Units

CGS

$$\vec{F} = q\vec{E} + q\,\frac{\vec{v}}{c}\times\vec{B} \qquad \text{Lorentz force}$$

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$\oint_S \vec{E}\cdot d\vec{a} = 4\pi Q_{encl}$ $\quad$ $\vec{\nabla}\cdot\vec{E} = 4\pi\rho$ $\quad$ $\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J}$ $\quad$ $\oint_C \vec{B}\cdot d\vec{s} = \frac{4\pi}{c} I_{encl}$

$\oint_C \vec{E}\cdot d\vec{s} = 0$ $\quad$ $\vec{\nabla}\times\vec{E} = 0$ $\quad$ $\vec{\nabla}\cdot\vec{B} = 0$ $\quad$ $\oint_S \vec{B}\cdot d\vec{a} = 0$

From Lorentz force we see that units of B are same as units of E, ie dyne/esu or equivalently statvolt/cm.

we can also see this from $\vec{\nabla}\cdot\vec{E} = 4\pi\rho \Rightarrow \frac{E}{cm} \sim \frac{esu}{cm^3} \Rightarrow E \sim \frac{esu}{cm^2}$

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} \Rightarrow \frac{B}{cm} \sim \left(\frac{s}{cm}\right)\left(\frac{esu}{cm^2\text{-}s}\right) = \frac{esu}{cm^2}$$

when measuring B fields one introduces the new unit

1 "gauss" = 1 statvolt/cm (only for B, not for E!)

MKS

$$\vec{F} = q\vec{E} + q\,\vec{v}\times\vec{B}$$

in MKS units of B defined so there is no $\frac{1}{c}$ in Lorentz force

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$\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\varepsilon_0}$ $\quad$ $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ $\quad$ $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ $\quad$ $\oint_C \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$

$\oint_C \vec{E}\cdot d\vec{s} = 0$ $\quad$ $\vec{\nabla}\times\vec{E} = 0$ $\quad$ $\vec{\nabla}\cdot\vec{B} = 0$ $\quad$ $\oint_S \vec{B}\cdot d\vec{a} = 0$

ε_0 is a constant of nature determined by historical units chosen for charge

μ_0 is a constant of nature determined by historical units chosen for B

units of E are volt/m, units of B are $\frac{volt}{m}\,\frac{s}{m} = \frac{volt\text{-}s}{m^2} = \frac{\text{"weber"}}{m^2}$

$= \text{"tesla"}$

What is value of μ_0? We can determine it by making the MKS equations agree with the CGS equations

in CGS $\quad \dfrac{\vec{\nabla}\cdot\vec{E}}{|\vec{\nabla}\times\vec{B}|} = \dfrac{4\pi\rho}{\frac{4\pi}{c}|\vec{J}|} = \dfrac{c\rho}{|\vec{J}|}$ is dimensionless

in CGS magnetic force is $q\frac{\vec{v}}{c}\times\vec{B}$
in MKS magnetic force is $q\,\vec{v}\times\vec{B}$ $\Big\} \Rightarrow (qB)_{MKS} = \left(\frac{qB}{c}\right)_{CGS}$
$\Rightarrow cB_{MKS} = B_{CGS}$

in MKS $\quad \dfrac{\vec{\nabla}\cdot\vec{E}}{c\,|\vec{\nabla}\times\vec{B}|} = \dfrac{\rho/\epsilon_0}{c\,\mu_0|\vec{J}|} = \dfrac{1}{c\epsilon_0\mu_0}\dfrac{\rho}{|\vec{J}|}$ is dimensionless

$$\Rightarrow \left[\frac{c\rho}{|\vec{J}|}\right]_{CGS} = \left[\frac{1}{c\epsilon_0\mu_0}\frac{\rho}{|\vec{J}|}\right]_{MKS} \Rightarrow \boxed{c^2 = \frac{1}{\epsilon_0\mu_0}}$$

That the constants of nature ϵ_0 and μ_0 defined experimentally in MKS units combined to give the speed of light as above was one of the indicators that light was an electro-magnetic phenomenon. CGS units were devised with this knowledge already in place, which is why "c" appears explicitly in the equations of electromagnetism in CGS units

$\epsilon_0 = 8.85\times10^{-12}\ \frac{\text{coul}}{\text{volt-m}} \qquad \mu_0 = \frac{1}{\epsilon_0 c^2} = 4\pi\times10^{-7}\ \frac{\text{volt}}{\text{coul}}\frac{s^2}{m}$

$$\frac{1}{\sqrt{\epsilon_0\mu_0}} = 2.999\times10^{8}\ m/s = c \quad \text{speed of light}$$

