Motion of a charged particle in a uniform B-field

Charge q, velocity $\vec{v} \perp \vec{B}$ initially

$\vec{B} = B\hat{z}$, $\vec{v}$ in xy plane say $\vec{v} = v\hat{x}$

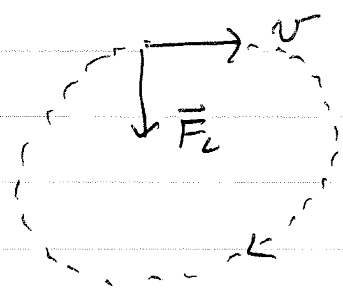
force on q is $\vec{F}_L = q\frac{\vec{v}}{c} \times \vec{B}$

$$= qB\frac{v}{c}(\hat{x} \times \hat{z}) = -q\frac{Bv}{c}\hat{y}$$

force is $\perp \vec{v}$ $\Rightarrow$ B can do no work on q

$\Rightarrow$ kinetic energy of q stays constant $\Rightarrow$ speed v is constant

same as for circular motion!



$$F_L = m\underset{\substack{\uparrow \\ \text{centripetal} \\ \text{acceleration}}}{a_c} = \frac{mv^2}{\underset{\substack{\uparrow \\ R \text{ is radius} \\ \text{of orbit}}}{R}}$$

so $$\frac{mv^2}{R} = \frac{qBv}{c}$$

$$R = \frac{mv^2 c}{qBv} = \frac{mvc}{qB}$$

Time to make one orbit is

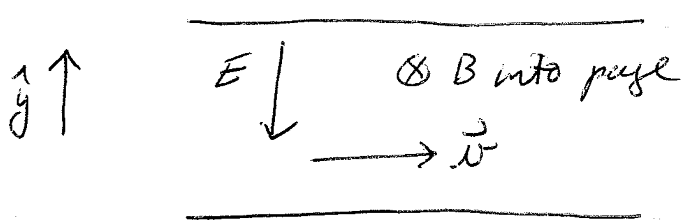
$$\tau = \frac{2\pi R}{v} = \frac{2\pi}{v}\frac{mvc}{qB} = 2\pi\frac{mc}{qB}$$

angular freq of orbit is $\omega = \frac{2\pi}{\tau} = \frac{qB}{mc}$ "cyclotron" frequency

Motion in perpendicular uniform $\vec{E}$ and $\vec{B}$

$$\vec{F} = q\vec{E} + q\frac{\vec{v}}{c}\times\vec{B}$$

Consider $\vec{E}$ down, $\vec{B}$ into page



for example, $\vec{E}$ could be field between two capacitor plates

$$\vec{F}_{elec} = q\vec{E} = -qE\hat{y} \qquad \text{down}$$

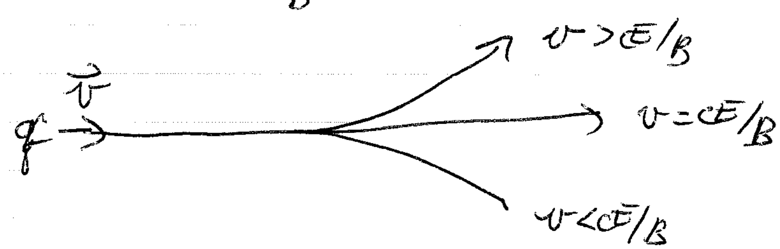
$$\vec{F}_{mag} = q\frac{\vec{v}}{c}\times\vec{B} = q\frac{v}{c}B\,\hat{y} \qquad \text{up}$$

$$\vec{F} = \vec{F}_{elec} + \vec{F}_{mag} = q\left(\frac{vB}{c} - E\right)\hat{y}$$

$$= 0 \quad \underline{\text{when}} \quad v = \frac{cE}{B}$$

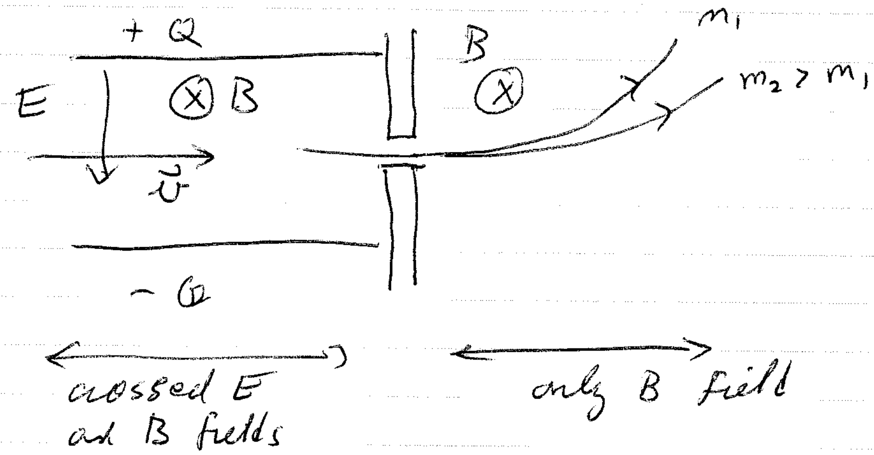
When $v = \frac{cE}{B}$, $F = 0$, and particle travels straight through at constant velocity $\vec{v}$.

1) For $v > \frac{cE}{B}$, $\vec{F}$ is in $+\hat{y}$ direction
2) For $v < \frac{cE}{B}$, $\vec{F}$ is in $-\hat{y}$ direction



particles is deflected upwards or downwards depending on the magnitude of its velocity.

mass spectrometer



For fixed $\vec{B}$, adjust $\vec{E}$ so that particle goes straight through and comes out opening into uniform B only.

