22. Atomic Physics and Quantum Mechanics

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Most of what you have been learning this semester and last has been 19th century physics. Indeed, Maxwell's theory of electromagnetism, combined with Newton's laws of mechanics, seemed at that time to express virtually a complete "solution" to all known physical problems. However, already by the 1890's physicists were grappling with certain experimental observations that did not seem to fit these existing theories. To explain these puzzling phenomena, the early part of the 20th century witnessed revolutionary new ideas that changed forever our notions about the physical nature of matter and the theories needed to explain how matter behaves. In this lecture we hope to give the briefest introduction to some of these ideas.

1 Photons

You have already heard in lecture 20 about the photoelectric effect, in which light shining on the surface of a metal results in the emission of electrons from that surface. To explain the observed energies of these emitted electrons, Einstein proposed in 1905 that the light (which Maxwell beautifully explained as an electromagnetic wave) should be viewed as made up of discrete particles. These particles were called *photons*. The energy of a photon corresponding to a light wave of frequency f is given by,

$$\boxed{E = hf} \quad ,$$

where $h = 6.626 \times 10^{-34}\,\mathrm{J \cdot s}$ is a new fundamental constant of nature known as *Planck's constant*. The total energy of a light wave, consisting of some particular number n of photons, should therefore be *quantized* in integer multiples of the energy of the individual photon, $E_{\mathrm{total}} = nhf$.

The idea that the energy of light must be quantized into discrete units, and the discovery of the constant h, were actually due to Max Planck in his earlier 1900 theory of *blackbody radiation*, which sought to explain the observed spectrum of frequencies of light emitted from an object in equilibrium at some fixed temperature T (see section 37-1 in the text). But it was Einstein who pushed forward the idea that photons are not just conceptual ideas, but rather behave just like other physical particles. In particular, photons carry not only energy, but they also carry momentum.

You have already learned that the Theory of Special Relativity (also due to Einstein!) changed the formulas we must use for the energy E and momentum p of a particle, and that these changes from Newtonian mechanics become important when the particles speed v approaches comparable to the speed of light in the vacuum c. Special relativity tells us that E and p are related to v by,

$$E = \frac{mc^2}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} \ , \qquad p = \frac{mv}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} \ . \tag{1}$$

Dividing one equation by the other then gives,

$$\frac{E}{p} = \frac{c^2}{v} \ , \qquad \text{or,} \qquad E = \frac{pc^2}{v}$$

Since a photon is a quantized particle of a light wave, we expect that photons must travel with the speed of light c! If we set $v = c$ in the above, we get the relation between energy and momentum for a photon,

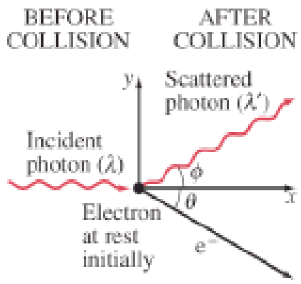
$$\boxed{E = pc} \ , \qquad \text{or,} \qquad p = \frac{E}{c} \ .$$

Using the relation, $E = hf$, between energy of the photon E and the frequency of the light wave f, and the relation, $f = c/\lambda$, between frequency and wavelength of a light wave, we can now get a relation between the photon's momentum and the wavelength of the light,

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{hc}{c\lambda}, \qquad \text{or,} \qquad \boxed{p = \frac{h}{\lambda}} \ .$$

Note, since photons move with the speed of light c, the above Eqs. (1) for E and p would diverge, unless the mass of the photon vanished, $m = 0$, and so it does! Photons carry energy and momentum, yet they are massless!

That photons do indeed carry momentum as do particles was beautifully born out in the 1923 experiments of A. H. Compton, who considered the scattering of light waves by a stationary electron.



Consider an incident light wave of wavelength λ that hits a stationary electron e^-, as sketched in the figure above. In Maxwell's classical theory of electromagnetism, the electron will feel the force of the electric field of the wave, which will then cause the electron to oscillate back and forth with the same frequency $f = c/\lambda$ as the incident wave. The oscillating electron then serves as a source of an outgoing spherical electromagnetic wave, also at frequency f with the same wavelength λ as the incident wave. Thus if one looks at the outgoing light at any angle ϕ with respect to the incident direction, one should see light with wavelength λ, the same as the incident wave.