Conversion between tesla and gauss. $(B)_{CGS} = (cB)_{MKS}$

$$1 \text{ tesla} = 1 \frac{\text{volt-s}}{m^2} = \frac{1}{300} \frac{\text{statvolt-s}}{(10^2 \text{cm})^2} = \frac{\text{statvolt-s}}{3\times10^6 \text{ cm}^2}$$

$$c\,(1 \text{ tesla}) = \left(3\times10^{10} \frac{\text{cm}}{\text{s}}\right)\left(\frac{\text{statvolt-s}}{3\times10^6 \text{ cm}^2}\right) = 10^4 \frac{\text{statvolt}}{\text{cm}} = 10^4 \text{ gauss}$$

so $\boxed{1 \text{ tesla} = 10^4 \text{ gauss}}$

In MKS, B from infinite straight wire with current I is

$$B = \frac{\mu_0}{4\pi} \frac{2I}{r}$$

I in amps, r in m
B in tesla

Force per length on parallel wires in MKS is

$$\vec{f} = \frac{\mu_0}{4\pi} \frac{2 I_1 I_2}{d}$$

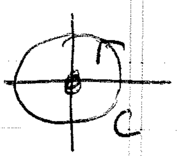
$\vec{f}$ in newtons/m

$$\oint_C \vec{B}\cdot d\vec{s} = \frac{4\pi}{c} I_{encl}$$

infinite straight wire along $\hat{z}$ axis with current I.

by symmetry $\vec{B}(\vec{r}) = B(r)\,\hat{\varphi}$

cylindrical radial coord



$$\oint_C \vec{B}\cdot d\vec{s} = B(r)\,2\pi r = \frac{4\pi}{c} I_{encl}$$

circle of radius r

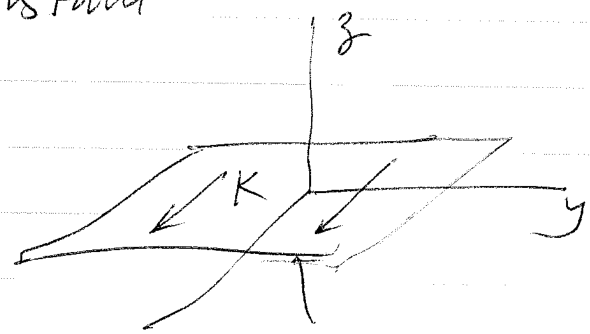
$$B(r) = \frac{2I}{rc}$$

$$\vec{B}(\vec{r}) = \frac{2I}{rc}\,\hat{\varphi}$$

Flat plane with ~~curre~~ surface current density $\vec{K}$ = constant

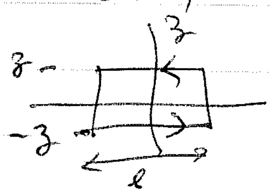
xy plane at $z=0$

$\vec{K} = K\hat{x}$



$\vec{B}$ curls around direction $\hat{x}$ of $\vec{K}$ $\Rightarrow$ $\vec{B} \sim \begin{cases} -\hat{y} & z>0 \\ +\hat{y} & z<0 \end{cases}$

$\vec{B}(\vec{r}) = B(z)\,\hat{y}$ with $B(-z) = -B(z)$



$$\oint_C \vec{B}\cdot d\vec{s} = lB(-z) - lB(z) = -2lB(z) = \frac{4\pi}{c} Kl$$

$\vec{B}(z) = -\frac{2\pi}{c} K\hat{y}$ $z>0$, $B(z) = \frac{2\pi}{c} K\hat{y}$ $z<0$

discontinuity in $\vec{B}$ across a current sheet

write $\vec{B}_{above} - \vec{B}_{below} = -\frac{2\pi}{c}K\hat{y} - \frac{2\pi}{c}K\hat{y} = -\frac{4\pi}{c}K\hat{y}$

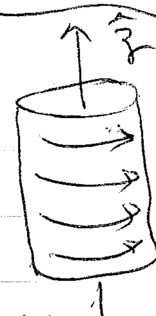
use $\hat{z}\times\hat{x} = \hat{y}$ or $-\hat{y} = \hat{x}\times\hat{z}$ use $\vec{K} = K\hat{x}$

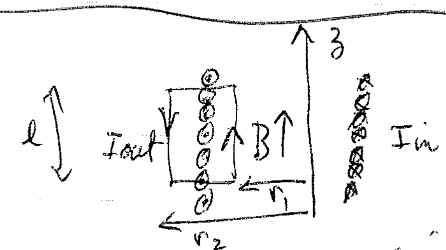
outward normal is $\hat{n} = \hat{z}$

$\vec{B}_{above} - \vec{B}_{below} = \frac{4\pi}{c}K(\hat{x}\times\hat{z})$

$$\boxed{\vec{B}_{above} - \vec{B}_{below} = \frac{4\pi}{c}(\vec{K}\times\hat{n})}$$ true in general for any surface current density

solenoid





cross sectional view

$\vec{B}(\vec{r}) = B(r)\hat{z}$ r is cylindrical radial coord

$$\oint \vec{B}\cdot d\vec{s} = B(r_1)l - B(r_2)l = \frac{4\pi}{c}INl$$

N = # turns of wire per unit length

solenoid can have any crosssectional shape - need not be circular

$$B(r_1)l - B(r_2)l = \frac{4\pi}{c}INl$$

assume $B = 0$ outside infinitely long solenoid

then $B(r_1) = \frac{4\pi}{c}IN$ indep of r_1

$$\Rightarrow \vec{B} = \begin{cases} 0 & \text{outside} \\ \frac{4\pi}{c}IN\hat{z} & \text{inside} \end{cases}$$

constant B inside solenoid

also gives $\vec{B}_{above} - \vec{B}_{below} = -\frac{4\pi}{c}IN\hat{z} = \frac{4\pi}{c}(\vec{K}\times\hat{n})$, $\vec{K} = IN\hat{\varphi}$