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Vortex lattice melting in 2D superconducting networks

M. Franz[a], S. Teitel[b,*]

[a] Institute for Materials Research and Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, L8S 4M1 Canada
[b] Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627, USA

Abstract

We use Monte Carlo simulations to study the melting of vortex lattices in two-dimensional square superconducting networks, for weak magnetic fields ($f = 1/n$), and for magnetic fields near full frustration ($f = \frac{1}{2} - 1/n$). We find distinct pinned vortex lattice, floating vortex lattice, and vortex liquid phases.

1. Introduction

When an external magnetic field is applied perpendicular to the plane of a two-dimensional (2D) superconducting network (such as a Josephson junction array, or wire net), it induces a fixed density of vortices into the phase angle of the superconducting wave function. At low temperature, the vortices form a periodic lattice; at high temperature, they form a vortex liquid. The free diffusion of vortices gives rise to "flux flow" resistance causing a transition between a true superconducting phase and a normal phase. An early conjecture [1] by Teitel and Jayaprakash (TJ) argued that the superconducting transition would be governed by commensurability effects. For a vortex density $f = p/q$, they predicted that the transition temperature would vary as $T_c(p/q) \sim 1/q$. We present here the results of Monte Carlo simulations which explore the phase transition in vortex structure for two special cases of a square network: (i) a dilute density $f = 1/n$, and (ii) a high density near full frustration $f = 1/2 - 1/n$. Our results have been presented in greater detail elsewhere [2]. Here we summarize these results, and add some additional comments.

* Corresponding author.

2. The model

Vortices in a square 2D superconducting network can be mapped [3] onto the one component *Coulomb gas*,

$$\mathscr{H} = \frac{1}{2} e^2 \sum_{i,j} (n_i - f) V(\boldsymbol{r}_i - \boldsymbol{r}_j)(n_j - f). \qquad (1)$$

Here, i and j label the *dual* sites of the network, where the vortices are located; *charges* $n_i = 0, +1$ indicate the absence or presence of a vortex at site i; $f = Ba^2/\Phi_0$ is the number of flux quanta (Φ_0) of applied magnetic field B, per unit cell of the square network with lattice constant a. $V(\boldsymbol{r})$ is the solution to the lattice Laplacian, $\Delta^2 V(r) = -2\pi\delta_{r,0}$, which gives the 2D logarithmic Coulomb potential with which vortices interact. The magnitude of the *charges* is given by $e^2 = 2\pi J$, where J is the coupling constant of the superconducting network bonds. Because of the long range nature of the interaction, one is constrained to configurations in which $\sum_i n_i = Nf$.

Our simulations will be carried out in this Coulomb gas formulation. Temperature will be measured in units such that $e^2 = 1$. The main quantity we calculate, in order to characterize the different vortex phases, is the structure function,

$$S(\mathbf{k}) = \frac{1}{Nf}\sum_{i,j} e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \langle n_i n_j \rangle, \tag{2}$$

where N is the number of sites in the network. $S(\mathbf{k})$ gives the diffraction pattern that would be obtained from scattering off the vortex positions.

3. Dilute vortex density

For a dilute vortex density $f = 1/n$, the ground state will be the closest approximation to a triangular vortex lattice that is commensurate with the square geometry of the underlying network [4]. We consider here the specific case $f = 1/51$. The ground state, for a 51×51 square network, is the almost-square vortex lattice shown in Fig. 1(a). Heating from this ground state, we show in Fig. 1(b)–(d) intensity plots of $S(\mathbf{k})$, for $k_x, k_y \in (-\pi/a, \pi/a)$, at three representative temperatures. At low T, Fig. 1(b), we see a periodic lattice of *sharp* Bragg peaks, reflecting the *long range* translational order of the vortices which remain *pinned* to the network in their ground state structure. In this phase, the network is truly superconducting with zero linear resistivity.

