Dielectric Materials - static electric fields

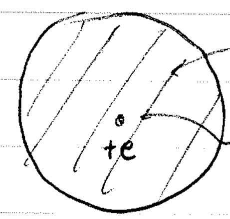
Up to now we have considered electric and magnetic fields in the vacuum, where all charges and currents are specified by the charge density ρ and current density $\vec{J}$.

But in most of everyday life we want to describe electric fields in matter - gases, liquids, solids, without having to consider the details of the positions and motions of each individual charge, such as the electrons and protons, that make up that matter.

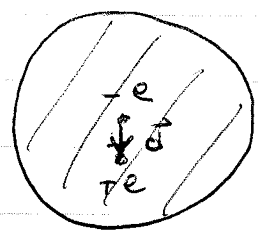
We consider here "dielectrics", which are materials in which the electrons remain bound to the atomic cores of the atoms (or molecules). Dielectrics are in general ~~insul~~ ~~con~~ electrical insulators.

Consider what happens when a neutral atom is placed in an electric field. From quantum mechanics our modern theory of the atom tells us we should regard the electrons as if they form a cloud of negative charge centered about the positive nucleus of the atom. For simplicity we will consider the simplest atom hydrogen.

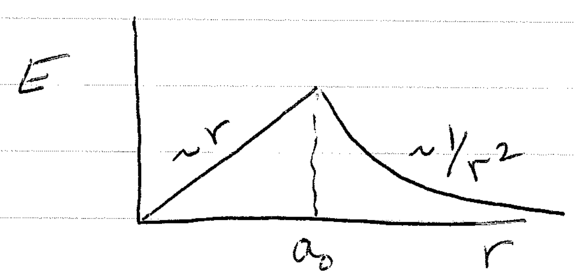
Here we have a single +e proton at center, surrounded by a spherically symmetric cloud of one electron −e.



Suppose we displace the +e proton from the center of the negative electron charge a distance $\vec{d}$



There will be restoring force pushing +e back towards center of −e. This is because the electric field inside the electron cloud increases linearly from the origin



← electric field from a uniform sphere of charge of radius a_0

So if the +e is displaced a distance $\vec{d}$ from center of electron sphere of −e charge it feels an electric force

$$\vec{F} = -k e \vec{d}$$

proportionality constant

pushing it back.

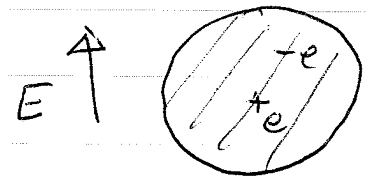
~~on a proton equal and opposite~~

~~be~~

~~restoring force $\vec{F}_{rest} = -ek\vec{d}$~~

~~on electron~~ ~~const of proportionality~~

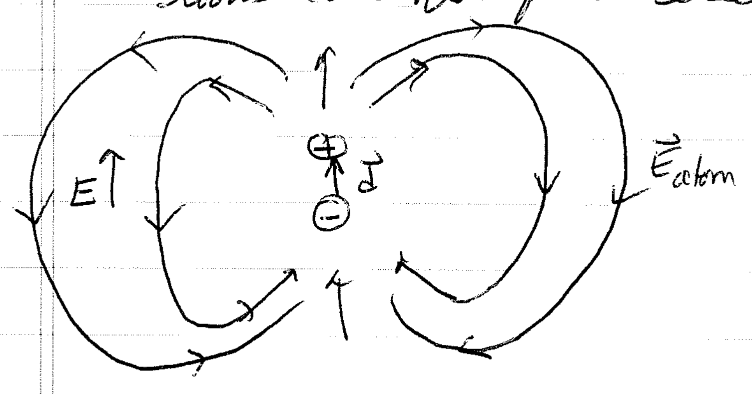
If we now put the atom in an external electric field $\vec{E}$, the proton will feel a force $+eE$ separating it from the electron. But as it displaces, it feels the restoring force $-ek\vec{d}$ pulling it back. The net displacement $\vec{d}$ will be determined when the total force vanishes,



$$+e\vec{E} - ek\vec{d} = 0$$

$$\Rightarrow \quad \vec{d} = +\frac{\vec{E}}{k}$$

Since for $r > a_0$ the $\vec{E}$ field of the electron sphere looks just like a point charge at the center of the sphere, the field from the atom in the external electric field looks like that of an electric dipole



we say that the external $\vec{E}$ has polarized the atom and given it an electric dipole moment

$$\vec{p} = +e\vec{d} = \frac{e}{k}\vec{E}$$

Define atomic polarizeability α by

$$\vec{p} = \alpha\vec{E} \qquad \alpha = e/k$$

dipole moment ~~p~~ induced by electric field $\vec{E}$.

