24. Nuclear Physics

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1 Structure of the Nucleus

Rutherford's experiments: positive charge of atom is confined to an extremely small central part of the atom called the *nucleus*.

Radius of atom is of order 10^{-10} m.

Radius of nucleus is of order 10^{-15} m.

Volume of nucleus is of order $(10^{-15}/10^{-10})^3 = 10^{-15}$ the volume of the atom.

Electrons in Bohr orbits at radii $r_n = n^2 a_0$ are responsible for the larger size of the atom as compared to the size of the nucleus.

Later experiments demonstrated the nucleus is not one solid mass, but made up of more basic building blocks. We expect this must be so because the atom is neutral, so the total positive nuclear charge is always an integer multiple of the magnitude of the electron charge. The basic building block of the nucleus that carries the positive charge is the *proton*. What is less obvious is that there is a second basic building block of the nucleus that has roughly the same mass as the proton but carries no electric charge. This is the *neutron*, discovered by James Chadwick in 1932.

proton: mass $m_p = 1.67262 \times 10^{-27}$ kg electric charge $+e$

neutron: mass $m_n = 1.67493 \times 10^{-27}$ kg electric charge zero

Protons and neutrons are know collectively as *nucleons*.

The mass of either the proton or the neutron is about 1800 times as large as the mass of the electron. Thus essentially all the mass of an atom is contained within the nucleus; the electrons contribute only a tiny fraction of the total atomic mass.

atomic number Z - integer equal to the number of protons in the nucleus (= number of electrons)

atomic mass number A - integer equal to the number nucleons, i.e. number of protons plus number of neutrons, in the nucleus. $N = A - Z$ is the number of neutrons.

isotopes: We group elements according to the atomic number Z of their atoms nucleus. This is because chemical activity is determined by the number of electrons in the atom, and this is just equal to Z. But different atoms of the same element X may have nuclei with different numbers of nucleons A. The group of nuclei (or the corresponding atoms) which have the same Z (i.e. same number of protons) but different A (i.e. different numbers of neutrons) are called *isotopes* of the given element X.

notation: One denotes nuclei by ${}^A_Z\mathrm{X}$, where X is the element symbol. For example, a typical carbon atom has a nucleus denoted ${}^{12}_{6}\mathrm{C}$, with 6 protons and 6 neutrons, so $A = 6 + 6 = 12$. But there are also isotopes of carbon with nuclei ${}^{11}_{6}\mathrm{C}$ and ${}^{13}_{6}\mathrm{C}$ having 5 and 7 neutrons, respectively. 98.8% of all natural carbon is ${}^{12}_{6}\mathrm{C}$; only about 1.1% is ${}^{13}_{6}\mathrm{C}$. Even the simplest atom, hydrogen, has isotopes: 99.9% of all naturally occurring hydrogen is ${}^{1}_{1}\mathrm{H}$ (only one proton in nucleus), but there is also ${}^{2}_{1}\mathrm{H}$ called *deuterium* (one proton + one neutron) and ${}^{3}_{1}\mathrm{H}$ called *tritium* (one proton + 2 neutrons). *Heavy water* is just water H_2O in which the hydrogen appears in the form of deuterium.

atomic mass unit: It is customary to measure nuclei masses in *atomic mass units* u. This is defined so that the mass of ${}^{12}_{6}\mathrm{C}$ has the exact value of 12.000000 u. Since mc^2 has units of energy, it is also common to give the mass of nuclei in units of energy/c^2. If one uses Mev= 10^6 eV as the unit of energy, this unit of mass is then MeV/c^2. The conversion factors are

$$1.0000\,\mathrm{u} = 1.66054 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV}/c^2$$

particle	**kg**	**u**	**MeV/c²**
electron	9.1094×10^{-31}	0.00054858	0.51100
proton	1.67262×10^{-27}	1.007276	938.27
${}^1_1$H atom	1.67353×10^{-27}	1.007825	938.78
neutron	1.67493×10^{-27}	1.008665	939.57

