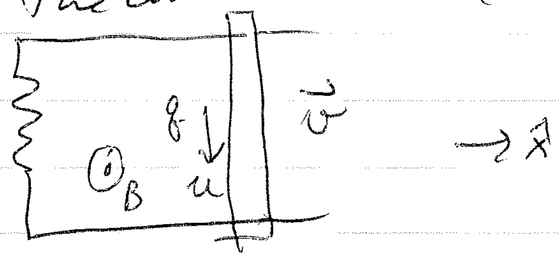
Where does the work come from that drives the current around the loop?



$\vec{F} = q\frac{\vec{u}}{c}\times\vec{B} = q\frac{u}{c}B\,(-\hat{x})$ force on one charge

force acts against direction of motion

total force on current:

$$\vec{F} = \underbrace{(nAL)}_{\text{number of charges in rod}}\, q\frac{uB}{c}\,(-\hat{x})$$

density of charges, cross sectional area of rod, length of rod

use $J = qun$, $I = qunA$

$$\vec{F} = -L\frac{IB}{c}\hat{x}$$

F_{mech} is force of person pushing rod - must balance out magnetic force so $F_{mech} = -F$

work to move rod Δx is $W = F_{mech}\Delta x$

$$W = L\frac{IB}{c}\Delta x$$

~~power dissipated in circuit is I^2R~~

Now $I = \frac{\mathcal{E}}{R} = \frac{vB}{c}\frac{L}{R}$

So $W = \frac{LB}{c} \Delta x \; \frac{vBL}{cR} = \left(\frac{LB}{c}\right)^2 \frac{v \Delta x}{R}$

The ~~tot~~ energy dissipated in this time is

$$P \Delta t = \cancel{I} \frac{\mathcal{E}^2}{R} \Delta t = \left(\frac{vBL}{c}\right)^2 \frac{\Delta t}{R}$$

$$= \left(\frac{LB}{c}\right)^2 \frac{v}{R} (v \Delta t)$$

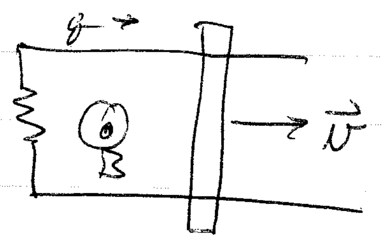
$$= \left(\frac{LB}{c}\right)^2 \frac{v \Delta x}{R}$$

So the work done moving the rod = energy dissipated by current in resistor.

The emf of a circuit is the work done by an external force moving a charge q around the circuit.

$$W = \oint_{\text{around circuit}} \vec{F} \cdot d\vec{s} \qquad \mathcal{E} = \frac{W}{q} = \oint \frac{\vec{F}}{q} \cdot d\vec{s}$$

For the circuit with sliding rod, a positive q moving clockwise around circuit in direction of the current feels force



$$\vec{F} = q \frac{\vec{v}}{c} \times \vec{B} = \frac{qvB}{c} \text{ down}$$

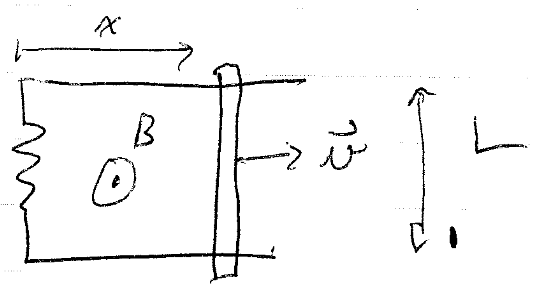
when traveling in the rod

$$\Rightarrow \quad \mathcal{E} = \oint \frac{\vec{F}}{q} \cdot d\vec{s} = \frac{vB}{c} L$$

just like we argued before.

Note: there is also a component of the charge's velocity $\vec{u}$, parallel to the path of the circuit, that represents the motion of the charge down the circuit. But since $\vec{u} \parallel d\vec{s}$, then $\vec{F} = q \frac{\vec{u}}{c} \times \vec{B}$ is always $\perp d\vec{s}$, so $\vec{F} \cdot d\vec{s} = 0$ from this component of the charges velocity.

Note: as the rod slides, the total magnetic flux passing through the circuit loop is changing.



magnetic flux through loop out of page is

$$\Phi = BLx$$

as rod moves, flux changes

$$\frac{d\Phi}{dt} = BL\frac{dx}{dt} = BLv = c\mathcal{E}$$

Since the current flows clockwise around loop, we will adopt convention that we should compute the flux in a direction consistent with the current via the right hand rule

