Conductors

Properties in electrostatics

1) $\vec{E} = 0$ inside a conductor – if $\vec{E} \neq 0$ then a current would flow $\Rightarrow$ not an electrostatic situation.
When first apply $\vec{E}$ to a conductor, charges flow until they result in a configuration such that the charges in the conductor produce an electric field that exactly cancels the applied field so that the total field inside the conductor is zero

2) $\rho = 0$ inside conductor – follows from $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$ and $\vec{E} = 0$ inside

3) (2) $\Rightarrow$ any net charge on a conductor must lie on the surface of the conductor

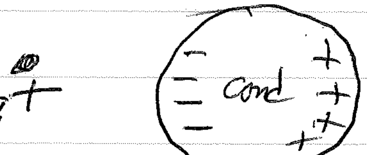
4) V = constant throughout the conductor – since $-\vec{\nabla} V = \vec{E}$ and $\vec{E} = 0$ inside

5) Just outside the conductor $\vec{E} \perp$ to surface
– since if $\vec{E}$ did have a component $\parallel$ to surface, it would exert forces on charges on surface causing them to flow – ~~same pr~~ so would not be electrostatics

Consequences of above properties

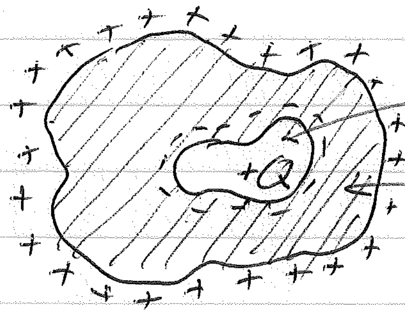
Induced charge

point charge in front of conducting sphere



conductor has no net charge $\Rightarrow$ induced charge on surface to cancel out $\vec{E}$ from q outside so $\vec{E} = 0$ inside sphere. But now there will be a net attractive force between q and the conductor even though conductor has no net charge

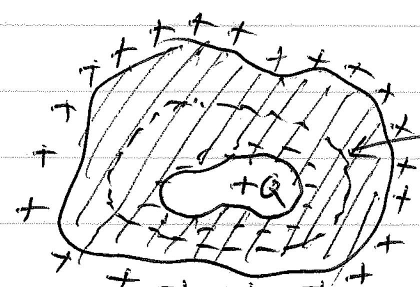
Cavities in conductors



+Q inside cavity attracts oppositely charged (−) electrons in conductor to ~~inner~~ inner surface, so that net field inside conductor is zero. Equal amount of (+) charge builds up on outside surface of conductor to give $\vec{E}$ field outside

Amount of induced charge on inner surface totals to $-Q$
" " " " " outer " " " $+Q$

to see, consider a Gaussian surface inside conductor that encloses the cavity



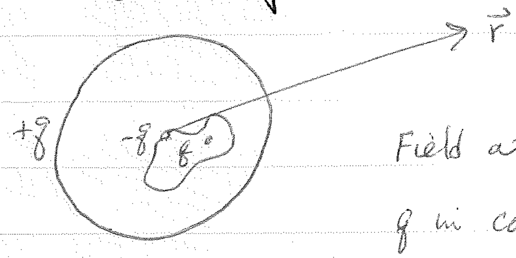
$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\varepsilon_0}$$

~~but~~ $\vec{E} = 0$ inside conductor so

$\oint \vec{E} \cdot d\vec{a} = 0 \Rightarrow Q_{encl} = 0$

$\Rightarrow$ induced charge on surface cavity must total to $-Q$

conducting ~~conductor~~ sphere with a cavity inside



Field at $\vec{r}$?

q in cavity induces $-q$ on inside surface that cancells out net electric field inside conductor ~~$\vec{E}$~~

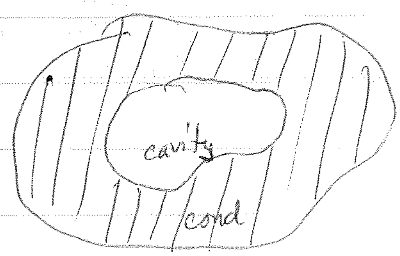
(field inside cavity ~~need no~~ is not in general zero)

$+q$ is induced on outer surface, spreads uniformly $\Rightarrow \vec{E}(\vec{r}) = \dfrac{q}{4\pi\varepsilon_0 r^2}\hat{r}$ ← gives $\vec{E} = 0$ inside sphere

uniform since $\vec{E} = 0$ inside $\Rightarrow$ no interaction with charges inside

True for any shape cavity at all!