2 Binding Energy and the Strong Interaction

If one adds up the masses of all the individual protons and neutrons in a nucleus, then for *stable* nuclei one finds that this is greater than the mass of the nucleus. For example,

mass of ${}^{12}_{6}$C is 12 u whereas $6m_p + 6m_n = 6(1.007276 + 1.008665)\,\mathrm{u} = 12.095646\,\mathrm{u}$

The difference, in this case, is $0.095646\,\mathrm{u} = 89.094249\,\mathrm{MeV}/c^2$. Multiplying by c^2 gives an energy, in this case 89.094249 MeV. This energy is the *total binding energy* of the nucleus. 6 protons and 6 neutrons infinitely separated from each other have an energy 89.094249 MeV *greater* than the energy of 6 protons and 6 neutrons sitting next to each other in the carbon nucleus. Alternatively, the total binding energy is the energy one must add to a nucleus in order to break it up into its individual nucleons well separated from each other.

The *binding energy per nucleon* is the total binding energy divided by the number of nucleons A. As a function of atomic mass number A, the binding energy per nucleon rises rapidly, reaches a maximum around $A \approx 50$, then slowly decreases. See Fig. 41-1 in the text.

The existence of the binding energy tells us there must be an *attractive* force between the nucleons. It is similar to the situation of, for example, the hydrogen atom. We know that in the ground state of hydrogen, the electron in orbit about the proton has an energy -13.6 eV *lower* than it has when the electron and proton are infinitely far away from each other. $E_0 = 13.6$ eV is the *ionization* energy of the ground state of hydrogen, i.e. the energy we must add to the hydrogen atom to strip off the electron and separate it far from the proton. Alternatively, E_0 is the binding energy that the electron and proton release when they bind together to form an atom. If we include in our energy balance the relativistic rest mass energy of particles then the energy of an electron and a proton separated far apart from each other is just

$m_pc^2 + m_ec^2$. The energy of the hydrogen atom is then $m_pc^2 + m_ec^2 - E_0$. If we could accurately measure the mass of the hydrogen atom, we would find it to be $m_\mathrm{H} = m_p + m_e - E_0/c^2$, or $13.6\,\mathrm{eV}/c^2$ lower than the mass of the separated electron plus proton! (In practice, this is too hard to measure since $m_p, m_e \gg 13.6/c^2$.)

The binding energy E_0 of the hydrogen atom arises from the attractive Coulomb interaction between the electron and proton. Similarly, the binding energy of a nucleus arises from an attractive interaction between the nucleons. That there should be such an attractive interaction is clear: without it there would be nothing to hold the nucleons together in the very small volume of the nucleus; they would be energetically just as happy to be separated from each other. But moreover, without an attractive nucleon interaction the electrostatic Coulomb interaction would cause the protons to strongly repel each other, breaking the nucleus apart.

This attractive interaction between nucleons used to be known as the *nuclear force*. Today it is understood as being a consequence of what is called the *strong force* that acts between *quarks*, the constituent building blocks out of which the nucleons (i.e. the proton and the neutron) are constructed. The nuclear force is always attractive, and it is essentially the same between all nucleons, whether proton or neutron. An important fact about the nuclear force is that, unlike the Coulomb or gravitational forces, the nuclear force is *short ranged*. Whereas two opposite charges will attract no matter how far apart they are, two nucleons will only feel an attractive force when they are separated by a distance of less then about $10^{-15}\,\mathrm{m}$. If they are separated further than that, the nuclear force is essentially zero. In the nucleus, a given nucleon will feel an attractive interaction to its immediate neighboring nucleons, but not to the nucleons that are further away from it. The nuclear force also *saturates*. Once a given nucleon is already interacting with some small maximum number of other nucleons, it will no longer interact with an additional nucleon – this is why there is no energetic advantage to cram all the nucleons into an ever tighter volume and so the volume of a nucleus is in general proportional to the number of nucleons A it contains. Because the nuclear force saturates, whereas the Coulomb repulsion between protons does not, one also finds that as A increases, the fraction of neutrons out of the total number of nucleons *increases*. One needs to pad the nucleus with more neutrons to keep the protons further apart from each other and reduce the electrostatic energy of the protons mutual repulsion.