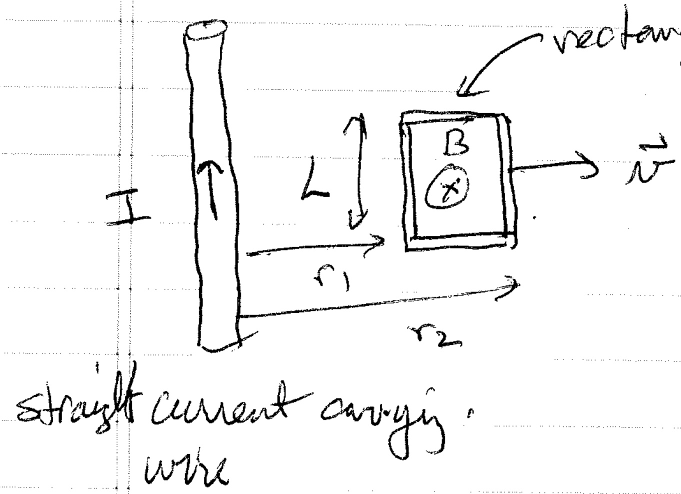
I $\Rightarrow$ flux computed <u>into</u> the page

with this sign convention we have

$$\Phi = -BLx$$

$$\frac{d\Phi}{dt} = -c\mathcal{E} \Rightarrow \boxed{\mathcal{E} = -\frac{1}{c}\frac{\partial\Phi}{\partial t}}$$

consider now a different situation



$r_2 - r_1 = w$ fixed

Magnetic field B from straight wire passes through loop, directed into the page

Consider q going around loop clockwise

Force on q in segment at r_2 is $\left| q \frac{\vec{v}}{c} \times \vec{B} \right| = \frac{q v B(r_2)}{c}$ directed <u>upwards</u>.

Similarly force on q in segment at r_1 is $\left| q \frac{\vec{v}}{c} \times \vec{B} \right| = \frac{q v B(r_1)}{c}$ directed upwards

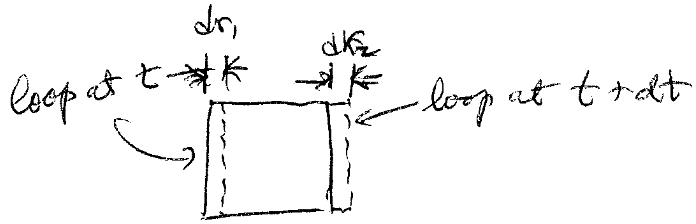
$$\oint \frac{\vec{F}}{q} \cdot d\vec{s} = L \left[\frac{v B(r_1)}{c} - \frac{v B(r_2)}{c} \right] = \mathcal{E}$$

clockwise

If B was <u>uniform</u>, we would have $\mathcal{E} = 0$.
But here B is <u>NOT</u> uniform.

since $r_1 < r_2$, $B(r_1) > B(r_2)$ for field from straight wire, and so $\mathcal{E} > 0 \Rightarrow$ current does circulate clockwise.

Now compute the magnetic flux through the loop, with flux directed into page, consistent with right hand rule.



$$\Phi = L \int_{r_1}^{r_2} dx\, B(x)$$

change in flux is $L B(r_2)\, dr_2 - L B(r_1)\, dr_1$

$$\frac{d\Phi}{dt} = \frac{d}{dt}\left[L \int_{r_1(t)}^{r_2(t)} dx\; B(x) \right]$$

$\vec{B}$ is indep of time, but positions r_1 and r_2 change with time as loop moves with velocity $\vec{v}$.

$$= L B(r_2) \frac{dr_2}{dt} - L B(r_1) \frac{dr_1}{dt}$$

$$= L v \left(B(r_2) - B(r_1) \right)$$

$$= -c\,\mathcal{E}$$

so again we find $\boxed{\mathcal{E} = -\frac{1}{c}\frac{d\Phi}{dt}}$

Read page 266 in text to see that above result is true for any shape loop moving in any manner (ie different segments of loop can have different velocities!) in any external magnetic field $\vec{B}$. In computing Φ it does not matter what surface we use to compute $\Phi = \int d\vec{a} \cdot \vec{B}$ provided it is bounded by the loop. Because $\vec{\nabla} \cdot \vec{B} = 0$

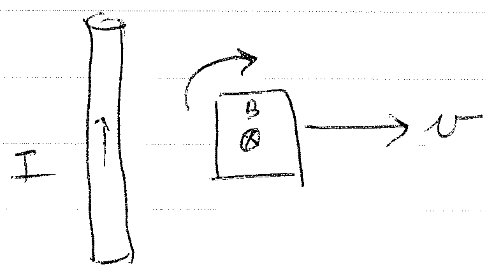
$\Rightarrow$ changing magnetic flux through a closed loop creates an emf around the loop. that drives a current around the loop just like a battery.

The minus sign — Lenz's Law

we have $\mathcal{E} = -\frac{1}{c}\frac{d\Phi}{dt}$ with the minus sign by our right hand rule convention.

But there is also a physical reason for the minus sign. This goes by the name of Lenz's Law

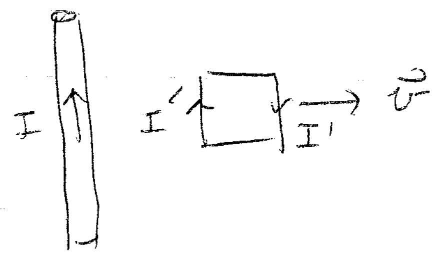
When a current is induced in a wire loop that is moving in a magnetic field, the direction of the current will be such as to create a magnetic field that opposes the change in flux.



magnetic flux from I decreases as loop moves to left

induced current flowing clockwise creates magnetic flux through the loop that is into page ie adds to flux from straight wire with I. So decrease in flux from I as loop moves to right is in part counter balanced by additional flux through loop created by the clockwise flowing induced current.