If, however, we regard the incident light as a beam of particles (photons) carrying energy $E = hf$ and momentum $p = c/\lambda$, the collision process will look very different! If the incident photon hits the electron and knocks it into motion, with the electron moving off at some angle θ with respect to the incident direction, the electron has acquired energy from the photon. Conservation of energy then tells us that the photon has lost energy. If the photon's energy E has decreased in the collision, then its frequency $f = E/h$ has similarly decreased; and if the photons frequency f has decreased, then its wavelength $\lambda = c/f$ has *increased*! Thus a photon colliding with the electron and coming out scattered at some angle ϕ should have a *larger* wavelength λ than the incident wave. By using the laws of conservation of energy and conservation of the vector momentum $\mathbf{p}$, Compton was able to derive an explicit formula that related the wavelength λ' of the scattered light at angle ϕ to the wavelength of the incident light λ,

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\phi) \ ,$$

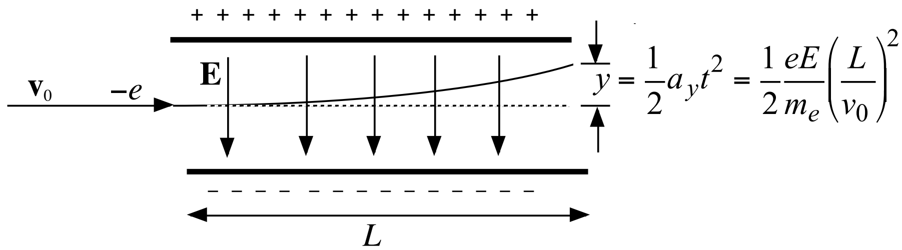
where m_e is the mass of the electron (you may read a derivation of this equation in section 37-4 of the text). Compton found that his data was in excellent agreement with the above formula based on the photon nature of light.

2 Atoms - the classical picture

We all know today that matter is made up of basic building blocks called *atoms*. Here we give a brief history of how our understanding of the atom developed. Explaining the physics of the atom led to one of the most revolutionary theories of modern physics, with *quantum mechanics* replacing Newtonian mechanics when one needs to describe physical phenomena that take place on the atomic scale.

The idea that matter is made up of small particles called *atoms*, that could be divided no further, was first put forth by the Greek philosopher Democritus around 400 BCE. But philosophy is not science! It took around 2000 years until chemists in the 1700-1800's put the theory of atoms on a solid scientific basis, by observing that elemental gases only reacted together in certain specific integer proportions by weight. John Dalton is credited with the first such atomic theory which he proposed in 1804. He stated: (1) Matter is composed of small particles called atom; (2) All atoms of an element are identical to each other, but are different from those of any other element; (3) During chemical reactions, atoms are neither created nor destroyed, but simple recombine into different arrangements; (4) Atoms always combine in whole number multiples of each other. Dimitri Mendeleev organized the elements into the periodic table in 1869.

The deduction by the chemists, based only on experiments with macroscopic volumes of gas, that matter must be made of basic units called atoms, is truly one of the outstanding achievements of science. But it did not provide any information about the physical nature of these basic units or how they were constructed. The first clues came from the experiments of Pierre and Marie Curie who found that certain "radioactive" elements seemed to decay by emitting particles. In 1899 Ernest Rutherford determined that the particles emitted in radioactive decay consisted of positively charged "alpha" particles, and negatively charged "beta" particles. He determined this by having these emitted particles pass through an electric field between two capacitor plates and observing their deflections. Rutherford determined that the alpha particles had twice the magnitude of charge as the beta particles, and were over 7000 times as massive! (Today we know that an alpha particles is just a doubly ionized helium atom consisting of two protons and two neutrons only, and that a beta particle is just the electron.) Rutherford's experiments were the first step in showing that atoms are comprised of positive and negatively charged components.