3 Radioactivity

When the total binding energy is positive, the nucleus is stable; its nucleons stay bound together for all time no matter how long one waits. However not all nuclei are stable. If one waits a sufficient time, some nuclei will decay into a different nuclei of lower energy, while spitting out some other particles (either nucleons, electrons, or photons). The process of decay of unstable nuclei is called *radioactivity.* Radioactivity was first discovered by Henri Becquerel in 1896. He found it in uranium salts. Pierre and Marie Curie isolated two other highly radioactive elements, polonium and radium in the early 1900's (Marie Curie died as a result of exposure to the radioactivity of her experiments since at that time the danger was not understood). Some radioactive nuclei occur naturally in nature. Others are created artificially in the laboratory. Radioactive decays are classified as *alpha decay*, *beta decay*, or *gamma decay*, according to the nature of the particles that get emitted in the decay. *When nuclei decay, the total number of nucleons and the total charge remain conserved.*

4 Alpha Decay

Because the nuclear force is short range and saturates, small groups of nucleons within the nucleus can form locally stable clusters. A particularly stable cluster is the combination of 2 protons and 2 neutrons. This combination is the same as the nucleus of ordinary helium, ${}^{4}_{2}\mathrm{He}$, and is called an *α particle* (it was so named *before* it was understood that it was in fact just ${}^{4}_{2}\mathrm{He}$).

Alpha decay is when a nucleus decays by emitting an alpha particle. When it does so, it converts into a more stable nucleus with atomic mass number A decreased by 4 and atomic number Z decreased by 2 from what it was originally. Radium 226 decaying into radon 222 is the alpha decay,

$${}^{226}_{88}\mathrm{Ra} \quad \rightarrow \quad {}^{222}_{86}\mathrm{Rn} \quad + \quad {}^{4}_{2}\mathrm{He}$$

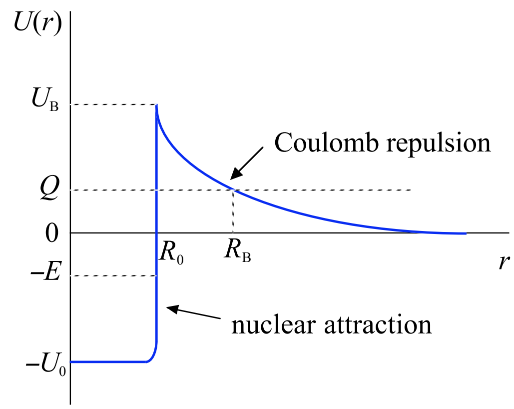
Because Z decreases by 2 in an alpha decay, the new nucleus formed is of a different element from the original one.

In an alpha decay, the mass energy of the products is less than the mass energy of the initial nucleus. If "P" denotes the original, or "parent" nucleus, "D" denotes the product, or "daughter" nucleus, and α denotes the alpha particle, then

$$m_{\mathrm{P}}c^2 = m_{\mathrm{D}}c^2 + m_\alpha c^2 + Q$$

where the mass energy difference Q is always positive and is called the *disintegration energy*. Physically, the energy Q is accounted for by the kinetic energy of the resulting daughter and alpha particle.

One can visual the process of alpha decay with the following simple model. Imagine the potential energy $U(r)$ due to the nuclear forces that the last alpha particle added in constructing a given nucleus sees. It looks something like the sketch below (Fig. 41-7).



If the alpha particle is initially far from the nucleus, it sees a repulsive interaction due to the Coulomb repulsion between the positively charged protons. Work must be done to push the alpha particle closer to the nucleus. However, when the alpha particle gets close enough, i.e. close on a length scale $R_0 \sim 10^{-15}$ m, then it sees the attractive nuclear interaction that binds it to the nucleus.

When the alpha particle binds to the nucleus, there are two possibilities. If its total energy inside the nucleus is *negative*, say $-E$, then the nucleus is stable and will never decay. To extract that last alpha particle from the nucleus one will need to add at least the finite energy E to the nucleus from some external source.

