see also Ref. [18]). Furthermore, Marchetti and Nelson [8] show that a *hexatic* vortex line liquid, believed [8,21] to lie in between the line lattice and normal line liquid states, can be described by a continuum elastic theory in which one includes free dislocation loops. Averaging over these dislocations, they find in the $q \to 0$ limit, that vortex correlations are described precisely by an effective elastic theory in which the elastic moduli c_{11} and c_{44} are largely unchanged from the vortex lattice state, however, the shear modulus is renormalized to $c_{66}(q_z = 0, \mathbf{q}_\perp) \sim q_\perp^2$. Thus, while $c_{66}(0) = 0$ as expected for a liquid, c_{66} remains finite for finite $\mathbf{q}_\perp$. Applying Eq. (10), we therefore find total screening of the perturbation, and hence superconducting coherence parallel to $\mathbf{H}$, in the hexatic line liquid state [22]. This is the main result of our paper.

Continuing the expansion as in Eq. (7) to next order in the strain, we find a correction only to λ_R of order $\lambda_R^2/\lambda^2 \sim (3.8T/\pi J_0)\sqrt{B/\phi_0}$ (using $B \simeq 0.2H_{c2}$). Evaluating at the vortex line lattice melting temperature, which we find to be $T_M \sim 1.7c_L^2\pi J_0\sqrt{\phi_0/B}$ (where $c_L \sim 0.15$ is the Lindemann parameter), we find a small correction to λ_R^2 of order 15%. Thus the conclusions above from the lowest order expansion continue to hold [23].

If one considers the above calculation in the limit of an extreme type-II superconductor where $\lambda \to \infty$, and all fluctuations of the gauge field $\mathbf{A}$ are frozen out, we return to the case analogous to an ordinary superfluid. Taking the $\lambda \to \infty$ limit in Eqs. (9)–(11), we find that $\Upsilon_z(\mathbf{q} \to 0) = J_0$ is finite, and hence the system has phase coherence in the $\hat{\mathbf{z}}$ direction, even in the vortex line liquid state (provided $c_{66} > 0$ for $\mathbf{q}_\perp \neq 0$). Υ_x, however, vanishes at all temperatures, as $B = H$ when $\lambda \to \infty$. This explains the recent numerical results of Li and Teitel [6] in a lattice $\lambda \to \infty$ model. There Υ_z was found to vanish at a T_{cz}, well into the vortex line liquid state, while Υ_x vanished at a much lower $T_{c\perp}$, where the vortex line lattice melted. The finite Υ_x for this model at low T is due to the effects of pinning introduced by the discrete numerical mesh, which creates a finite energy barrier to small q elastic distortions. This effectively adds a q independent constant to the denominator of the second term in Eq. (9), so that as $q \to 0$, $\Upsilon_x = J_0$. Only when the vortex line lattice melts will thermal fluctuations dominate over pinning, and one recovers Eq. (12) with the resulting $\Upsilon_x = 0$.

Much work has been done using an analogy between fluctuating vortex lines, and the imaginary time world lines of two dimensional bosons [1,5]. Feigel'man *et al.* [5] have used this analogy to argue that in the large λ limit, there will exist a boson normal fluid phase intermediate between the boson lattice and the boson superfluid phases. They predict that the corresponding phase of the superconductor is characterized by coherence parallel to the applied magnetic field, but not transverse to it. It is interesting to examine this analogy within the above elastic approximation. A convenient expression for the 2D boson superfluid density has been given by Ceperley and Pollock [24] in terms of the "winding number" $\mathbf{W}$ of boson world lines, $\rho_s = mT\langle W^2\rangle/2\hbar^2$. If there are *only* magnetic field induced vortex lines fluctuating in a directed fashion [25] [i.e., a single valued displacement $\mathbf{u}_i(z)$],

$$\langle W^2\rangle = \frac{1}{L_\perp^2}\left\langle \left| \sum_i [\mathbf{u}_i(L_z) - \mathbf{u}_i(0)] \right|^2 \right\rangle = \frac{1}{L_\perp^2}\langle \mathbf{n}_{q=0}^\perp \cdot \mathbf{n}_{-q=0}^\perp \rangle, \qquad (13)$$

where $\mathbf{n}_{q=0}^\perp$ is the average vortex line density transverse to the average magnetic induction $B\hat{\mathbf{z}}$. Translating [1] from 2D bosons to vortex lines ($\hbar/T_{\text{boson}} \to L_z$, $\hbar \to T_{\text{vortex}}$, $m \to \epsilon_1 \sim \pi J_0$ the single vortex line tension), and evaluating $\langle W^2\rangle$ within the elastic approximation, one finds [26]

$$\rho_s = \lim_{q_\perp \to 0} \lim_{q_z \to 0} \frac{\epsilon_1 f^2}{2} \left\{ \frac{q_z^2}{c_{44}q_z^2 + c_{66}q_\perp^2} + \frac{q_z^2}{c_{44}q_z^2 + c_{11}q_\perp^2} \right\}. \quad (14)$$

The second term above always vanishes when one takes $q_z \to 0$ first, as c_{11} is always finite. The first term is just the same factor as appears in Eq. (10) for Υ_z. Hence in the vortex line liquid, if $c_{66}(\mathbf{q}_\perp \neq 0) > 0$, we have both total Meisner screening of perturbations $\delta A_{q_\perp}\hat{\mathbf{z}}$, and $\rho_s = 0$, consistent with the predictions of Feigel'man *et al.*

When c_{66} vanishes identically, Eq. (14) gives $\rho_s = \epsilon_1 f^2/2c_{44}(0)$. This result also follows from a direct evaluation of Eq. (13) within a hydrodynamic approximation [18]. If the 2D boson normal to superfluid transition is of the Kosterlitz-Thouless (KT) type, then the universal jump in ρ_s at the transition may be written [24] as $\langle W_c^2 \rangle = 4/\pi$. This gives a transition temperature for the vortex lines, $T_{\mathrm{KT}} = (\phi_0^2/\pi^2 L_z) dH_\perp/dB_\perp$. For $H \simeq H_{c1}$, this result gives $T_{\mathrm{KT}} \sim 1/B$ in good agreement with an earlier estimate by Nelson [27]. At larger B, where $dH_\perp/dB_\perp \simeq 1$, T_{KT} is independent of B. For sufficiently large L_z, however, this KT transition is presumably preempted by the transition to the hexatic vortex line liquid, in which $c_{66}(\mathbf{q}_\perp) > 0$.

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[26] The limit $q_z \to 0$ in Eq. (14) is not meant to imply that we are only interested in the $L_z \to \infty$ case, but only to indicate the correct way to obtain the $\mathbf{q} = 0$ correlation which defines W^2 in Eq. (13). That this is the correct order for the limits is seen by comparing the result of Eq. (14) when $c_{66} \equiv 0$, with the direct evaluation of Eq. (13) in the high T hydrodynamic approximation [18].

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