At roughly the same time as Rutherford did his experiments above, J. J. Thomson did experiments with *cathode ray* tubes in which he determined that an electron current would flow between a negative metallic cathode and a positive metallic anode when placed in an evacuated sealed tube. By placing the tube between charged capacitor plates, and in a perpendicular magnetic field, and measuring how the current between cathode and anode was deflected as a function of the electric and magnetic fields in the capacitor, Thomson determined that the particles carrying the current were negatively charged and he was also able to determine their charge to mass ratio. Thomson found that this charge to mass ratio stayed constant independent of the type of metal that was used to make the cathode and anode. He concluded that these particles were basic negatively charged subatomic components that must be contained within all atoms. This marked the discovery of the *electron*. Since atoms were known to be electrically neutral, Thomson further proposed that atoms must contain an equal amount of positive charge. Thomson proposed his "plum pudding" model of the atom in which the positive charge was smeared out uniformly over a sphere of size equal to that of that atom, while the negative charge was made of smaller discrete particles embedded in this positive charge ball.

The next important step was taken again by Rutherford in 1909 with colleagues Hans Geiger and Ernest Marsden, who wanted to test Thomson's model that the atom was a ball of solid mass. Rutherford used a radioactive source to emit positively charged alpha particles and aimed them at a very thin foil of gold. If the plum pudding model of the atom was correct, Rutherford expected that the high energy alpha particles would all penetrate through the thin foil, receiving only small deflections as they passed though the electric fields of the smeared positive charge and the discrete electric charges. Instead of this Rutherford observed that, while most alpha particles did pass through the foil with small deflections, a small fraction

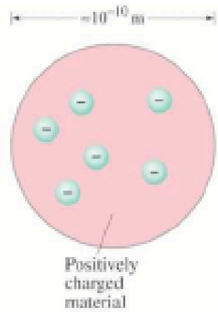


FIGURE 37–16 Plum-pudding model of the atom.

of the alpha particles underwent very strong deflections, some even coming straight backwards. As Rutherford later said, "It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you." Rutherford concluded that the only consistent explanation was if the positive charge was not smeared over the entire volume of the atom but was rather densely concentrated in a very small region at the core of the atom. Alpha particle passing close to this dense core of positive charge would receive the strong deflections. From earlier experiments it was known that the radius of an atom was on the order of 10^{-10} m. Using his data, Rutherford's calculations estimated that the positive charge of the atom should be confined to a *nucleus* of about $10^{-15} - 10^{-14}$ m. Thus only a fraction of about 10^{-8} of the volume of the atom was filled with the heavy massed positive charge. The remainder of the atom was filled with the much lighter massed electrons.

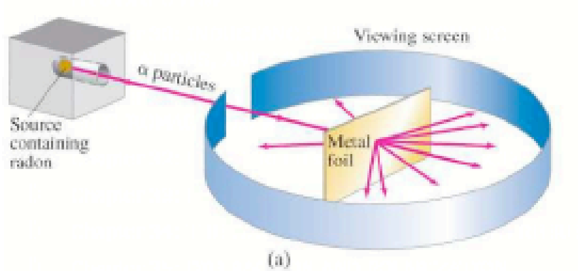
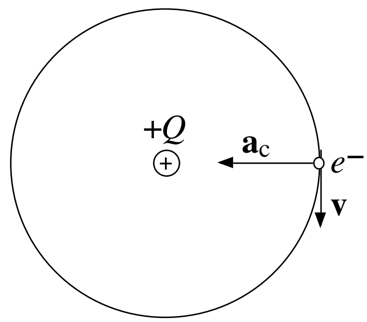


FIGURE 37–17 (a) Experimental setup for Rutherford's experiment: α particles emitted by radon are deflected by a thin metallic foil and a few rebound backward; (b) backward rebound of α particles explained as the repulsion from a heavy positively charged nucleus.

If the positive charge of the atom was concentrated at the center of the atom, what kept the electons from being attracted to it and collapsing to the center? If the electrons were at rest, that is what they would do. But not necessarily if they are in motion. This lead to the "solar system" model of the atom, in which the negatively charged electrons orbit the positively charged nucleus just like the planets orbit around the sun, with the Coulomb attraction providing the centripetal acceleration of the orbital motion.



For motion in an orbit of radius r we have,

$$a_c = \frac{v^2}{r} = \frac{F}{m_e} = \frac{eQ}{4\pi\epsilon_0 r^2 m_e} \quad \Rightarrow \quad v^2 = \frac{eQ}{4\pi\epsilon_0 r m_e} \tag{2}$$

The total kinetic plus potential energy of an electron in such an orbit is thus,

$$E = K + V = \frac{1}{2}m_e v^2 - \frac{eQ}{4\pi\epsilon_0 r} = -\frac{eQ}{8\pi\epsilon_0 r} \tag{3}$$

The negative sign indicates that the energy of the electron in an orbit of radius r is *lower* than that of an electron which has escaped the nucleus and is at $r \to \infty$. This is the *binding energy* of the electron to the nucleus.