If however the total energy of the alpha particle inside then nucleus is *positive*, say Q, but still less than the maximum potential energy U_{B}, then this nucleus is unstable. If we could somehow get the alpha particle far away from the nucleus, it would have a finite kinetic energy Q. (If $Q > U_{\rm B}$ then the alpha particle would never bind to the nucleus no matter how close it got, so this third possibility is not interesting).

According to *classical* Newtonian mechanics, an alpha particle inside the nucleus, $r < R_0$, with positive total energy $Q < U_{\rm B}$ as in the sketch above, would always stay bound inside the nucleus. It could never enter the region or space $R_0 < r < R_{\rm B}$. To do so would require it to have a potential energy U that was *greater* than its total energy Q, i.e. it would have to have a *negative kinetic energy!* Classically, that can *never* happen, and so the alpha particle would always be trapped inside the nucleus.

However, once we have to take into account the *wave-like* nature of the alpha particle (just as we had to take into account the wave-like nature of the electron inside the atom), then the rules change! It becomes possible for the alpha particle to *tunnel* though the classically forbidden region of negative kinetic energy, and escape outside the nucleus $r > R_{\rm B}$ where it will have a positive kinetic energy $Q - U(r)$. The probability for such *quantum mechanical tunneling* depends on the width of the barrier $R_{\rm B} - R_0$, which depends on the energy Q of the alpha particle inside the nucleus. This probability for tunneling then determines the rate at which the nucleus will decay via alpha particle emission, i.e. the alpha-decay rate.

To see that the wave-like nature of the alpha particle is indeed important here, we can estimate its de Broglie wavelength when it has a kinetic energy Q.

$$Q = \frac{p^2}{2m_\alpha} \quad \Rightarrow \quad p = \sqrt{2m_\alpha Q} \quad \Rightarrow \quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_\alpha Q}} = \frac{hc}{\sqrt{2m_\alpha c^2 Q}}$$

Since $m_\alpha c^2 \simeq 3728\,\mathrm{MeV}$, and typically $Q \simeq 5\,\mathrm{MeV}$, and $hc = (4.136 \times 10^{-15}\,\mathrm{eV}\cdot\mathrm{s})(3 \times 10^8\,\mathrm{m/s}) = 1.24 \times 10^{-6}\,\mathrm{eV}\cdot\mathrm{m}$ one has,

$$\lambda \simeq \frac{1.24 \times 10^{-6}\,\mathrm{eV}\cdot\mathrm{m}}{\sqrt{(2)(3728 \times 10^6\,\mathrm{eV})(5 \times 10^6\,\mathrm{ev})}} \simeq 0.9 \times 10^{-14} \sim R_0$$

So its de Broglie wavelength is indeed of the same length scale as the nuclear size.

5 Beta Decay and the Weak Interaction

In ordinary *beta decay* (also called β^- decay) the nucleus decays by emitting an electron rather than an alpha particle. Since the charge of the electron is $-e$, the net positive charge of the daughter nucleus D must be one unit greater than that of the parent nucleus P, ie. $Z_{\mathrm{D}} = Z_{\mathrm{P}}+1$. Since no nucleons are emitted, $A_{\mathrm{D}} = A_{\mathrm{P}}$.

Beta decay is qualitatively different from alpha decay. In alpha decay, the nucleus just ejects a number of its constituent building blocks, the nucleons (in particular it ejects 4 nucleons in the form of an alpha particle). In β^- decay, the nucleus ejects an electron, even thought the nucleus did not originally contain any electrons! It must be that a neutron of the original nucleus gets converted into a proton and an electron. The resulting proton stays inside the nucleus, while the resulting electron gets ejected! The interaction by which a neutron can convert to a proton plus an electron is known as the *weak interaction*. The strong interaction and the weak interaction are the two new interactions that arise at the nuclear scale. The weak interaction is called "weak" because the kinetic energy of the electron emitted in a typical β^- decay (which we can think of as the binding energy of the electron and proton in forming a neutron!) is about 10 or more times smaller than the kinetic energy of the alpha particle emitted in a typical alpha decay (a measure of the binding energy of nucleons).