At the intermediate, T, Fig. 1(c), we see a triangular lattice of peaks of *finite width*. This is characteristic of the *quasi-long range* translational order expected for a 2D vortex lattice in a uniform continuous film [2, 5]. Note that the peaks in Fig. 1(c) are in distinctly different locations compared to the peaks in Fig. 1(b). We conclude that a transition has occurred from a pinned commensurate almost-square vortex lattice, to a floating incommensurate triangular vortex lattice. Although the translational coupling to the network has been destroyed, there remains a strong orientational coupling [5]. The minimum energy corresponds to the case where one of the three basis directions of the triangular vortex reciprocal lattice aligns with one of the two diagonal directions of the square network. This results in two possible distinct orientations for the floating vortex lattice (one of which is clearly seen in Fig. 1(c)) which break the cubic symmetry of the network geometry. In this floating phase, the vortex lattice is free to diffuse as a whole, and will therefore give "flux flow" resistance in the presence of any applied D.C. current. Thus the floating lattice phase is no longer truly superconducting. However, the breaking of cubic symmetry due to the orientational coupling of the vortex lattice to the network, will lead to an anisotropic mobility for the vortex lattice. The result will be an angular dependent resistivity, and in the case that the applied current is not aligned with a symmetry direction, a non-zero Hall voltage.

Finally, at high T, Fig. 1(d), we see the approximately circular intensity rings of a vortex liquid. The 4-fold asymmetry in these rings is due to the square geometry of the underlying network. As the cubic symmetry, broken by the floating vortex lattice, is now restored, we expect to see an isotropic resistivity in this phase, with a vanishing Hall voltage. The vanishing of the Hall voltage therefore should serve as a clear experimental signal for the melting of the floating vortex lattice.

We have carried out similar simulations for other values of $f = 1/n$. The resulting phase diagram is shown in Fig. 2. $T_c(f)$ denotes the transition between the pinned and floating vortex lattices. As $f \to 0$, we find that $T_c(f) \sim f$ vanishes, in agreement with the TJ conjecture. $T_m(f)$ denotes the melting temperature of the floating vortex lattice. As $f \to 0$, we find that $T_m \simeq 0.007$ approaches a finite constant, in agreement with the theory of vortex lattice melting in a continuous film [6]. When the vortex density is too large, $f > 1/30$, T_c and T_m merge, and there is only a single transition from pinned vortex lattice to vortex liquid.

