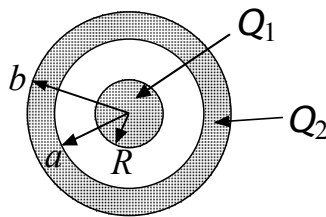


Please put a box around around your final answer. Cross off any work you do not want the grader to consider. Each problem has several parts - not all parts are related to the preceding parts. Many parts do not require an involved calculation to get the answer.

**Problem 1** [32 points total]

a) [8 points] Consider a conducting sphere of radius  $R$  that is held at a potential  $\phi_0$  statvolts with respect to infinity (i.e. the potential at infinity is zero). What is the total charge on the conductor?

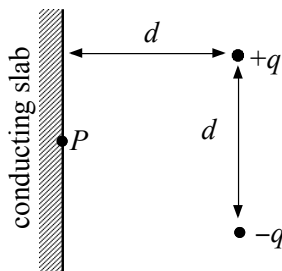
b) [8 points]



Consider a solid conducting sphere of radius  $R$  that holds a total charge  $Q_1$ . It is surrounded by a concentric spherical shell of inner radius  $a > R$  and outer radius  $b$  as shown in the figure above. The total charge on the shell is  $Q_2$ . What is the electric field  $\mathbf{E}$  everywhere in space? i.e. for  $r < R$ ,  $R < r < a$ ,  $a < r < b$ ,  $b < r$ ? How are the charges  $Q_1$  and  $Q_2$  distributed?

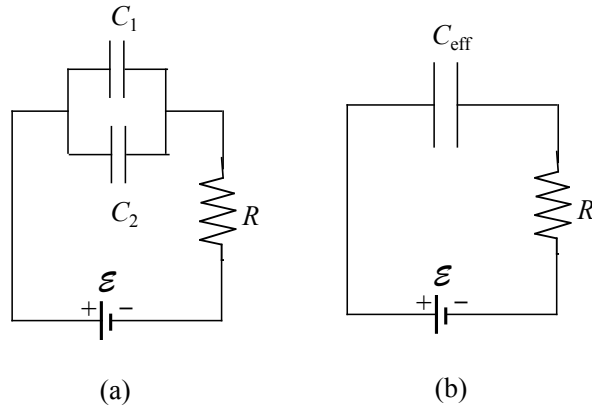
c) [8 points] Consider a positive point charge  $q$  a distance  $r$  from the center of a neutral conducting sphere of radius  $R$  ( $r > R$ ). Is there a force between  $q$  and the sphere? If so, is it attractive or repulsive? Give a *brief* explanation of the physics of your answer (i.e. don't just guess!).

d) [8 points]

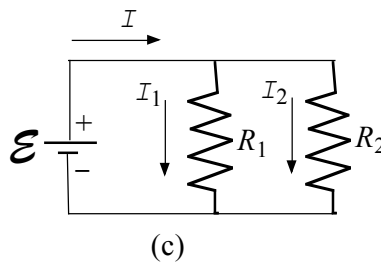


Consider two charges  $+q$  and  $-q$  positioned a distance  $d$  in front of a grounded infinite flat conducting slab. The two charges are a distance  $d$  apart from each other, as shown in the figure above. What is the induced surface charge density  $\sigma$  at the point  $P$  on the surface of the slab exactly in the middle of the two charges?

**Problem 2.** [34 points total]



- a) [6 points] Consider two capacitors, with capacitances  $C_1$  and  $C_2$  respectively, wired together in the circuit shown in figure (a) above. The battery has emf  $\mathcal{E}$  and the resistor has resistance  $R$ . In steady state (i.e. after a long time when nothing is changing) what is the value of the charges  $Q_1$  and  $Q_2$  on the positive plates of each capacitor?
- b) [8 points] If we wish to replace the circuit in figure (a) by the one in figure (b), what should be the effective capacitance  $C_{\text{eff}}$  of the single capacitor in figure (b) so that the electrical properties of the circuit remain the same as that in figure (a)?
- c) [6 points] If the battery in circuit (a) is suddenly short circuited (i.e. replaced by a wire) at time  $t = 0$ , what will be the time constant with which the voltage across capacitor  $C_1$  decays to zero? What will be the time constant with which the voltage across capacitor  $C_2$  decays to zero?



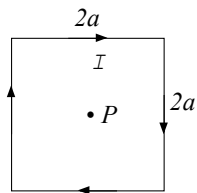
- d) [8 points] Consider the circuit in figure (c) above. What fraction of the total current  $I$  flows through resistor  $R_1$  and what fraction flows through resistor  $R_2$ ?
- e) [6 points] Again considering the circuit in figure (c) above, if  $R_1 > R_2$ , which resistor dissipates more power?

**Problem 3.** [34 points total]

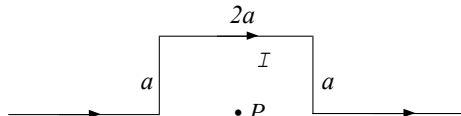


(a)

a) [8 points] Consider an infinite straight wire with a circular cross-sectional area of radius  $R$  as shown in figure (a) above. The wire carries a total current  $I$  flowing down its length. The current is distributed uniformly over the cross-sectional area. Find the resulting magnetic field  $\mathbf{B}$  everywhere, i.e. both outside and inside the wire.



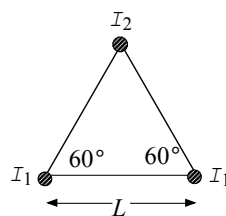
(b)



(c)

b) [10 points] Consider a square wire loop as shown in figure (b) above. The sides of the square have length  $2a$ . The loop carries a current  $I$  flowing clockwise. What is the magnetic field  $\mathbf{B}(P)$  at point  $P$  directly in the center of the loop? You only need to write down the integral from which one would evaluate  $\mathbf{B}(P)$ . You do not need to do the integral (but make sure your answer is in an explicit enough form that the integral could be looked up in a table or online). Be sure to specify the direction of  $\mathbf{B}$ .

c) [8 points] Consider a wire as shown in figure (c) above. The left most and right most straight segments of the wire continue off to infinity. The kink in the middle has length  $2a$  and height  $a$  as shown. The wire carries a current  $I$  flowing to the right. What is the magnetic field at the point  $P$ ? How does it compare to the result in part (b)?



(d)

d) [8 points] Consider three infinitely long straight current carrying wires configured as in figure (d) above. The wires are running perpendicular to the page. The bottom two wires lie along the ground and carry equal currents  $I_1$  flowing out of the page. The upper wire carries a current  $I_2$  and is suspended vertically above the ground, with the downward force of gravity balanced by the magnetic force from the two bottom wires. What is the direction of the current  $I_2$ ? If the upper wire has a mass density per unit length of  $m$ , what is the magnitude of the current  $I_2$ ?