Electrostatics

Charge and Coulomb's law

Matter possesses various properties that cause forces of interaction between particles.

You already know one such property — the mass. The mass of a particle determines the magnitude of the gravitational force it exerts on other particles. Mass is always a positive scalar (ie real number).

Another such property is charge. Charge comes in two flavors, positive and negative. It is observed that like charges (charges of the same sign) repell, while opposite charges attract — this is unlike gravitation which is always attractive. The charge of a particle is a scalar quantity that can be positive or negative.

Charge is conserved

The total charge of all particles in a box remains constant in time, provided no particles pass through the walls bounding the box. Particles can be created and destroyed in the box; particles can combine and separate, but the total charge remains constant.

Charge is quantized

It is found that all charges observed freely in nature are integer multiples of the charge on the electron. We denote the magnitude of this charge by e.

The charge on the electron is $-e$

The charge on the proton is $+e$

[protons, and other strongly interacting particles (hadrons) are now believed to be made up of subparticles called quarks. The charge on a quark can be either $\pm\frac{1}{3}e$ or $\pm\frac{2}{3}e$. However it is not possible to observe a *free* quark. Quarks always come bound in triplets or pairs whose total charge is an integer multiple of e]

~~Coulomb's Force Law~~

Although charges are quantized in units of e, ~~a fact of great importance in particle physics~~ in *classical* electromagnetic theory we will generally take the charge q of a particle to be any real number.

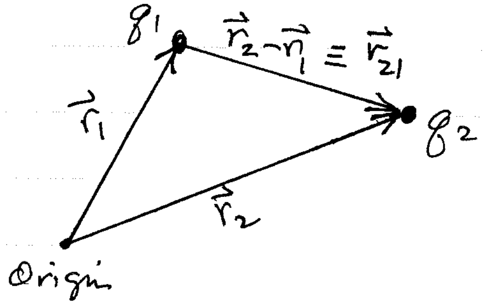
Coulomb's Force Law

Consider a particle 1 with charge q_1 at position $\vec{r}_1$ and a particle 2 with charge q_2 at position $\vec{r}_2$.

When both charges are at <u>rest</u> (velocities $\vec{v}_1 = \vec{v}_2 = 0$) the force between the two charges is given by Coulomb's Law

The force on particle 2 due to particle 1 is

$$\vec{F}_2 = \frac{k\, q_1 q_2}{r_{21}^2}\, \hat{r}_{21}$$



Here $\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$ displacement from q_1 to q_2

$r_{21} \equiv |\vec{r}_{21}|$ distance from q_1 to q_2

$\hat{r}_{21} \equiv \dfrac{\vec{r}_{21}}{r_{21}}$ unit vector in direction from q_1 to q_2

Direction of force $\vec{F}_2$ is along line connecting q_1 to q_2

If $q_1 q_2 > 0$ (charges have same sign) then $\vec{F}_2$ is in direction $+\hat{r}_{21}$ $\Rightarrow$ points away from q_2

$\Rightarrow$ force is <u>repulsive</u>

If $q_1 q_2 < 0$ (charges have opposite signs) then $\vec{F}_2$ is in direction $-\hat{r}_{21}$ $\Rightarrow$ points into q_1

$\Rightarrow$ force is <u>attractive</u>

$|\vec{F}_2| \sim \frac{1}{r_{21}^2}$ decreases as square of distance between charges "inverse square" force law.

Similarly, force on q_1 due to q_2 is

$$\vec{F}_1 = \frac{k q_1 q_2}{r_{12}^2} \hat{r}_{12} = -\vec{F}_2 \quad \text{since} \quad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = -\vec{r}_{21}$$

$$\Rightarrow |\vec{r}_{12}| = |\vec{r}_{21}| \text{ and } \hat{r}_{12} = -\hat{r}_{21}$$

So force on q_1 from q_2 is equal but opposite to force on q_2 from q_1 (Newton's 3rd law)

In the above Coulomb's law, k is a universal constant of nature. It does NOT depend on the magnitudes of the charges that are interacting nor on their positions.

The numerical value of k depends on what system of units we are using. Since charge is a new physical quantity, we could always choose the units of charge so that the constant k comes out simple.

This is what is done in CGS units

CGS units:

			conversion to MKS
length	cm	centimeters	cm $= 10^{-2}$ m
time	s	seconds	s = s
mass	g	grams	g $= 10^{-3}$ kg
force	dyn	dyne	dyn $= 10^{-5}$ N
energy	erg	erg	erg $= 10^{-7}$ J

In CGS units one measures the unit of charge so that $k = 1$. The unit of charge in CGS is the "esu" or electrostatic unit. The esu is defined so that 2 charges, each with charge 1 esu, separated by a distance 1 cm, will feel a force of 1 dyne.

$$1 \text{ dyne} = \frac{(1\,\text{esu})^2}{(1\,\text{cm})^2}$$

$$\Rightarrow \quad 1 \text{ esu} = \text{cm}\sqrt{\text{dyn}}$$

In MKS (also called SI, or standard) units, the charge was defined historically to be the "Coulomb" or coul. In MKS units

$$k = 8.988 \times 10^{9} \, \frac{\text{N}\cdot\text{m}^2}{\text{coul}^2}$$

Conversion of charge from MKS to CGS

$$\boxed{1 \text{ coul} = 2.998 \times 10^{9} \text{ esu}}$$

proof: Consider two 1 coul charges separated by 1 m. The force between them has magnitude

$$F = k\frac{(1\,\text{coul})^2}{(1\,\text{m})^2} = 8.988\times10^9\ \frac{\text{N m}^2}{\text{coul}^2}\cdot\frac{\text{coul}^2}{\text{m}^2}$$
$$= 8.988\times10^9\,\text{N}$$