For any two charges $+q$ and $-q$, the <u>electric dipole moment</u> is

$$\boxed{\vec{p} = q\vec{d}}$$

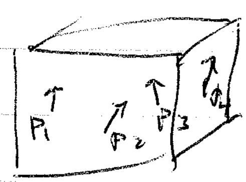
where $\vec{d}$ is distance from $-q$ to $+q$

More generally, for a set of charges q_i, that are net neutral ie $\sum_i q_i = 0$, the dipole moment for the charges is

$$\vec{p} = \sum_i q_i \vec{r}_i$$

$\vec{r}_i$ is position of q_i

A dielectric material is made up of a density n of such atoms, each of which gets polarized when the material is in an electric field $\vec{E}$. The <u>polarization density</u> $\vec{\mathbb{P}}$ is defined as the electric dipole moment per unit volume in the dielectric.



vol ΔV

$$\vec{\mathbb{P}} = \frac{\sum_i \vec{p}_i}{\Delta V}$$

$\vec{p}_i$ dipole moment in vol ΔV

If n is the density of polarizable atoms (or molecules) in the dielectric then

$$\vec{\mathbb{P}} = n\vec{p} = n\alpha\vec{E} = \chi_e \vec{E}$$

↑ electric susceptibility

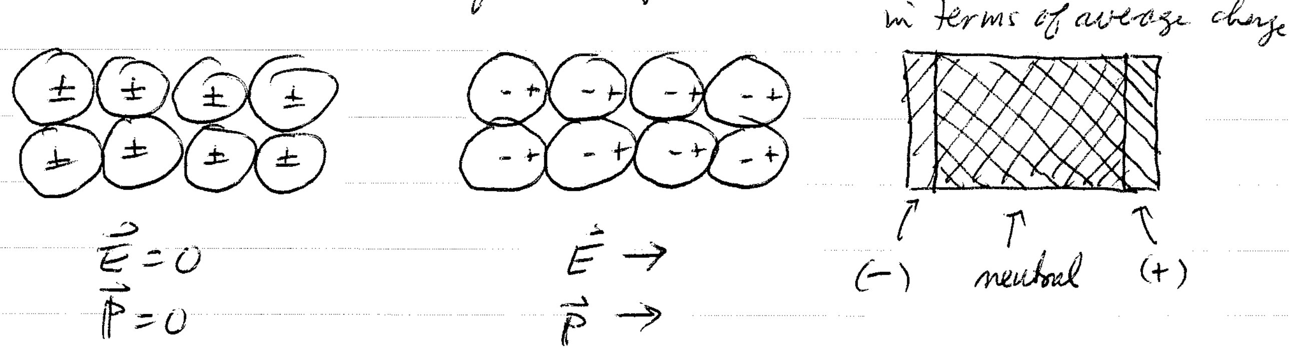
$$\vec{\mathbb{P}} = \chi_e \vec{E} \qquad \chi_e \simeq n\alpha$$

"linear dielectric" $\Rightarrow \vec{\mathbb{P}} \propto \vec{E}$

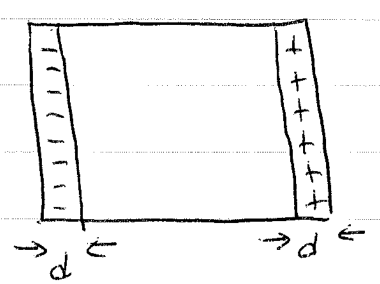
↑ atomic polarizability

Although the dielectric is <u>neutral</u> with just as much positive as negative charge, the polarization of the dielectric $\vec{P}$ in the presence of an ~~applied~~ electric field $\vec{E}$ can lead to regions of net charge, called the <u>bound</u> charge (since it is bound to the dielectric and not free to move about), that can contribute to creation of electric fields.

Consider a lattice of polarizeable atoms



<u>uniform</u> polarization of the dielectric gives rise to surface charge on its surface!



How big is surface charge?

Total charge at ~~the~~ right surface is

$(dA)qn$

↑ vol of non neutral region

<u>bound</u> surface charge density $\sigma_b = \frac{(dA)qn}{A} = dqn$

$= |\vec{p}|n$

$= |\vec{P}|$

or $\sigma_b = \vec{P}\cdot\hat{n}$ ← outward unit vector