The electron in orbit at radius r is orbiting in circular motion with a frequency,

$$f = \frac{v}{2\pi r} = \frac{1}{2\pi r}\sqrt{\frac{eQ}{4\pi\epsilon_0 r m_e}} \sim \frac{1}{r^{3/2}} \tag{4}$$

so the smaller the radius r, the larger the frequency of the orbital motion.

3 Atoms - the quantum picture

As soon as the solar system model of the atom was introduced, it was understood that it was in serious conflict with classical physics. A key success of Maxwell's theory of classical electromagnetism was that it predicted the existence of electromagnetic waves. In Maxwell's theory, the source of such electromagnetic waves is the *acceleration* of charges. In particular, a charge that oscillates with a frequency f is found to radiate outgoing spherical electromagnetic waves with the same frequency f. Therefore, an electron orbiting the nucleus with frequency f would be radiating electromagnetic waves. You have learned that electromagnetic waves carry energy, therefore such an electron orbiting the nucleus would be steadily radiating away energy in outgoing electromagnetic waves. As the electron energy E decreases, Eq. (3) tells us that the radius of the orbit r should decrease. Ultimately the electron would spiral inwards and crash into the nucleus. Moreover, this would happen extremely fast; classical mechanics, combined with Maxwell's electromagnetism, would predict that an electron orbiting at a radius $r \sim 10^{-10}\,\mathrm{m}$ would hit the nucleus in less than $10^{-6}\,\mathrm{s}$. This is not good news for a making a theory of stable atoms!

The above picture creates yet another problem. As the electron spirals into the nucleus, with its radius r continuously decreasing, Eq. (4) tells us that the frequency of the orbital motion f would be steadily increasing. Since an electron with orbital frequency f should emit electromagnetic waves with frequency f, one should observe light waves radiated from the decaying atom that spanned a continuous spectrum of frequencies f. This, however, was in direct contradiction to experiments.

3.1 Atomic spectra

When one heats dilute atomic gases, one excites the atoms to high energy states. As the atoms decay back down to lower energy states they lose energy by emitting electromagnetic waves, i.e. light. In the classical picture, one would expect this radiated light would be found with a continuous range of frequencies. What is found, however, is that for atoms of any particular element, the frequencies of the radiated light do not span a continuous band, but rather come at specific discrete values − these are called the *spectral lines* of the atom. The discrete frequencies of the spectral lines from atoms of a given element are unique and specific to that element; indeed they serve as a sort of atomic fingerprint. By observing the spectral lines emitted from a heated atomic gas, one can tell which elements the atoms of the gas belong to. If one looks at light from a distant star, and analyses the frequency spectrum, one can tell what elements the gas of that star is comprised of!

Hydrogen is the simplest of all atoms, having only one electron. It was found that the spectral lines from hydrogen obeyed a very simple looking empirical formula, For frequencies starting in the *visible* range of the light spectrum, these spectral lines are known as the *Balmer series*, after J.J.

Balmer who found this formula in 1885. It is usually stated in terms of the wavelengths of the emitted light,

$$\frac{1}{\lambda} = \frac{f}{c} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \;, \qquad n = 3, 4, 5, \ldots \qquad \text{the Balmer series}$$

Here $n > 2$ is an integer, and R is called the *Rydberg constant* and has the value $R = 1.0974 \times 10^7\,\mathrm{m}^{-1}$. The values $n = 3, 4, 5, 6$ give rise to spectral lines at 656 nm, 486 nm, 434 nm, and 410 nm. Later experiments showed that there were similar spectral lines for hydrogen that lay in the ultraviolet (UV) and infrared (IR) regions of the light spectrum. In the UV region, these are the *Lyman series* and they obey the formula,

$$\frac{1}{\lambda} = \frac{f}{c} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \;, \qquad n = 2, 3, 4, \ldots \qquad \text{the Lyman series}$$

In the IR, these are the *Paschen series* and the obey the formula,

$$\frac{1}{\lambda} = \frac{f}{c} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \;, \qquad n = 4, 5, 6, \ldots \qquad \text{the Paschen series}$$

In general, we can thus conclude that the spectral lines of hydrogen obey the formula,

$$\frac{1}{\lambda} = \frac{f}{c} = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) \;, \qquad n > n', \quad n, n' \text{ positive integers}$$

But these formulae were purely empirical with no physical model that explained where they came from!

