20. More Optics

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March 31, 2009

In most everyday situations we can treat light as if it goes along rays. So people used to think that light was made of tiny particles. But there are some phenomena and many devices that depend on the wave properties of light. When the wavelength and the size of the devices are about the same, wave phenomena like interference become important.

But the idea that light is made of particles is not entirely wrong. There are phenomena in quantum physics where light behave as if it is made of particles, each carrying an amount of energy $h\nu$ proportional to its frequency ν . This explains the photoelectric effect, for which Einstein got his Nobel Prize.

1 Wavelength Depends on Refractive Index

The frequency does not. As light passes from one medium to another, its frequency remains the same. But its wavelength changes. If the refractive index is n, the wave length in that material is $\lambda_n = \frac{\lambda}{n}$ where λ is the wavelength in vacuum.

2 The color of light depends on wavelength

Red light has a long wavelength of about 750 nm and blue light has shorter wavelength of about 450 nm. The eye cannot distinguish all the possibilities in between. It is only sensitive to three "primary colors" of wavelengths 420 nm (violet), 564 nm (yellowish green) and 534nm (green). All other colors are percieved as combinations of these.People with deficiency in one of these kinds of receptors, will have some degree of color blindness.

Most devices like computer screens are made to work with primary colors like Red Green Blue or Red Yellow Green. Thus perceived color is not so much a physical property of light as a property of our eyes. Other species have different numbers and kinds of color receptors.

3 Interference

If two waves of the same wavelength come together, they can either reinforce each other (constructive interference) or cancel each other out (destructive interference). The first happens if the maxima of the waves coincide, in which case the minima will coincide also. In the latter case, the maximum of one happens at the minimum of the other and vice versa. Suppose A_1 and A_2 are both positive numbers, describing the amplitude of two waves. When the waves are "in phase" the maxima occur at the same point and the combined wave has amplitude $A_1 + A_2$:

$$A_1 \sin[kx - \omega t] + A_2 \sin[kx - \omega t] = (A_1 + A_2) \sin[kx - \omega t]$$

If the second wave has maxima displaced relative to the first, there will be a number ϕ called the phase that describes this. When we combine

$$A_1 \sin[kx - \omega t] + A_2 \sin[kx - \omega t + \phi]$$

we can get a more complicated answer depending on ϕ . Recall that

$$\sin[\theta + \pi] = -\sin\theta.$$

So if $\phi = \pi$, we get destructive interference.

 $A_1 \sin[kx - \omega t] + A_2 \sin[kx - \omega t + \pi] = (A_1 - A_2) \sin[kx - \omega t].$

If the waves are of equal strength $A_1 = A_2$ they will completely cancel each other! Thus we get constructive interference if ϕ is an even multiple of π and destructive interference if it is an odd multiple. A phase change of 2π has the same effect as the wave moving a full wavelength.

4 The Young Slit Experiment

Light of a fixed wavelength passes through two slits, and then allowed to fall on a screen. If light moved strictly alongs rays you would expect to see two bright spots. Instead we see alternate bright and dark spots. At some points the two beams of light are in phase and at others they are out of phase. *Read the detailed explanation in the textbook* where a formula is derived for the locations of the dark and bright fringes.

5 Change of phase during reflection

There is something peculiar about reflection of light from a surface of higher refractive index: the reflected light has a phase chaneg of π relative to the transmitted light. This does not happen when light is reflected from a surface of lesser refractive index. It is beyond the scope of this course to explain why this peculiar phenomenon happens. (Follows from Maxwell's theory.) It makes some subtle corrections to interference on films.

6 Example: Color on Thin films

We have all seen some colored patterns on thin oil films or soap bubbles. The thickness of these films is about half a micron which is about the wavelength of light. At different angles and thicknesses light of different wavelengths will interefere constructively.

Suppose light is incident normally on oil film. Some of the light is reflected at the top layer. This reflected light will change its phase by π because oil has higher refractive index than air. Some of the light will also be reflected by the bottom layer. This light does not change its phase. To have constructive interference, so that the light from the two layers add up if

$$2t = \frac{1}{2}\frac{\lambda}{n}.$$

where t is the thickness of the film.

Light reflected at the bottom layer travels twice through the oil, so the distance it travels through oil is 2t. Its wavelength in oil is $\frac{\lambda}{n}$. If the distance that light has travelled is a half a wavelength, it will interfere constructively because there is a phase change of π (or half a wavelength) due to reflection at the first surface.

Thus

$$t = \frac{\lambda}{4n}.$$

This is the minimum thickness. If the distance travelled is

$$2t = \frac{1}{2}\frac{\lambda}{n} + \frac{\lambda}{n}$$

again you get the same effect: a complete wavelength later the light is in the same phase.

Thus in general the condition is

$$2t = \left[m + \frac{1}{2}\right]\frac{\lambda}{n}, \quad m = 0, 1, 2, \cdots$$

6.1 Example: Non-Reflective Coating

We can get the opposite effect if the light is made to interfere destructively. This is how non-reflective coating on eyeglasses or camera lenses work. The condition is

$$2t = m\frac{\lambda}{n}, \quad m = 0, 1, 2, \cdots$$

The smallest thickness is then

$$t = \frac{\lambda}{2n}.$$

7 Diffraction

If you place an obstruction in front of light, there is a shadow behind it: makes perfect sense if light goes along straight rays. But remarkably, if the size of the object is comparable to the wavelength, there is no perfect shadow: the edges show an interference pattern. This phenomenon is called diffraction. Poisson even showed in a remarkable experiment that there is a very small bright spot right in the center of the shadow of a disk: the light from the edges interfere constructively there.