Suppose 1 coul = Q esu then we have

$$F = 8.988\times10^9\,\text{N} = 8.988\times10^9\times10^5\ \text{dyne} = \frac{(Q\ \text{esu})^2}{(10^2\,\text{cm})^2}$$

$$(Q\ \text{esu})^2 = 8.988\times10^{18}\ \text{dyn cm}^2$$

$$Q\ \text{esu} = 2.998\times10^9\ \text{cm}\sqrt{\text{dyn}}$$

since esu = cm$\sqrt{\text{dyn}}$ we have $Q = 2.998\times10^9$

So $\underline{1\ \text{coul} = 2.998\times10^9\ \text{esu}}$

In this course ~~we~~ will be using <u>CGS units</u>

In MKS units it is customary to write the constant k as $k = \frac{1}{4\pi\varepsilon_0}$ so Coulomb's Law becomes

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

ε_0 is called the "permittivity of free space".

The value of the magnitude of the electron charge is

$$e = 4.8023 \times 10^{-10} \text{ esu} = 1.6022 \times 10^{-19} \text{ coul}$$

Note Coulomb's Law has exactly the same form as Newton's law of gravitation

$$\vec{F}_2 = -G \frac{m_1 m_2}{r_{21}^2} \hat{r}_{21}$$

m_1 mass of particle 1 at $\vec{r}_1$
m_2 mass of particle 2 at $\vec{r}_2$

$$G = 6.67 \times 10^{-11} \; m^3/kg\, s^2$$
$$= 6.67 \times 10^{-11} \cdot \frac{(10^2 cm)^3}{(10^3 g)\, s^2} = 6.67 \times 10^{-8} \frac{cm^3}{g\, s^2}$$

Since $m_1 > 0$, $m_2 > 0$, the $(-)$ sign gives always an attractive gravitational force

Interesting to compare the Coulomb and gravitational force between an electron and a proton

electron mass $\quad m_e = 9.11 \times 10^{-28}\, g$

proton mass $\quad m_p = 1.67 \times 10^{-24}\, g$

$$\frac{F_{elec}}{F_{grav}} = \frac{e^2}{r^2} \frac{r^2}{G m_e m_p} = \frac{e^2}{G m_e m_p}$$

since both Coulomb and gravity $\sim \frac{1}{r^2}$ the ratio is independant of the separation r.

$$\frac{F_{elec}}{F_{grav}} = \frac{(4.80 \times 10^{-10}\, esu)^2}{\left(6.67 \times 10^{-8} \frac{cm^3}{g\, s^2}\right)(9.11 \times 10^{-28}\, g)(1.67 \times 10^{-24}\, g)}$$

$$= 2.27 \times 10^{39} \frac{esu^2}{\left(\frac{cm^3 g}{s^2}\right)} = 2.27 \times 10^{39} \; !$$

to see that units are correct, use $esu^2 = cm^2 dyn$

$$= cm^2 \left(\frac{g\, cm}{s^2}\right) = \frac{cm^3 g}{s^2}$$

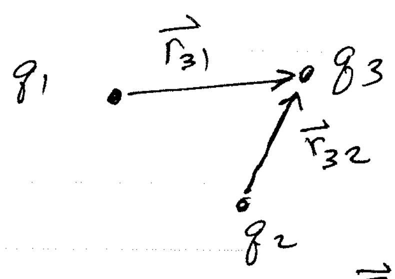
electric force is $\sim 10^{39}$ times stronger than gravitational force. This is why we can ignore gravity when considering problems at the atomic scale

Only when we are considering "large" objects (balls, planets) where m is large and the object is electrically neutral or close to it (equal numbers of electrons and protons) will F_{grav} dominate over F_{elec}.

Coulomb's force is directed along the direction from q_1 to q_2.
This is reasonable since for <u>stationary</u> charges in <u>uniform space</u> the only direction that is uniquely defined is $\hat{r}_{21}$. There are simply no other vector quantities around, from which one could try to make the vector force $\vec{F}$. This will no longer be so if one or both of the charges are <u>moving</u>. Then the velocities $\vec{v}_1$ and $\vec{v}_2$ give other vector directions; similarly accelerations $\vec{a}_1$ and $\vec{a}_2$ need not be zero.

Principle of Superposition

when there are more than two charges, the total electric force on a given charge q_i is the sum of Coulomb forces from all other charges q_j



$$\vec{F}_3 = \frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} + \frac{q_2 q_3}{r_{32}^2}\hat{r}_{32}$$

similarly,

$$\vec{F}_2 = \frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{q_3 q_2}{r_{23}^2}\hat{r}_{23}$$

$$\vec{F}_1 = \frac{q_2 q_1}{r_{12}^2}\hat{r}_{12} + \frac{q_3 q_1}{r_{13}^2}\hat{r}_{13}$$

For N charges q_i $\quad i = 1, N$ $\quad$ force on a particular q_i is

$$\vec{F}_i = \sum_{j \neq i} \frac{q_j q_i}{r_{ij}^2}\hat{r}_{ij} = q_i \sum_{j \neq i} \frac{q_j}{r_{ij}^2}\hat{r}_{ij}$$