Some nuclei decay not by emitting an electron, but by emitting a *positron*. This is known as β^+ decay. A positron is a particle that in all respects is just like an electron (same mass, point-like) except its electric charge is $+e$. It is called the *antiparticle* to the electron. In
bet^+ decay, a proton inside the nucleus is converted via the weak interaction into a neutron and a positron. The neutron remains inside the nucleus while the positron is ejected.

When looking at the tracks made by the particles emitted in beta decay, it became apparent that there was a problem. One could compute the energy and momentum of the initial and final pieces, and it appeared that energy was not conserved! That is, if K is the kinetic energy of the ejected electron in a β^- decay, it was very often found that $m_{\mathrm{P}}c^2 - m_{\mathrm{D}}^2 - m_e c^2 > K$ (where P is the parent and D the daughter nucleus). Similarly, momentum and angular momentum were not conserved. Wolfgang Pauli proposed in 1930 the following solution. It must be that yet another particle gets emitted in the beta decay, and it is this particle that carries off the missing energy and momentum. This new particle was named the *neutrino* ("little neutral one") by Enrico Fermi. The neutrino carries no charge.

An example of a β^- decay is thus

$$^{14}_{6}\mathrm{C} \quad \rightarrow \quad ^{14}_{7}N \quad + \quad e^- \quad + \bar{\nu}$$

where $\bar{\nu}$ is an *anitneutrino* (it is called the antineutrino and not the neutrino for historical reasons).

An example of a β^+ decay is

$$^{19}_{10}\mathrm{Ne} \quad \rightarrow \quad ^{19}_{9}\mathrm{Fe} \quad + \quad e^+ \quad + \quad \nu$$

where ν is the neutrino.

For a long time it was thought that the neutrino was massless like the photon. It is now known that it does have a very small but finite mass $< 0.14\,\mathrm{eV}/c^2$.

6 Gamma Decay

The last means of nuclear decay is *gamma decay* in which the emitted particle has zero electric charge. It is now understood that the particles of this emitted "gamma ray" are just photons of light. Just as electrons in the atom can be in excited states, with energy higher than the ground state, so nucleons in the nucleus can be in excited states. A nucleus can get put into an excited state by a collision with another particle, that transfers energy into the nucleus. Once in the excited state, it can decay back down to a lower energy state by emitting a photon (a "γ-ray"). Such γ-rays can carry energy spanning a range of keV to MeV.

7 Half Life and Decay Rate

The decay of a radioactive nuclei is a probabilistic event, that is we cannot say a given nuclei will have definitely decayed after some specific time has elapsed. Rather we can only give the probability that it will have decayed within a certain time period. The probability to decay within a short time period Δt is observed to be proportional to the length of that time period. One thus defines the *decay rate* λ as the probability per unit time for a decay to occur.

If one starts with a sample of N such nuclei, the number $|\Delta N|$ that will have decayed within a short time interval Δt is just N time the probability of decay, i.e. $|\Delta N| = N\lambda\Delta t$. If ΔN is the change in N (where ΔN is negative because N is decreasing) then

$$\Delta N = -\lambda N \Delta t \quad \Rightarrow \quad \frac{\Delta N}{\Delta t} = -\lambda N \quad \Rightarrow \quad \frac{dN}{dt} = -\lambda N$$

The solution to this differential equation is just a simple exponential decay,

$$\frac{dN}{dt} = -\lambda N \quad \Rightarrow \quad N(t) = N(0)\mathrm{e}^{-\lambda t}$$

with λ the decay rate (just substitute $N(0)\mathrm{e}^{-\lambda t}$ back into the differential equation to see that it works!).

