Dielectric Materials - Static Electric Fields

Up to now we have considered electric and magnetic fields in the vacuum, where all charges and currents are specified by the charge density \( \rho \) and current density \( \mathbf{j} \).

But in most of everyday life we want to describe electric fields in matter - gases, liquids, solids, without having to consider the details of the positions and motions of each individual charge, such as the electrons and protons, that make up that matter.

We consider here "dielectrics", which are materials in which the electrons remain bound to the atomic cores of the atoms (or molecules). Dielectrics are in general non-conductors.

Consider what happens when a neutral atom is placed in an electric field. From quantum mechanics and modern theory of the atom tells us we should regard the electrons as if they form a cloud of negative charge centered about the positive nucleus of the atom. For simplicity we will consider the simplest atom, that of hydrogen...
Here we have a single +e proton at center, surrounded by a spherically symmetric cloud of an electron -e.

Suppose we displace the +e proton from the center of the negative electron charge a distance \( d \).

There will be restoring force pushing +e back toward center of -e. This is because the electric field inside the electron cloud increases linearly from the origin.

\[ E \sim \frac{1}{r^2} \]

So if the +e is displaced a distance \( d \) from center of electron sphere of -e charge +e feels an electric force \( F \propto \frac{1}{r^2} \) pushing it back. The restoring force \( F = k_e \frac{e_d}{r^2} \) balances the net electric force between the proton and electron.
If we now put the atom in an external electric field $\vec{E}$, the proton will feel a force $+e \vec{E}$ separating it from the electron. But as it displaces, it feels the restoring force $-e \vec{F}$, pulling it back. The net displacement $\vec{d}$ will be determined when the total force vanishes:

$$+e \vec{E} - e \vec{F} = 0$$

$$\Rightarrow \vec{d} = +\frac{\vec{E}}{k}$$

Since for $r > a$, the field of the electron sphere looks just like a point charge at the center of the sphere, the field from the atom in the external electric field looks like that of an electric dipole $\vec{q}$.

We say that the external field $\vec{E}$ has polarized the atom and given it an electric dipole moment $\vec{p} = +e \vec{d} = \frac{e}{k} \vec{E}$.

Define atomic polarizability $\alpha$ by

$$\vec{p} = \alpha \vec{E}$$

$$\alpha = \frac{e}{k}$$

dipole moment induced by electric field $\vec{E}$.
For any two charges \( q \) and \(-q \), the electric dipole moment \( \vec{P} \) is given by:

\[
\vec{P} = q \vec{d}
\]

where \( \vec{d} \) is the distance from \(-q\) to \(+q\).

More generally, for a set of charges \( q_i \), that are net neutral \( \sum q_i = 0 \), the dipole moment for the charges is:

\[
\vec{P} = \sum q_i \vec{r}_i
\]

\( \vec{r}_i \) is the position of \( q_i \).

A dielectric material is made up of a density \( n \) of such atoms, each of which gets polarized when the material is in an electric field \( \vec{E} \). The polarization density \( \vec{P} \) is defined as the electric dipole moment per unit volume in the dielectric.

\[
\vec{P} = \frac{\sum \vec{P}_i}{\text{vol } \Delta V}
\]

\( \vec{P}_i \) is the dipole moment in vol \( \Delta V \).

If \( n \) is the density of polarizable atoms (or molecules) in the dielectric, then

\[
\vec{P} = n \vec{P} = n d \vec{E} = n \varepsilon_0 \vec{E}
\]

\( \varepsilon_0 \) is the electric susceptibility.

If \( \varepsilon \) is the material's dielectric constant, then

\[
\vec{P} = \varepsilon_0 \varepsilon \vec{E}
\]

\( \varepsilon \approx n d \)

"Under dielectric" \( \Rightarrow \vec{P} \propto \vec{E} \) is the atomic polarizability.
Although the dielectric is neutral, with just as much positive as negative charge, the polarization of the dielectric $\vec{P}$ in the presence of an applied electric field $\vec{E}$ can lead to regions of net charge, called the bound charge (since it is bound to the dielectric and not free to move about), that can contribute to creation of electric fields.

Consider a lattice of polarizable atoms in terms of average charge:

$$\vec{E} = 0 \quad \vec{P} = 0$$

uniform polarization of the dielectric gives rise to surface charge on its surface:

$$\text{Total charge at right surface is } (dA) \sigma_m$$

how big is surface charge?

$$\text{Total charge at right neutral region}$$

$$\text{bound surface charge density} \quad \sigma_b = \frac{(dA) \sigma_m}{A} = \frac{d \sigma_m}{A}$$

or $\sigma_b = \frac{\vec{P}}{\mid \vec{P} \mid} \quad \text{surface unit vector}$

$$\mid \vec{P} \mid = \sqrt{\vec{E} \cdot \vec{E}}$$