

7. Electric Currents

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An electric current is produced by the movement of electric charges. In most cases these are electrons. A conductor is a material through which an electric charge can move easily. Metals are conductors, copper being the most commonly used to make electrical wires.

Consider electrons moving through a copper wire. Even in a good conductor like copper they will encounter many obstacles to their movement: other electrons, and atoms get in the way. So unlike in empty space, an electron does not keep moving once it starts: it is brought to a stop by friction, much like a person caught in a crowd. To keep an electron moving we must exert a force on it, usually with an electric field.

If we connect the two ends of the wire to the poles of a battery, we can create a potential difference and hence an electric field inside the wire. The electrons will then be able to move to the positive end of the battery from the negative end, completing a circuit. The larger the potential difference, the larger the current.

1 Ampere

To make matters more precise we will need a unit of electric current. One way would be to just count how many electrons cross some point of the wire per unit time. It would be like counting how many cars go by a bridge with one way traffic. That is likely to be a huge number; so we instead measure how many Coulombs of charge cross a point per sec. This is the unit of electric current called an *Ampere*; often abbreviated to amp or just A .

While one Coulomb is a huge amount of free charge, it is not unusual for one Coulomb (about 6×10^{19} electrons) to go through a wire in a second. A car battery provides about 200 A of current for few seconds to start the car.

Electric current is usually denoted by the letter I , just as electric potential is denoted by V .

2 Ohm's Law

The larger the potential difference the larger the current passing through a conductor. To a good approximation they are proportional:

$$V = RI$$

The constant of proportionality R is a property of the conductor: it is called resistance. It depends on the length and area as well as on the material it is made of.

An insulator has very large resistance, while good conductors like metals have low resistance.

The unit of resistance is *Ohm*, often abbreviated to Ω (the Greek letter Omega, but pronounced Ohm in this context.)

$$\text{Ohm} = \frac{\text{Volt}}{\text{Amp}}.$$

The human body has a resistance of about a 1000 Ω ; men usually have lower resistance than women, as thicker arms and legs allow passage of electricity easier. A current of 10 A is usually lethal. So even a potential difference of a 100 V can give you an unpleasant electrical shock.

Ohm's Law is Not A Fundamental Law of Nature Unlike Coulomb's law or Gauss' law, Ohm's law is not a fundamental law of nature. It is merely an approximation. If you pass a large enough current you will heat the wire and its resistance will go up, so V and I are not strictly proportional. Gauss's law is however, a fundamental law which always holds: we have not found even the slightest variation from it in a hundred and fifty years of careful observation.

3 Power

Suppose there is a current I passing through a wire; the two ends of the wire are at a potential difference of V . Now recall that when a charge Q moves over a potential difference V it gains an energy of QV . Current is just the amount of charge moving across the conductor per unit time. Thus the amount of energy gained by the charges per unit time is IV : this is the power.

Now, this power is dissipated or wasted: it is turned into heating the conductor. There is no simple way of converting it back into electricity. In fact electrical resistance is caused by this conversion of electrical energy into heat.

The power dissipated in a conductor is, using Ohm's law

$$P = IV = RI^2 = \frac{V^2}{R}.$$

At a fixed current the higher the resistance the greater the dissipated power; at a fixed voltage, power is greater for low resistance.

4 Alternating Current

Most household devices (unlike portable ones) run on a alternating current. In these devices, the current is not a constant: it even reverses direction many times a second. The current is driven in one direction then the other, so that the current is a periodic function of time. The period is $\frac{2\pi}{\omega}$; the frequency is $\frac{\omega}{2\pi}$. The voltage

$$V(t) = V_0 \sin \omega t$$

driving the current is also a function of time, and the resistance is also a function of time. The resistance of appliances does not change with time. Because the power dissipated is proportional to $V^2(t)$, it is useful to find the average of the square of the voltage: $\frac{1}{2}V_0^2$. The square root of the mean of the square (r.m.s) is thus $\frac{V_0}{\sqrt{2}}$. This is what people usually mean when they talk of the “voltage” of an AC power source.

The frequency of the electrical grid in the US is 60 Hz; that is the current reverses itself 60 times a second. The peak voltage V_0 at an outlet is 170V; people usually refer instead to the r.m.s. voltage which is 120V. In other countries (Europe and Asia) it is usually 50 Hz and an rms voltage of 240V.