<u>Field inside charge free cavity = 0</u>



$-\nabla^2 V = \rho/\varepsilon_0 = 0$ inside cavity

$V(\vec{r}) = V_0 =$ constant <u>on surface</u> of cavity

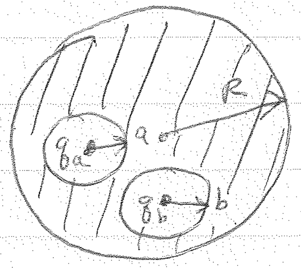
$\Rightarrow V(\vec{r}) = V_0$ inside cavity is solution

$(-\nabla^2 V_0 = 0)$

$\Rightarrow \vec{E} = -\vec{\nabla} V = 0$ inside

<u>Faraday cage</u>: putting apparatus inside a grounded conducting cage screens out all external electric fields

Prob 2.39



q_a, q_b at centers of spherical cavities

a) surface charge σ_a, σ_b, σ_R

uniform
$$\begin{cases} \sigma_a = -q_a/4\pi a^2 \\ \sigma_b = -q_b/4\pi b^2 \\ \sigma_R = (q_a+q_b)/4\pi R^2 \end{cases}$$
– take gaussian surface about cavity a. $\oint \vec{E}\cdot d\vec{a} = 0 \Rightarrow Q_{encl} = 0$

b) $\vec{E}$ outside is $\vec{E} = \dfrac{(q_a+q_b)}{4\pi\varepsilon_0 r^2}\hat{r}$ ie σ_R is uniform

c) $\vec{E}$ inside cavities is – field has spherical symmetry about center of cavity $\Rightarrow \oint \vec{E}\cdot d\vec{a} = \dfrac{q_{encl}}{\varepsilon_0}$ easy to get $\vec{E}$

$$\vec{E}_a = \frac{q_a\,\hat{\mathscr{r}}}{4\pi\varepsilon_0 \mathscr{r}^2} \qquad \vec{\mathscr{r}} = \vec{r}-\vec{r}_a$$

$$\vec{E}_b = \frac{q_b\,\hat{\mathscr{r}}}{4\pi\varepsilon_0 \mathscr{r}^2} \qquad \vec{\mathscr{r}} = \vec{r}-\vec{r}_b$$

d) force on q_a and q_b $= 0$

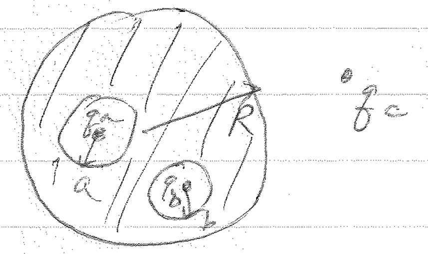
Consider force on q_a: $\vec{E}$ field at $\vec{r}_a$ from $\sigma_R = 0$ as we are inside uniformly charged shell

$\vec{E}$ field from q_b exactly cancelled by $\vec{E}$ field from σ_b

$\vec{E}$ field at $\vec{r}_a$ from $\sigma_a = 0$ as we are inside uniformly charged shell

$\Rightarrow \underbrace{\vec{E}(\vec{r}_a)}_{\text{field from all charges excluding } q_a} = 0$. Similarly $\underbrace{\vec{E}(\vec{r}_b)}_{\text{field from all charges excluding } q_b} = 0 \Rightarrow$ force on a, b $= 0$

e) What changes if q_c ~~brout~~ brought near outside of conductor?



a) σ_a, σ_b unchanged. σ_R still has total charge $+q_a+q_b$ but now charge is more concentrated on side of sphere closest to q_c

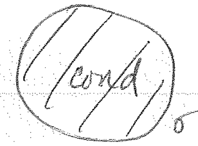
b) $\vec{E}$ outside changes, as result of redistribution of charge σ_R

c) $\vec{E}_a$ + $\vec{E}_b$ do not change

d) Force on $q_a + q_b$ do not change. σ_R must redistribute itself so that field from σ_R and q_c cancel inside the conductor + so give no force on q_a or q_b. Force on q_a from σ_a, σ_b and q_b is as before, so total force on $q_a = 0$. Same for q_b

Conductor acts to insulate the cavities from each other and from the outside.

(2.5.3) Surface charge on conductor



$\vec{E}_{above} - \vec{E}_{below} = \frac{\sigma}{\varepsilon_0}\hat{n}$ as cross surface charge density σ.