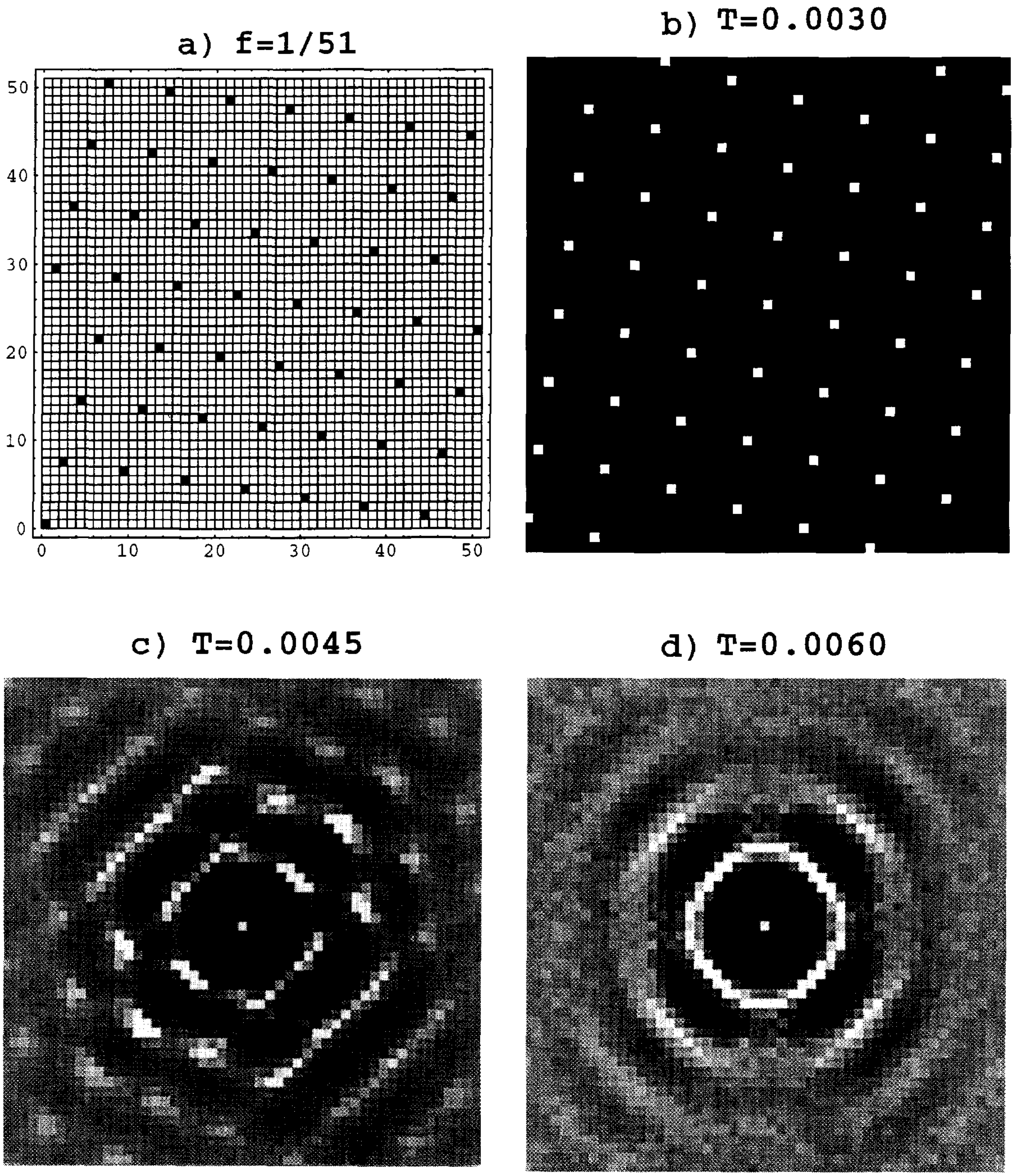

Fig. 1. Dilute vortex density $f = \frac{1}{51}$: ground state vortex structure (a) – a solid square denotes the location of a vortex; structure function $S(\boldsymbol{k})$ at (b) $T = 0.003$; (c) $T = 0.0045$; (d) $T = 0.006$.

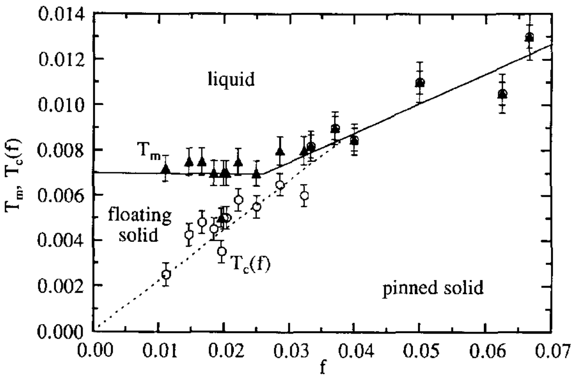

Fig. 2. Phase diagram for densities $f = 1/n$. T_c denotes the unpinning transition, while T_m denotes the melting transition.

4. Vortex density near full frustration

For a vortex density $f = \frac{1}{2} - 1/n$, the ground state is almost everywhere like the $f = \frac{1}{2}$ checkerboard pattern of a vortex on every other site, except with a superlattice of missing vortices (*defects*). We consider here the specific case $f = \frac{5}{11}$, whose ground state [7] is shown in Fig. 3(a). Heating from this ground state, we show in Fig. 3(b)–(d) intensity plots of $S(\boldsymbol{k})$ at three representative temperatures. At low T, Fig. 3(b), we find a periodic structure of sharp Bragg peaks. Note that the peaks in the corners arise from the $f = \frac{1}{2}$-like background; these are brighter than the other peaks, which arise from the defect superlattice.

At an intermediate T, Fig. 3(c), we continue to see sharp Bragg peaks in the corners; however, the other peaks have been replaced by circular rings. The defect superlattice has melted into a defect liquid, but the $f = \frac{1}{2}$-like background remains ordered. Finally, at high T, Fig. 3(d), the peaks in the corners broaden, the $f = \frac{1}{2}$-like background has melted, and one finds an isotropic vortex liquid.