Alternating current is more convenient to transport over an electrical grid: it is possible to change the voltage of AC using a device called a transformer. This is harder to do with direct current (current that is constant in time). The large electrical towers that bring the electricity into a city from a power plant are at very high voltage (120kV or more). This would be too dangerous to use in homes, so transformers are used to step it down to a lower voltage in several stages.

Why would you want the voltage to be so high in the grid? It is a way to minimize the losses due to resistance in the electrical transmission wires while still supplying a constant amount of power to the consumers.

We will see how transformers work later in the course.

5 Resistivity

Electric current is much like traffic: the conducting wire is like a highway. If the wire is broader (has larger cross-sectional area) it is like having more lanes of traffic. So the resistance is inversely proportional to the area A . Also, the longer the wire is the greater the resistance. To get a quantity that is independent of the shape of the wire and only depends on the material it is made of, we must take the combination

$$\rho = R \frac{A}{l}.$$

This is called resistivity. Its inverse $\sigma = \frac{1}{\rho}$ is called conductance. For example the resistivity of Cu is $17.2 \times 10^{-9} \Omega m$. An insulator like glass has much larger $\rho \approx 10^{10}$ to $10^{14} \Omega m$. The only difference between a conductor and an insulator is in the value of the resistivity. But notice the enormous difference.

6 Temperature Dependence of Resistivity

The hotter a material gets, the more its atoms move around. So the greater the chances that an electron moving through it will be knocked off its path. Thus resistance increases with temperature. For small changes in temperature this change in resistivity will be linear:

$$\rho(T) = \rho(T_0) [1 + \alpha(T - T_0)].$$

The constant α is a property of the material and is called the thermal coefficient of resistance. For Cu, $\alpha = 3.0 \times 10^{-3}/\text{Celsius}$.

This temperature dependence is useful to make thermometers: small changes in resistance are easy to measure.

7 Resistors in Series

If we have a current I that passes through a resistance R_1 the potential difference across it is $V_1 = R_1 I$. If the same current passes through another resistance R_2 , there is an additional potential difference of $V_2 = R_2 I$. Thus the total potential difference is $V = (R_1 + R_2)I$. The two resistances are equivalent to a single resistance

$$R = R_1 + R_2.$$

In this case they are said to be in series.

A long conductor can be thought of as two shorter ones in series. You can now see why the resistance is proportional to the length: imagine the conductor as cut into pieces of equal length, connected in series.

8 Resistors in Parallel

Suppose two resistors are connected so that the potential difference across them is the same. The current will split into two: the larger current will pass through the lower resistance

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}$$

The total current is thus

$$I = V \left[\frac{1}{R_1} + \frac{1}{R_2} \right] = \frac{V}{R}.$$

It is as if the highway splits into two and rejoins later: the traffic will split into two, more cars will go to the branch that has more lanes.

The net resistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

In this case the resistances are said to be in parallel.

A broad conductor can be thought as in parallel: divide it up into pieces with equal area to see that the resistance is inversely proportional to the area.

9 Contrast Capacitors and Resistors

Note that the rules for adding capacitors in series is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

while adding them in parallel is

$$C = C_1 + C_2.$$

These are the opposite of the rule for conductors. Beware of this in solving problems.

10 Kirchoff's Laws

If an electrical wire is split into two, the sum of the currents flowing through each branch will be equal to the original. More generally,

1. The total current coming into any junction will equal to what flows out.

This is an example of a conservation law. It is a fundamental law of nature that net electrical charge cannot be created or destroyed: the total charge flowing in must equal what flows out.

The next law of Kirchoff is essentially the statement that the electric force is conservative.

2. The sum of the potential differences around any closed loop is zero.

We can understand the above formulas for series and parallel resistors as a consequence of these laws. Any complicated circuit of resistors and capacitors can be analyzed (taken apart and put together mentally) using these laws.

11 The Direction of Electric Current

Most electric currents are caused by the flow of electrons. Thus they flow from the negative side of a battery to the positive side. Before they understood this our ancestors arbitrarily decided that the direction of the current should be considered as from positive to negative. It didn't matter as long as you don't get down to the level of the smallest charges, the elementary particles. They had a 50% chance of getting it right and they goofed. So we have to live with the consequences. Always remember that the direction of the current in a wire is the opposite of the direction in which electrons are actually moving.

There are materials in which both positive and negative charges move. In an electrolyte (like a salt solution or acid) there are positive and negative ions which can both move. In this case the total current is the number of positive ions that cross a surface - the number of negative ions that cross in the same direction. If the negative ions are crossing in the opposite direction, they add to the current.