Another way to view it

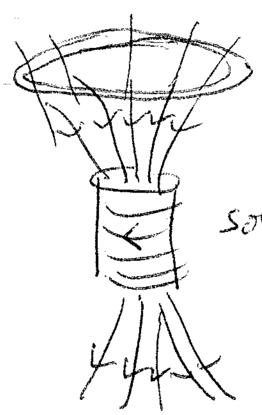


When current flows clockwise in loop, there is a net force of attraction between loop and wire

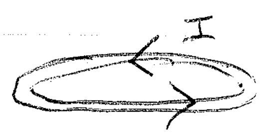
(since segment with ∥ current is closer to wire than the segment with the anti ∥ current)

The attractive force tends to oppose the force moving the loop to the right. The person pulling the loop does work against this force. This is the work that drives the current around the loop

<u>Another example</u> ~~[illegible]~~ ring



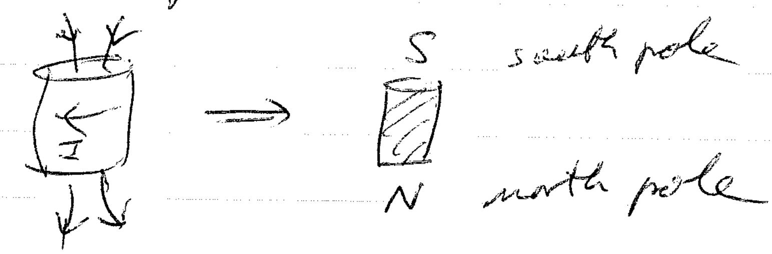
ring is falling in field of solenoid
Magnetic flux computed downwards through ring is increasing as ring falls.



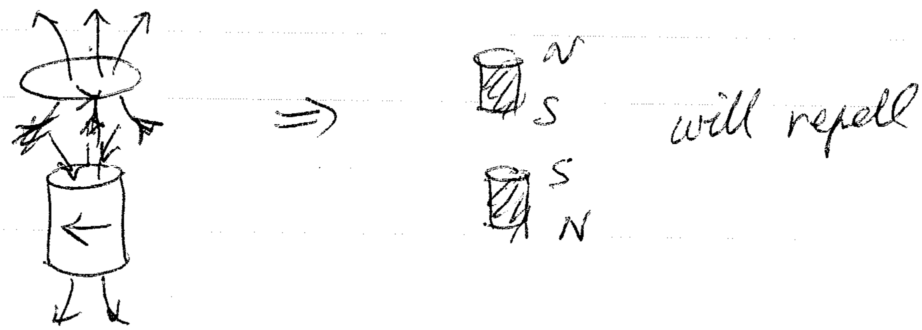
Current will be induced in ring to try and reduce this increasing flux $\Rightarrow$ current circulates counterclockwise to create magnetic flux penetrating upwards through ring

The ring with oppositely oriented current flowing acts like a solenoid of opposite orientation

Think of a solenoid like it behaves like a bar magnet



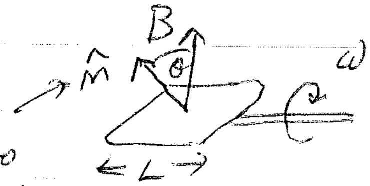
so



depending on the mass of the ring and the force of gravity pulling it down, the repulsive magnetic force might only slow its fall, or if balancing gravity might levitate the ring, or if greater than gravity might repel the ring.

The induced current will always be in a direction to create forces on the loop that will oppose the change in magnetic flux, ie one will have to do work on the ring to keep the magnetic flux through it changing.

Rotating ~~loop~~ square loop of length L

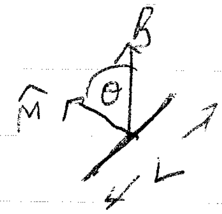


normal to plane of loop

magnetic flux through loop is

$$\Phi = BL^2\cos\theta = \vec{B}\cdot\vec{A} = L^2\vec{B}\cdot\hat{n}$$

side view



$$\frac{d\Phi}{dt} = -BL^2\sin\theta\,\frac{d\theta}{dt}$$

$$= -BL^2\omega\sin\omega t$$

$$\mathcal{E} = -\frac{1}{c}\frac{\partial\Phi}{\partial t} = \frac{BL^2}{c}\omega\sin\omega t$$

If loop has resistance R, a current flows

$$I(t) = \frac{\mathcal{E}}{R} = \frac{BL^2\omega}{cR}\sin\omega t$$

oscillating "ac" (alternating current) current in loop.

This is the principle by which electric generators work. Some form of mechanical work rotates the loop and that work is transformed into electrical energy of the induced ac current