3.2 de Broglie wavelength

An important step in understanding how to fix the classical problems with the atom came from Louis de Broglie in 1923. He argued that if a light wave with wavelength λ could sometimes behave like a particle (the photon) with momentum $p = h/\lambda$, then perhaps a particle with momentum p could sometimes behave like a wave with wavelength,

$$\boxed{\lambda = \frac{h}{p}} \;.$$

This is now known as the *de Broglie wavelength* of a particle.

For macroscopic sized particles that we encounter in everyday life, the de Broglie wavelength is extremely small, and so we do not notice the wave-like aspect of such particles. For example, for ball of mass $m = 0.2\,\mathrm{kg}$ moving with a speed $v = 15\,\mathrm{m/s}$, the de Broglie wavelength is,

$$\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}\,\mathrm{J \cdot s}}{(0.2\,\mathrm{kg})(15\,\mathrm{m/s})} = 2.2 \times 10^{-24}\,\mathrm{m}$$

This is much small than even the radius of the atomic nucleus!

On the other hand, for particles on the atomic scale, the de Broglie wavelength can be comparable to the scales of interest. For example, the typical size of an atom is about $10^{-10}\,\mathrm{m}$. Consider an electron which orbits the at this radius $r = 10^{-10}\,\mathrm{m}$. For an atom with nuclear charge $Q = Ze$, the electron's velocity is given by Eq. (2), which then gives for the de Broglie wavelength,

$$\lambda = \frac{h}{m_e v} = \frac{h}{m_e}\sqrt{\frac{4\pi\epsilon_0 r m_e}{Ze^2}} = \sqrt{\frac{4\pi r\epsilon_0 h^2}{Ze^2 m_e}}$$

$$\lambda = \sqrt{\frac{4\pi}{Z}}\sqrt{\frac{(10^{-10}\,\mathrm{m})(8.85 \times 10^{-12}\,\mathrm{C^2/Nm^2})(6.626 \times 10^{-34}\,\mathrm{Js})^2}{(1.602 \times 10^{-19}\,\mathrm{C})^2(9.11 \times 10^{-31}\,\mathrm{kg})}}$$

$$\lambda = \sqrt{\frac{4\pi}{Z}}\sqrt{(10^{-10}\,\mathrm{m})(1.66 \times 10^{-10}\,\mathrm{m})} = \frac{4.57 \times 10^{-10}\,\mathrm{m}}{\sqrt{Z}}$$

We thus see that in this case the de Broglie wavelength is the same size as the electron's orbit! The wave-like aspects of the electron should therefore be important at this atomic size scale.

In general, whenever the de Broglie wavelength of a particle is very much smaller than other lengths that enter the physical problem, one will not see any of the wave-like nature of the particle. Only when the de Broglie wavelength of a particle is comparable in size to other lengths in the physical problem will the wave-like nature of the particle make itself known!

3.3 The Bohr model of the atom

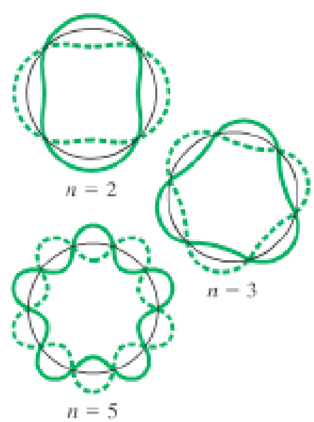
The idea of the de Broglie wavelength now gives us a means to explain some of the puzzling behaviors of atoms that is in contradiction to classical physics. This leads to what is known as the *Bohr model* of the atom. Niels Bohr actually developed this model in 1912-13, *before* de Broglie proposed his idea about the wave-like nature of particles. However de Broglie's later re-explanation of Bohr's model is more physically appealing, so we will follow that approach.