We are more familiar with diffraction for sound waves, because there wavelengths are close to the size of most objects in everyday life (1 or 2 meters). Everyone knows that you can hear sound even if your ears are not pointed right at the source. Sound can go around obstructions that are of a size smaller than its wavelength.

But visible light has shorter wavelength (half a micron) than most everyday objects. So we don't see this usually. There are some effects of diffraction in engineering devices though. A microscope cannot see objects that are smaller than the wavelength of light it is using. Also, the best angular resolution (the smallest angle at which two objects can no longer be distinguished) of a microscope is about $\frac{\lambda}{D}$, where D is the aperture (diameter of the lens which collects the light).

8 Doppler Effect

The frequency of light is not changed when it moves from one medium into another. (The wavelength does.) But if the source is moving towards you the frequency you measure will increase: more maxima and minima happen in a fixed amount of time. If the source is moving away the frequency is shifted down. This is called the Doppler effect. It is true of all waves, like sound waves.

An application of this is to the radar speed gun that the police use to measure the speed of a car. Nowadays they use a laser reflected off the car also. Why is the laser more effective?

Doppler radar measures not only the position of a thunderstorm but the speed of the rain drops carried in it.

The ultimate application of the Doppler effect is to Cosmology. The light coming to us from atoms in faraway galaxies has lower frequency than when it is emitted. (We know the natural frequencies of atoms-they can be measured in the lab.) This means that those galaxies are moving away from us: the whole universe is expanding. The farther away two points in the universe are, the faster they are moving apart from each other!

9 Coherent Light

If all the light waves in a beam move together, we have coherent light. But in most natural sources of light (the Sun, fluorescent lamps, incandescent lamps) the light is incoherent. Like a platoon of soldiers running out of step with eahc other, the light waves oscillate with random phases.

It used to be difficult to produce coherent beams of light. But then about fifty years ago the laser was invented. Light from a laser has the same frequency, the same direction and the same phase: it is coherent. It is like soldiers running in step: everyone takes the right step together then the left step. Such a well behaved source of light is very useful in may devices: CD players, laser pointers etc.

There is even a laser beam that is reflected back to the Earth from a mirror left behind on the Moon by the Appolo astronauts. Using this we have been measuring the distance to the moon to an accuracy of six inches for the last thirtyfive years. Very valuable data.

Laser beams dont spread out vey little with distance. For a while people thought that powerful lasers can be used to shoot down missiles and such. This idea is correct in principle, but has not worked satisfactorily yet in practice. It takes a lot of work to connect physics to engineering.

10 Newton's Rings

The wave nature of light is also evident in a famous experiment of Isaac Newton. A lens is placed on a glass plate . Light passes through the lens and is reflected from the plate then passes through the lens again to your eye (or a camera). If the little gap of air between the glass and lens is $0, \frac{1}{2}\lambda, \frac{3}{2}\lambda$, we get constructive interference- a bright ring. (In this case both light rayes are reflected at a denser material, so both are shifted by π in phase.) If the gap is $\lambda, 2\lambda$ etc. we get destructive interference: a dark ring.

Newton studied this phenomenon, which was pointed out to him by his less famous contemporary, Robert Hooke. But he did not make the correct conclusion from it: he wanted to explain everything in terms of particle mechanics. So instead of the obvious explanation in terms of waves, Newton came up with a complicated (and wrong) one in terms of particles. Newton got almost everything else about physics right, though.

11 Photons

But in a remarkable turnaround, about hundred years ago it was discovered that light does behave like particles in some situations. The energy of a pulse of light of frequency ν can only increase in multiples of $h\nu$ where h is a very small number called the Plank's constant. Its value is $6.63 \times 10^{-34} Js$. Thus, a pulse of light can be thought of as made up of a number N of photons (the particle of light); its total energy will be

$$E = Nh\nu.$$

This number is very large (like 10^{20} or more) for everyday situations. So we don't notice that the energy jumps by $h\nu$; each jump is very small. But there are some effects that make it clear that this is the situation.

For example, the photoelectric effect. When light is shined on metals, electrons absorb that energy and are ejected from the metal. Each metal has a characteristic energy W (call the Work function for historical reasons) that is the amount that must be given to liberate an electron. It was found that light of too low a frequency cannot liberate electrons even if it has high intensity. Einstein's explanation was that the electrons can only absorb one photon at a time. (In principle, there is a very small chance that two or more photons can be absorbed, but it is so small that it is hard to see in those experiments.) If each photon has an energy $h\nu$ the energy of the liberated electron will be

$$E = h\nu - W.$$

When $h\nu < W$ the electron cannot escape. Later experiments measured the energy of the liberated electrons and confirmed this prediction. Modern electronic cameras work on this principle.

12 Quantum Physics

So when does light behave as a wave and when as a particle? If it is a particle how can it also be a wave? This was the question that confused physicists about a hundred yaers ago. To add to the confusion, it was found that the electron behaves as a wave inside an atom.

All this was resolved by a new physical theory called quantum mechanics. We will only be able to skim the surface of this strange new world in this course.