Comment on "Longitudinal Superconductivity in Vortex-Line Phases: A Monte Carlo Study"

In a recent Letter [1], Carneiro reports on new simulations of vortex-line fluctuations in a London type II superconductor with a finite magnetic penetration length λ. He concludes that superconductivity parallel to the applied magnetic field **H** ("longitudinal superconductivity") vanishes at the same temperature as vortex-line lattice melting. This conclusion contradicts similar finite λ simulations we reported on earlier [2], where we found evidence that (for $L_z < \phi_0^2/2\pi^2 T_m$) longitudinal superconductivity vanished at a T_c distinctly *above* the melting T_m. One main difference between the two simulations is that Carneiro uses an ensemble at fixed applied **H**, while we used an ensemble at fixed average internal.

Carneiro's simulations are for a higher vortex-line density than our own; this alone might lead to the apparent merger of T_c and T_m, as we have observed in earlier $\lambda \to \infty$ simulations [3]. However, we believe there is a more fundamental error in Carneiro's analysis.

Carneiro uses as his criterion for the presence of longitudinal superconductivity the vanishing of the transverse magnetic susceptibility $\mu_2 \equiv (\partial B_2/\partial H_2)|_{\mathbf{H}=H\hat{x}_3}$. μ_2 is proportional to the transverse fluctuation in vortex-line density $\mathbf{n} = \mathbf{B}/\phi_0$. Carneiro states that "If the system behaves like a superconductor for currents parallel to the external field..., a field perpendicular to it is shielded by the Meissner effect and μ_2 vanishes." This, however, is incorrect. For a periodic system with no true surface, such as Carneiro's model, adding a uniform transverse magnetic field does not imply the existence of current flowing parallel to the original field. It represents merely a tilting of the original field, and thermodynamic arguments [4] show that μ_2 is related to the tilt modulus at zero wave vector, $\mu_2 = B^2/4\pi c_{44}(0)$. For a uniform superconductor (no pinning), $c_{44}(0)$ is finite in both vortex-line lattice and vortex-line liquid. Hence μ_2 should *nowhere* vanish, and so is not a measure of superconductivity at all. The rapid rise in μ_2 that Carneiro observes at T_m is, we believe, a consequence of the fact that, for such high vortex-line densities, vortex-lattice melting coincides with a depinning of the vortex lines from the numerical grid of sites that is used to discretize the continuum in the simulation. In the vortex lattice, the pinned vortex lines are stiff to tilting, so c_{44} is large and μ_2 is small; in the vortex liquid (or a vortex lattice in a *continuum*), $c_{44} \sim B^2/4\pi$ [5] and $\mu_2 \sim 1$.

Carneiro says that his computed μ_2 is related to the vortex-line winding number W^2 that determines the superfluid density $\rho_s^{2\mathrm{D}}$ [6] in the analog 2D boson system [7]. However, Carneiro's scheme for allowing vorticity in the $\hat{x}_2$ direction to fluctuate, by inserting half vortex loops in from the planes on the $\hat{x}_1$ sides of the system. Carneiro's scheme corresponds to *porous* walls which freely allow bosons to enter and leave the system, rather than impermeable walls which viscously clamp the normal component of the 2D boson system. With such boundary conditions, there is no reason to believe that Carneiro's μ_2 is measuring $\rho_s^{2\mathrm{D}}$.

A correct criterion for longitudinal superconductivity has been derived in Refs. [2,8]: For a translationally invariant ensemble at fixed $\mathbf{B} = B\hat{x}_3$, superconductivity is indicated by the vanishing of the vortex correlation $n_0 \equiv \lim_{q\to 0}\langle n_2(q\hat{x}_1)n_2(-q\hat{x}_1)\rangle$. In contrast, μ_2 is proportional to $\lim_{q\to 0}\langle n_2(q\hat{x}_3)n_2(-q\hat{x}_3)\rangle$, and our computation of μ_2 by this expression (denoted as "$1-\gamma_y$" and shown in Fig. 2 of Ref. [2]) agrees qualitatively with Carneiro's result, i.e., there is a rapid rise towards unity near T_m. The direction in which $\mathbf{q} \to \mathbf{0}$ is crucial to the distinction between n_0 and μ_2. n_0, defined as the above $q \to 0$ limit, also agrees precisely with the path integral formulation of the usual definition of $\rho_s^{2\mathrm{D}}$ in terms of the transverse momentum correlation function [9]. The value of this finite q correlation should be independent of the choice of fixed **B** versus fixed **H** ensemble; hence, in the limit $q \to 0$, we should recover the correct value of W^2 that one would find in the fixed **H** ensemble.

To conclude, Carneiro's results, when properly interpreted, are consistent with our own. However, they do not correctly address the question of longitudinal superconductivity.

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