

13. Faraday's Law

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1 Electromotive Force

If a coil moves (or rotates) near a magnet, a current is induced on it, even if it is not connected to a battery. That means an electric field is produced by a time-dependent magnetic field. Only static (time-independent) electric fields are conservative: the electric field produced by a time-dependent magnetic field is not the gradient of a potential. A positive charge moved in a complete loop in such a magnetic field can have a net gain or loss of energy. This energy gained in a complete circuit, per unit charge, is called the electromotive force. This is what Faraday, who discovered the phenomenon called it. But it is a confusing name: the electromotive force is not a force at all. It is instead the energy per unit charge gained or lost around a complete loop:

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l}.$$

2 Magnetic Flux

The crossing a small rectangle is equal to the area of the rectangle times the magnetic field at its location times the cosine of the angle between the magnetic field and the normal to the area. In other words,

$$d\Phi_M = \mathbf{B} \cdot d\mathbf{A}.$$

Recall that the area of a rectangle is a vector pointed in the direction of the normal.

The total flux is obtained by dividing up the area into small rectangles and adding up the flux from each. This is the same as the integral

$$\Phi_M = \int \mathbf{B} \cdot d\mathbf{A}.$$

The same idea as the electric flux. The flux through a coil of electrical wire can change with time if

1. the magnetic field changes, or if

2. the area changes, (a coil of wire expands or shrinks or moves in and out the field) or if
3. the angle between the area and the magnetic field: the coil of wire rotates.

In each of these cases a current will be induced on the wire,

$$I = \frac{\mathcal{E}}{R}$$

where \mathcal{E} is the electromotive force and R is the resistance of the wire. Faraday's law says that

$$\mathcal{E} = -N \frac{d}{dt} \Phi_M$$

where N is the number of turns of the coil and Φ flux through each turn. Or, in terms of the integral formula for the electromotive force,

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}.$$

Here on the left we have the magnetic flux through a surface; on the right we have the electromotive force around its boundary. This is known as Faraday's Law.

3 Lenz's Law

Lenz's law determines the sign of the induced e.m.f. The negative sign in the above equation says that the induced current will try to stop the magnetic flux from changing. If the flux is growing, the induced current will be in the direction such that its magnetic field is decreasing; if the flux is decreasing, the magnetic field of the induced current will be in the direction to increase the flux. Lenz's law says that magnetic fields have inertia: they don't like to change.

4 Induction

We can create a magnetic field using an electrical current passing through a coil. If the current varies with time, the magnetic field will vary with time also. Thus it will induce a current on a nearby coil, even if there is no physical connection between the two. This is the phenomenon of induction. We can magnify the effect by placing an iron rod that passes through each coil: it magnifies the magnetic field. We can calculate the induced e.m.f. on the secondary coil by the primary coil using Faraday's law. If there are N_S turns in the secondary and N_P turns in the primary, the e.m.f. induces on each by the same change in the magnetic flux is

$$V_P = N_P \frac{d\Phi}{dt}, \quad V_S = N_S \frac{d\Phi}{dt}.$$

Thus, whenever $\frac{d\Phi_M}{dt} \neq 0$ (the magnetic flux is changing)

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}.$$

This is the idea behind a transformer. We can step up the voltage using a pair of coils where the secondary has more turns; and step down the voltage by another pair where the primary has more windings.

The power carried by the current remains the same across a transformer: power is energy per unit time and it cannot be created or destroyed (but can be converted to some wasteful form like heat). Thus

$$I_S V_S = I_P V_P.$$

Thus as the current in the secondary is smaller if the voltage is stepped up and vice versa.

$$\frac{I_S}{I_P} = \frac{N_P}{N_S}.$$

5 Alternating Current

The simplest way to have a changing electric current is to let it be periodic in time: then the same thing can be made to happen over and over again. Engineers like to make devices be predictable easily. Alternating current has a simple periodic dependence:

$$I(t) = I_0 \sin \omega t$$

and similarly for voltage

$$V(t) = V_0 \sin \omega t.$$

The power is the product

$$P(t) = I_0 V_0 \sin^2 \omega t.$$

The average power is

$$P_{av} = \frac{1}{2} I_0 V_0.$$

The peak values of current and voltage I_0, V_0 give the peak power. But the average power is the more useful thing to know. The average current and voltage are both zero, so we must use the r.m.s. value. Here, r.m.s. stands for the root mean square: the square root of the mean of the square.

$$I_{rms} = \frac{1}{\sqrt{2}} I_0, \quad V_{rms} = \frac{1}{\sqrt{2}} V_0.$$

Note that the average power is the product of the two:

$$P_{av} = I_{rms}V_{rms}.$$

The period of the current is $\frac{2\pi}{\omega}$. The frequency is $\frac{\omega}{2\pi}$.
In the US, the values of these at a standard outlet are

$$V_{rms} = 120V, \quad \frac{\omega}{2\pi} = 50 \text{ Hz}.$$

6 Faraday's Law has many applications

Faraday discovered a fundamental law of physics. But it is also the foundation of many of the devices essential to modern life.

To convert mechanical energy into electrical energy, you can rotate a coil in a magnetic field. The induced e.m.f. can be transmitted by a wire to power an electric bulb for example. This is the principle of the dynamo.

Conversely, you can put a coil in a magnetic field and pass an alternating current through it. A torque will be exerted on the coil which can be used to turn an appliance (e.g., a fan). This is the principle of the electric motor.

It took some years and clever ideas to make such inventions possible. Edison invented many of the electrical appliances (e.g., the light bulb, movies, the electric chair) while Tesla invented or improved methods for the production (dynamo, hydroelectric power stations) and the transmission of electricity over long distances (the transformer).



Edison favored direct current, while Tesla emphasized that alternating current was a better way of transmitting electricity over long distances without losing power: using the transformer to step up the voltage in intermediate stages.

7 The Ohm's Law Again

Suppose we want to transmit electricity from the place where it is produced (Niagara Falls) to a city where it will be used to run devices (Buffalo). The power wasted (through heating the wires) during transmission is

$$P_T = V_T I = R_T I^2.$$

Here, V_T is the voltage drop across the transmission wire and R_T its resistance.

Thus a large current would mean a large loss in the wires. Now, the complete circuit includes the wire and the devices connected to it in series. With D. C. the same current would pass through the devices and the transmission wires, so there would be no way to minimize losses.

Suppose we could make the current large near the city, but keep it small over the transmission lines. Then we would decrease the losses. Tesla realized that this was possible using alternating current and transformers.

References

- [1] The picture of the transformer is from Wikipedia