Looking more closely at Fig. 3(c) for the defect liquid, we see that $S(\boldsymbol{k})$ is symmetric with respect to the Bragg planes that bisect the diagonals from the origin to the corners. This indicates that the defects, while freely diffusing, are still constrained to sit on only one sublattice of the original square network of sites; equivalently, one never has two vortices on two nearest neighbor sites. ,Since this sublattice has $\frac{1}{2}$ the number of sites as the original network, and since the defects interact with the same logarithmic interaction as to vortices, the problem of $f = \frac{1}{2} - 1/n$ at low temperatures becomes equivalent to that considered in the previous section: the $f = \frac{1}{2}$-like background remains ordered and can be ignored; the dilute density of mobile defects, behaves in the same way as a dilute density of vortices with $f' = 2/n$. This leads us to expect that for n sufficiently large, one will find an additional phase not observed for $f = \frac{5}{11}$. Upon heating, one will first have a transition T_c from a pinned to a floating defect superlattice, followed by a melting transition T_m to a defect liquid, followed finally by the melting $T_{m'}$ of the $f = \frac{1}{2}$-like background into an isotropic vortex liquid. The phase boundaries for T_c and T_m may be inferred from Fig. 2. T_c will thus again with the TJ conjecture. The boundary for $T_{m'}$ remains in general unknown.

5. Discussion

In light of the behavior observed for $f = \frac{1}{2} - 1/n$, it is interesting to speculate what might happen near other simple rational f. For example, the ground state for $f = \frac{1}{3} - 1/n$, for n sufficiently large, is likely to consist of an $f = \frac{1}{3}$-like background (which has diagonal stripes of vortices at every third site), with a superlattice of missing vortices (defects). A schematic of such a case is shown in Fig. 4. At low T, we expect that this defect superlattice is pinned, and the network is truly superconducting. Increasing T, we expect that the defect superlattice will unpin. For large enough n however, the floating defect lattice is most likely to be free to move only in the diagonal direction of the $f = \frac{1}{3}$-like background: i.e. vortices in the $f = \frac{1}{3}$-like background may hop to a vacancy site within a given diagonal stripe, but will not hop to a site between adjacent stripes. The result will be a state which remains superconducting in one direction, but has flux flow resistance in the orthogonal direction. At higher T, the floating defect lattice will melt into a defect liquid with very anisotropic mobility,

