**PHY 217** **Final Exam** **Fall 2016**

1) [25 points total] You should be able to give a short answer, or short calculation, to each part of this question. No lengthy calculations should be needed. But make sure to explain your answer clearly. [5 pts each part]

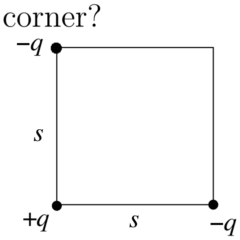
a) A point charge $q$ is situated a large distance $r$ from a neutral atom of atomic polarizability $\alpha$. Is the resulting force attractive or repulsive? How does the force depend on the distance $r$, i.e. if the force is $F \sim 1/r^n$, what is $n$?

b) An electrostatic potential is given in spherical coordinates by

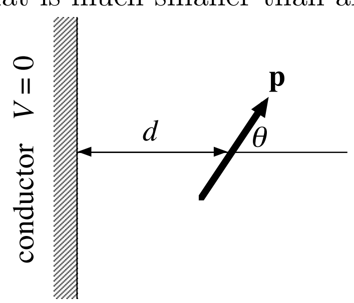
$$V(r,\theta,\varphi) = \frac{a}{r} + \frac{b\cos\theta}{r^3}, \qquad \text{should have been} \quad \boxed{V(r,\theta,\varphi) = \frac{a}{r} + \frac{b\cos\theta}{r^2}}.$$

Describe the charge distribution which produced this potential. [There was a typo in this problem, it should have been as in the box above. So everyone got 5 pts for this part.]

c) Three charges are situated at the corners of a square of side $s$ as shown below. How much work does it take to bring in another charge $+q$ from infinity and place it at the fourth corner?




d) A point dipole **p** is oriented at angle $\theta$ a distance $d$ in front of an infinite flat, grounded ($V = 0$) conducting plane, as shown in the figure below. Explain how you would compute the surface charge which is induced on the surface of the conductor. You do not have to do the calculation, but you must give sufficient detail to set the solution up. By "point" dipole one means that the charge distribution that gives rise to **p** is spread out on a length scale that is much smaller than any other length in the problem.




e) Two circular wire loops, each of radius $R$, are centered about the origin in the $xy$ plane. One loop is at height $z = +a$ and has a current $I$ circulating clockwise. The other loop is at height $z = -a$ and has a current $I$ circulating counterclockwise. For large distances $r \gg R$ the magnetic field will, to leading order, decay as $1/r^n$. What will be the value of $n$. You must explain your reasoning.

2) [25 points total]

Consider a thin spherical shell of radius $R$, with a fixed surface charge density given by $\sigma(\theta) = \sigma_0 \cos^2\theta$. Here $\theta$ is the usual polar angle is spherical coordinates.

a) (15 pts) Find the electrostatic potential $V(\mathbf{r})$ both inside and outside the shell. [Hint: at some point in the calculation you might wish to rewrite $\cos^2\theta$ as an expansion in Legendre polynomials; it is easiest to do this by inspection.]

b) (10 pts) Consider the multipole expansion for $V(\mathbf{r})$ in the large $|\mathbf{r}|$ limit, as applied to this problem. Putting the origin at the center of the shell, state which of the multipole moments which will be non-zero. You do not need to do an explicit calculation if you can answer clearly and convincingly without one.

Legendre polynomials:

$$P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = (3x^2 - 1)/2$$

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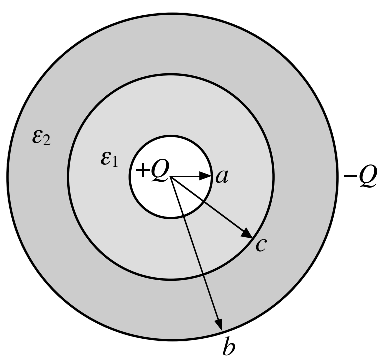
3) [25 points total]

A conducting sphere of radius $a$ is concentric with a thin conducting shell of radius $b$. The region between the two conductors is filled with two dielectric layers of permittivities $\varepsilon_1$ and $\varepsilon_2$, as shown in the sketch below. A total charge $+Q$ is placed on the conducting sphere, and a total charge $-Q$ is placed on the conducting shell.



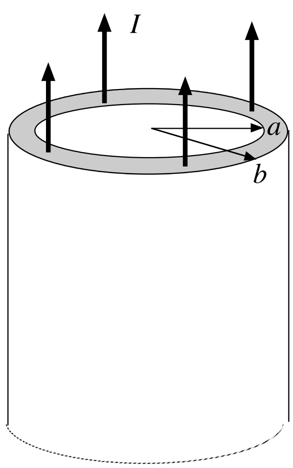

a) (10 pts) Find the electric field $\mathbf{E}(\mathbf{r})$ in all regions of space.

b) (8 pts) Find the voltage drop $\Delta V$ between the surface of the conducting sphere and the conducting shell. What is the capacitance $C$ of this system?

c) (7 pts) What is the total bound charge at the interface between the two dielectrics? Is there any bound charge inside the dielectrics? Be sure to explain your answers.

4) [25 points total]

Consider an infinitely long cylindrical metal tube of finite thickness, with inner radius $a$ and outer radius $b$. A total current $I$, flowing parallel to the cylinder axis, is uniformly distributed over the crosssectional area of the tube.




a) (15 pts) Find the magnetic field $\mathbf{B}(\mathbf{r})$ outside the tube ($r > b$), inside the tube ($r < a$), and within the thickness of the tube ($a < r < b$). Here $r$ is the cylindrical radial coordinate. Make a graph of $|\mathbf{B}|$ vs $|\mathbf{r}|$.

b) (10 pts) Consider an infinitely long straight wire, parallel to the tube's axis, which is a distance $r$ from the center of the tube. The wire carries a total current $I'$, flowing in the same direction as the current in the cylindrical tube. Assume the wire is infinitesimally thin. What is the force per unit length that acts on the wire? Is it attractive or repulsive? Consider both the case where the wire is outside the tube ($r > b$) and where the wire is inside the tube ($r < a$).