We have seen that above that, according to de Broglie, particles have a wave-like aspect to them with wavelength related to particle momentum by,

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad .$$

We have seen that for an electron orbit around the atomic nucleus, the de Broglie wavelength of the electron is of comparable size to the radius of the orbit. It is thus necessary to consider the wave-like nature of the electron in treating its behavior within the atom. De Broglie argued that in order for an electron orbit at radius r to be stable, there must be an integer number of de Broglie wavelengths going around the circumference of the orbit. In not, than as the wave traveled around the circumference of the orbit, it would return to its starting position out of phase with itself and destructively interfere with itself. Only if there were an integer number of wavelengths around the circumference could one come back in phase and set up a stable standing wave.

FIGURE 37–29 Standing circular waves for two, three, and five wavelengths on the circumference; n, the number of wavelengths, is also the quantum number.



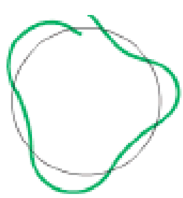


FIGURE 37–29 Standing circular waves for two, three, and five

De Broglie's condition is,

$$\boxed{2\pi r = n\lambda = n\frac{h}{p} = n\frac{h}{m_e v}} \quad , \qquad n \text{ a positive integer}$$

From Eq, (2) we know the relation between the speed v and the radius r of an orbiting electron. Considering the specific case of hydrogen where the nuclear charge $Q = e$ is the same magnitude as that of the electron, and substituting in for v above, gives,

$$2\pi r = n\frac{h}{m_e}\sqrt{\frac{4\pi\epsilon_0 r m_e}{e^2}} \quad \Rightarrow \quad 4\pi^2 r^2 = n^2\frac{h^2 4\pi\epsilon_0 r}{m_e e^2}$$

$$\Rightarrow \quad r = n^2\left(\frac{h^2\epsilon_0}{\pi m_e e^2}\right) = n^2 a_0$$

where,

$$a_0 \equiv \frac{h^2\epsilon_0}{\pi m_e e^2} = \frac{(6.626\times10^{-34}\,\mathrm{Js})^2(8.85\times10^{-12}\,\mathrm{C^2/Nm^2})}{(3.14)(9.11\times10^{-31}\,\mathrm{kg})(1.602\times10^{-19}\,\mathrm{C})^2} = 0.529\times10^{-10}\,\mathrm{m}$$

is called the *Bohr radius.*

The wave picture of the electron thus says that the electron is not stable in an orbit of general radius r, but only stable for orbits of certain specific discrete radii,

$$\boxed{r_n = n^2 a_0} \ , \qquad n = 1, 2, 3, \ldots$$

Similarly, the energy of the electron in orbit around the nucleus cannot be any general value, but only those discrete values that correspond to the above radii. For hydrogen with nuclear charge $Q = e$, Eq. (2) gives,

$$E_n = -\frac{e^2}{8\pi\epsilon_0 r_n} \ .$$

using the values of r_n determined above then gives,

$$E_n = -\left(\frac{e^2}{8\pi\epsilon_0}\right)\left(\frac{\pi m_e e^2}{h^2\epsilon_0}\frac{1}{n^2}\right) = -\left(\frac{e^4 m_e}{8\epsilon_0^2 h^2}\right)\frac{1}{n^2}$$

$$\Rightarrow \boxed{E_n = -\frac{E_0}{n^2}}$$

where

$$E_0 = \frac{(1.602\times10^{-19}\,\mathrm{C})^4(9.11\times10^{-31}\,\mathrm{kg})}{8(8.85\times10^{-12}\,\mathrm{C^2/Nm^2})^2(6.626\times10^{-34}\,\mathrm{Js})^2} = 2.18\times10^{-18}\,\mathrm{J}$$

In dealing with problems in atomic physics, it is usually the custom to express energy in units of *electron volts* rather than Joules. One electron volt (1eV) is the energy acquired by an electron in passing through one volt of potential. Thus,

$$1\,\mathrm{eV} = e(1\,\mathrm{V}) = (1.602\times10^{-19}\,\mathrm{C})(1\,\mathrm{V}) = 1.602\times10^{-19}\,\mathrm{J}$$