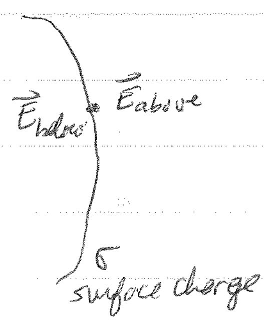
for conductor, $\vec{E}_{below} = \vec{E}_{inside} = 0 \Rightarrow \vec{E}_{above} = \boxed{\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}}$

field ~~and~~ at surface is in normal direction

or $\vec{E}\cdot\hat{n} = -\vec{\nabla}V\cdot\hat{n} = \boxed{-\frac{\partial V}{\partial n} = +\frac{\sigma}{\varepsilon_0}}$

$\Rightarrow$ can determine σ on surface of conductor if know V (or $\vec{E}$) outside

Electrostatic pressure at surface charge layer



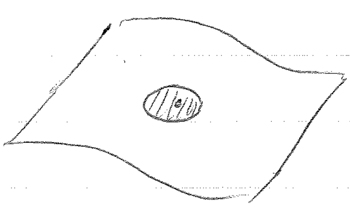
electric field $\vec{E}$ exerts force $\vec{f}$ on charge density σ.

$\vec{f} = \sigma\vec{E}$ ← but do we use $\vec{E}_{above}$ or $\vec{E}_{below}$?

↑ force/area

ans: $\vec{E} = \frac{1}{2}(\vec{E}_{above} + \vec{E}_{below})$ average value

proof:



write $\vec{E} = \vec{E}_{patch} + \vec{E}_{other}$

↑ electric field from σ on small patch at $\vec{r}$ on surface

$\vec{E}_{patch}$ does not exert force on the charge on the patch itself ie we do not want any self forces. patch feels force only from the other charges off the patch, ie σ outside patch + any external charges off the surface — these are the charges that determine $\vec{E}_{other}$

So $\boxed{\vec{f}_{patch} = \sigma_{patch} \vec{E}_{other}}$ where $\vec{E}_{other}$ is continuous as move accross patch, as it is field that would exist if a hole were cut out of the patch.

$$\text{Now } \vec{E} = \underbrace{\vec{E}_{other}}_{\text{continuous}} + \underbrace{\vec{E}_{patch}}_{\text{discontinuous}}$$

$$\Rightarrow \vec{E}_{above} = \vec{E}_{other} + \vec{E}_{patch\,above}$$

$$\Rightarrow \vec{E}_{below} = \vec{E}_{other} + E_{patch\,below}$$

But $\vec{E}_{patch}$ is discontinuous

$$\vec{E}_{patch\,above} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

$$E_{patch\,below} = \frac{-\sigma}{2\varepsilon_0}\hat{n}$$

just above center of patch, it looks like infinite flat plane

$$\text{or } \vec{E}_{above} = \vec{E}_{other} + \frac{\sigma}{2\varepsilon_0}\hat{n}$$

$$\vec{E}_{below} = \vec{E}_{other} - \frac{\sigma}{2\varepsilon_0}\hat{n}$$

$$\Rightarrow \boxed{\frac{1}{2}\left(\vec{E}_{above} + \vec{E}_{below}\right) = \vec{E}_{other}}$$

$$\text{so } \vec{f}_{patch} = \frac{1}{2}\sigma(\vec{r})\left(\vec{E}_{above}(\vec{r}) + \vec{E}_{below}(\vec{r})\right)$$

position of "patch"

For a conductor, $\vec{E}_{below} = 0$, $\vec{E}_{above} = \vec{E}$ on surface

$$\Rightarrow \vec{f} = \frac{1}{2}\sigma\vec{E} \qquad \text{but } \vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

$$\Rightarrow \boxed{\vec{f} = \frac{1}{2}\frac{\sigma^2}{\varepsilon_0}\hat{n}}$$

or equivalently

$$\boxed{\vec{f} = \frac{\varepsilon_0}{2}(\vec{E}\cdot\hat{n})\vec{E} = \frac{\varepsilon_0}{2}E^2\hat{n}}$$

Electrostatic pressure

$$p = \vec{f}\cdot\hat{n} = \frac{\varepsilon_0}{2}E^2 = \frac{\sigma^2}{2\varepsilon_0}$$

as $\vec{E} = E\hat{n}$

Note: $\vec{f}$ is always along outward $\hat{n}$ no matter what sign of σ

ie force always draws conductor into the field.