Instead of giving the decay rate λ, one usually gives the <u>*half life*</u> of the radioactive nuclei. The half life, $T_{\frac{1}{2}}$ is the time it takes for an initial population of such nuclei to decrease by half. We can relate $T_{\frac{1}{2}}$ to λ as follows using the decay law above. At time $T_{\frac{1}{2}}$ we have by definition that

$$N(T_{\frac{1}{2}}) = \frac{N(0)}{2}$$

but

$$N(T_{\frac{1}{2}}) = N(0)\mathrm{e}^{-\lambda T_{\frac{1}{2}}} \quad \Rightarrow \quad \frac{N(T_{\frac{1}{2}})}{N(0)} = \frac{1}{2} = \mathrm{e}^{-\lambda T_{\frac{1}{2}}} \quad \Rightarrow \quad \ln\left(\frac{1}{2}\right) = -\lambda T_{\frac{1}{2}} \quad \Rightarrow \quad \ln 2 = \lambda T_{\frac{1}{2}}$$

So

$$\lambda = \frac{\ln 2}{T_{\frac{1}{2}}} \quad \text{or} \quad T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

<u>*carbon 14 dating*</u>: A common method for dating archeological artifacts is carbon 14 dating. Since living organisms generally contain carbon as one of the fundamental elements of organic life, a living organism will have in it isotopes of carbon according to their natural abundance in nature. Most

carbon in nature is ${}^{12}_{6}$C, but a small fraction, about 1.3×10^{-12}, is the isotope ${}^{14}_{6}$C. The half life of ${}^{14}_{6}$C is 5730 years. As long as the organism is alive, ${}^{14}_{6}$C that the organism contains that undergoes radioactive decay is replaced by new ${}^{14}_{6}$C that is absorbed from the environment, and so the balance of ${}^{14}_{6}$C to ${}^{12}_{6}$C within the organism stays the same. Once the organism dies, the ${}^{14}_{6}$C that decays is no longer replaced from the surrounding environment and the ratio of ${}^{14}_{6}$C to ${}^{12}_{6}$C will steadily decrease in time. By measuring this fraction in some artifact, say a piece of wood, and knowing what it is in a still living organism, one can use the decay law to estimate how long it has been since the organism died, i.e. the age of the wood since it was cut down.

8 Fission

We have seen that unstable nuclei, in which the last alpha particle added to make that nuclei has a finite positive energy, will undergo radioactive decay. Other nuclei may be unstable in the following sense: the mass of the nucleus may be larger than the sum of the masses when it is split into two roughly equal pieces. Such a breakup may not occur naturally - the tunneling barrier for such a decay being too large for tunneling to happen in reasonable time - yet if one imparts energy to the nuclei to help it over the barrier, one can then stimulate this break up and receive back more energy than was put in. This is the process of nuclear *fission* . The most famous example is

$$n +^{235}_{92} \mathrm{U} \rightarrow^{141}_{56} \mathrm{Ba} +^{92}_{36} \mathrm{Kr} + 3n$$

By hitting the uranium with the neutron one adds energy to it and excites it to a high energy state. In that excited state it then becomes easier for it to decay into barium and krypton plus three new neutrons, with a net release of energy. The three emitted neutrons can then collide with other uranium nuclei to cause more fission reactions. In an atomic bomb this happens as a chain reaction resulting in a rapid release of enormous amounts of energy. In a nuclear reactor, one uses graphite rods to absorb some of the neutrons so that the fission reactions continue in a controlled way.

9 Fusion

The mass of every stable nucleus is *less* than the mass of its constituent parts when separated. If two protons and two neutrons combine to form ${}^4_2\mathrm{He}$, the result is a loss of mass and an accompanying release of a large amount of energy. This is a nuclear *fusion* reaction. Yet such fusion does not necessarily happen spontaneously. For example, if the nucleons are far apart one needs to add considerable energy to move the nucleons together against the repulsive Coulomb interaction of the protons. Only when the nucleons are close enough will the attractive nuclear forces take over, the nucleons will bind, and more energy than was put in will now be released. Such fusion reactions are the energy source behind the hydrogen bomb (fusing hydrogen to make helium) as well as the energy source of the sun and other stars.