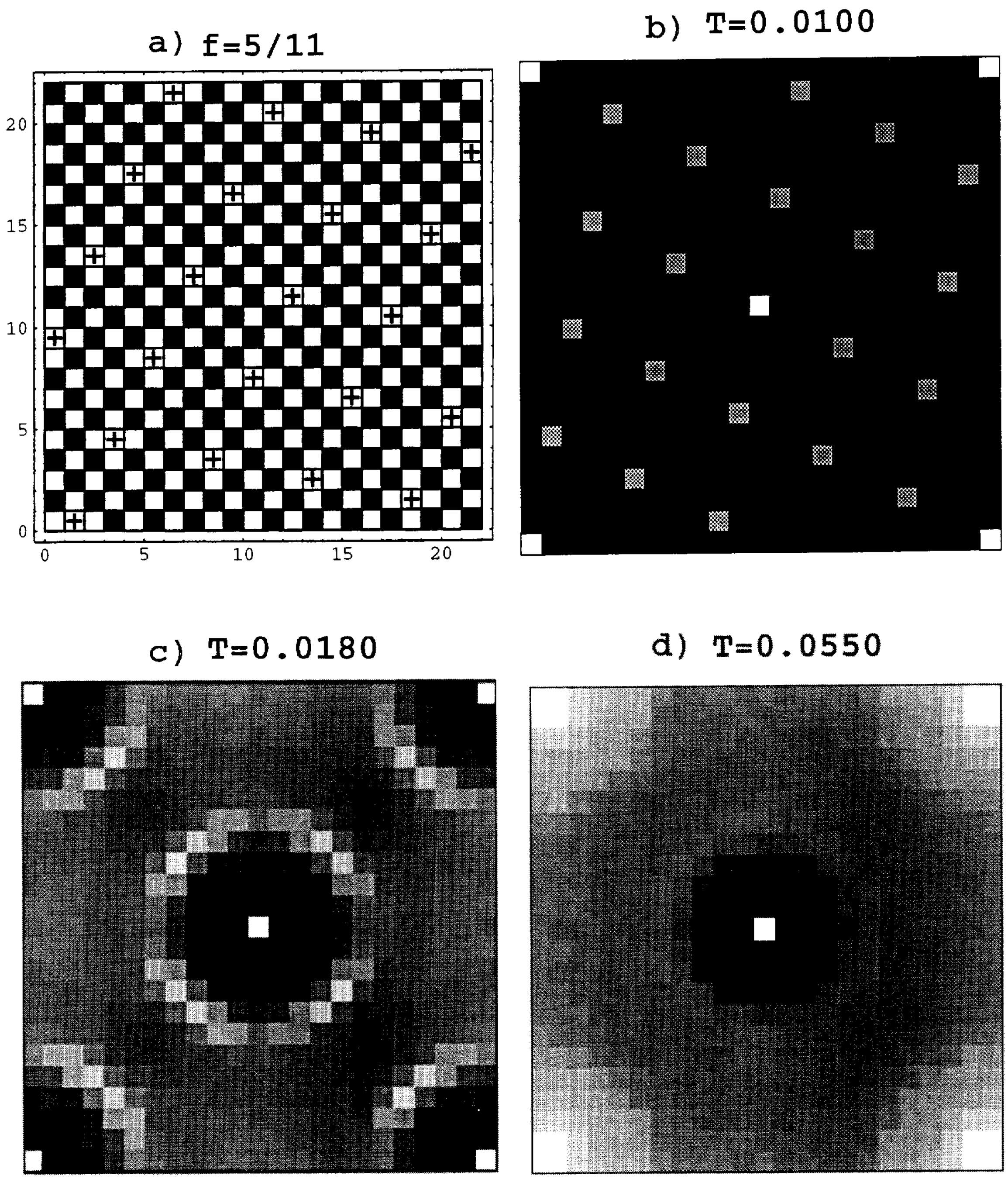

Fig. 3. Vortex density near full frustration, $f = \frac{5}{11}$: ground state vortex structure (a) – a solid square denotes the location of a vortex, a " + " denotes a missing vortex (defect) in the $f = \frac{1}{2}$-like background; structure function $S(\boldsymbol{k})$ at (b) $T = 0.01$; (c) $T = 0.018$; (d) $T = 0.055$.

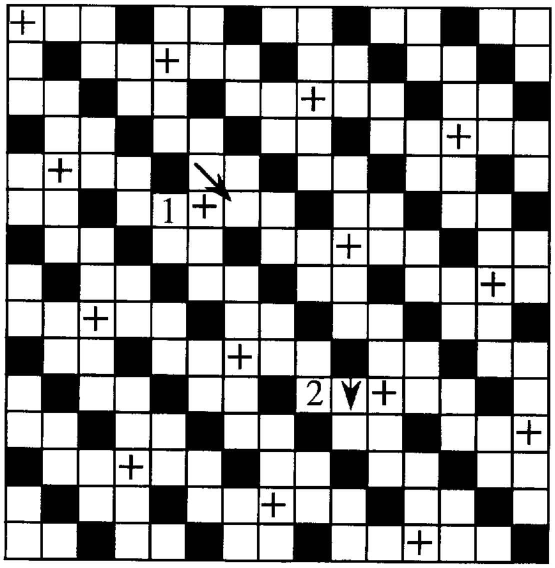

Fig. 4. Possible vortex ground state for $f = \frac{1}{3} - 1/n$; a solid square denotes the location of a vortex, a " +" denotes a missing vortex (defect) in the $f = \frac{1}{3}$-like back-ground. When the defect superlattice unpins, vortex hops of type 1 will dominate over hops of type 2. (In Ref. [4] it is argued that, near $f = \frac{1}{3}$, a defect superlattice will in fact be the correct ground state only at a larger value of n than is illustrated above.)

and only at a still higher temperature will one find an isotropic vortex liquid. The unpinning transition is likely follow the TJ conjecture.

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