In units of eV, Planck's constant is,

$$h = (6.626 \times 10^{-34}\,\mathrm{J \cdot s}) \frac{(1\,\mathrm{eV})}{(1.602 \times 10^{-19}\,\mathrm{J})} = 4.136 \times 10^{-15}\,\mathrm{eV \cdot s}$$

and,

$$E_0 = 2.18 \times 10^{-18}\,\mathrm{J} \frac{(1\,\mathrm{eV})}{(1.602 \times 10^{-19}\,\mathrm{J})} = 13.6\,\mathrm{eV}$$

With this we can write for the allowed energy levels of the hydrogen atom,

$$\boxed{E_n = -\frac{13.6}{n^2}\,\mathrm{eV}} \quad , \qquad n = 1, 2, 3, \ldots$$

If the electron in the hydrogen atom can only be in an orbit with one of the discrete energies E_n, then when an excited atom decays to a lower energy state, its change in energy is also a discrete amount. Specifically, if the atom in the state with energy E_n decays to the state with energy $E_{n'}$, the loss in energy is $\Delta E = E_n - E_{n'}$. If this loss in energy occurs as a photon emitted by the atom, that photon will have energy ΔE and hence a frequency f given by,

$$hf = \Delta E = E_n - E_{n'} = -E_0 \left(\frac{1}{n^2} - \frac{1}{n'^2} \right)$$

The wavelength λ of the emitted photon is then given by,

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{\Delta E}{hc} = \frac{E_0}{hc} \left(\frac{1}{n'^2} - \frac{1}{n^2} \right) \tag{5}$$

where

$$\frac{E_0}{hc} = \frac{13.6\,\mathrm{eV}}{(4.136 \times 10^{-15}\,\mathrm{eV \cdot s})(3 \times 10^8\,\mathrm{m/s})} = 1.096 \times 10^7\,\mathrm{m}^{-1}$$

But this is just the Rydberg! So $R = E_0/(hc)$ and Eq. (5) gives the exact empirically observed formula for the spectal lines of hydrogen (i.e. the Balmer, Lyman and Paschen series).

So de Broglie's wave idea explains the discrete spectral lines of hydrogen and produces the exactly correct formula that predicts the wavelengths of these spectral lines!

It also, in a sense, provides an answer to the stability of the atom, and why the electron does not spiral into the nucleus. If the only allowed radii for electron orbits are $r_n = n^2 a_0$, and these have energies $E_n = -E_0/n^2$, then the state with $n = 1$ orbits with the smallest radius a_0 and has the lowest energy, $E_0 = -13.6\,\text{eV}$. This lowest energy state is called the *ground state*. States with $n > 1$ are called the *excited states*. The electron cannot spiral into the nucleus because the wave-like nature of the electron does not allow it to be in an orbit with radius less than a_0.

This does not really explain the physical reason *why* an accelerating electron in orbit around the nucleus does not radiate electromagnetic waves as Maxwell's theory says it should. All we can say, at this stage of understanding, is that, once we have to treat the electron as a wave and not as a point particle, all bets based on classical physics about what should happen are off! As a hint of what is going on, you may think of the following: when the electron is in orbit with an integral number of wavelengths around the orbit circumference, we can thing that the electron is a *standing wave*. A standing wave is not traveling anywhere, so perhaps the electron is not accelerating after all. The correct, self consistent, understanding of what is really happening requires the development of an entirely new theory of particles and how they move under the influence of forces. This new theory, which replaces Newtonian mechanics, should agree with Newton when we are dealing with phenomena that happen on the macroscopic size scales of everyday life, but be drastically different when it comes to describing phenomena at the atomic size scales. This new theory, developed by Heisenberg and Schrodinger around 1925, is known as ***Quantum Mechanics!***

Note, de Broglie's condition $2\pi r = n\lambda = nh/p$ can be rewritten as $rp = nh/(2\pi)$. For circular orbital motion, rp is just the angular momentum L. De Broglie's condition can therefore be stated as,

$$L = n\hbar, \qquad \text{where} \qquad \hbar \equiv \frac{h}{2\pi}$$

i.e. angular momentum is quantized in integer multiples of the basic unit $\hbar = h/(2\pi)$. It was starting from the ad hoc assumption of quantization of angular momentum that Bohr developed his model of the atom even before de Broglie had presented his ideas about the wave-like nature of particles.