$\Rightarrow \quad v = \dfrac{cE}{B}$

once in magnetic field B with $E = 0$, particle will undergo cyclotron motion, orbiting in a circle with

radius $\quad R = \dfrac{mvc}{qB}$

substitute in $v = \dfrac{cE}{B} \quad \Rightarrow \quad R = \dfrac{mc^2 E}{qB^2}$

we know B, E, can measure R, so we can get

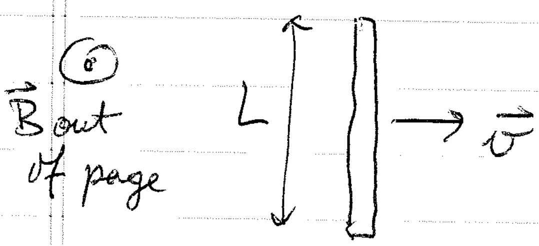
$$m = \frac{RqB^2}{c^2 E}$$

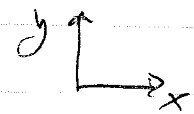
we can then measure m if we know q.

Electromagnetic Induction

changing magnetic fields create electric fields!

Consider motion of conducting rod of length L that moves with velocity $\vec{v}$ perpendicular to its length in a uniform magnetic field $\vec{B}$ that is perpendicular to the plane in which the rod moves



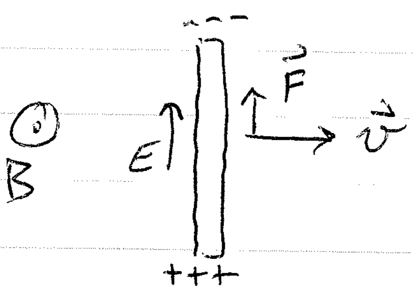


$$\vec{v} = v\hat{x}$$
$$\vec{B} = B\hat{z}$$

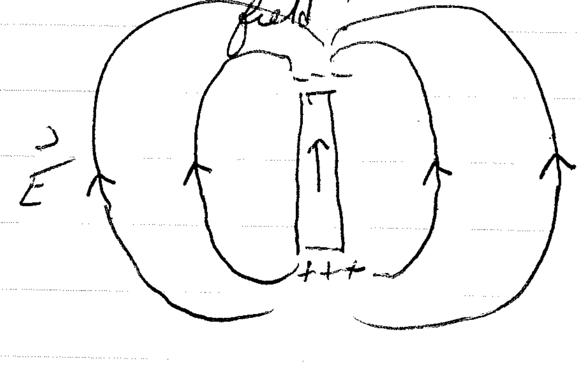
an electron of charge $-e$ in the conducting rod will feel a Lorentz force

$$\vec{F} = -e\frac{\vec{v}}{c}\times\vec{B} = -e\frac{v}{B}B\,(\hat{x}\times\hat{z}) = \frac{evB}{c}\,\hat{y}$$

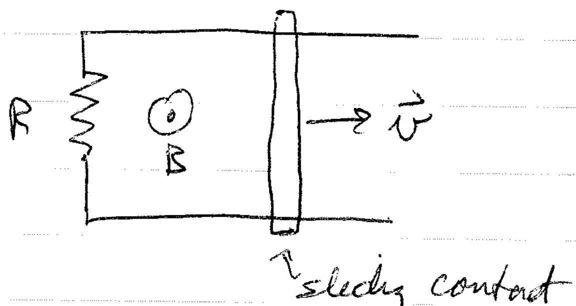
upwards



electrons accumulate at top of rod, leaving excess of positive charge at bottom of rod. Result is an electric field!



Now suppose a sliding electric contact connects the rod to an electric circuit



The magnetic force on the electrons in the rod will cause an electric current to flow around the circuit

electrons flow counterclockwise
$\Rightarrow$ electric current flows clockwise

The moving rod is playing a role just like a battery!

As the electron moves from the bottom of the rod to the top, work is done on the electron

$$W = \int_0^L d\vec{s} \cdot \vec{F} = \frac{evB}{c} L$$

The work per unit charge is the potential difference

$$\phi(L) - \phi(0) = \frac{W}{-e} = -\frac{vB}{c} L$$

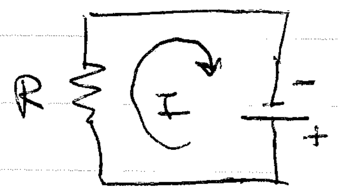
electrostatic potential at bottom of rod

electrostatic potential at top of rod

There is thus a voltage drop

$$\Delta V \equiv \phi(0) - \phi(L) = \frac{vB}{c} L$$

down the rod - just like across the terminals of a battery. The moving rod is therefore equivalent to the circuit



$\mathcal{E} = \frac{vBL}{c}$ electromotive force "emf"

$$\frac{vBL}{c} = \mathcal{E} = IR \Rightarrow I = \frac{vBL}{cR}$$ current around loop

This sort of emf, due to the motion of a conductor in a magnetic field, is called a <u>motional emf</u>

~~down the rod~~

But how does the $\vec{B}$-field do work on the electron? $\vec{v}\cdot(\vec{v}\times\vec{B}) = 0$!

The rod constrains the charges to move vertically instead of in a circle as they otherwise would like to. It is the sides of the rod that exert forces on the electrons to keep the motion vertical. This force cancels part of the magnetic force.

The energy creating the emf really comes from the force which pushes the rod through the $\vec{B}$ field at velocity $\vec{v}$. If the pushing force stopped, the rod would slow to a stop even if the sliding contacts were frictionless. The kinetic energy of the moving rod gets transfered to the emf of the electrons until all the kinetic energy of the rod is exhausted and the rod stops moving (at which point the current stops flowing).

The magnetic field in this case is just serving to transfer the mechanical energy of pushing the rod into electrical energy. emf.