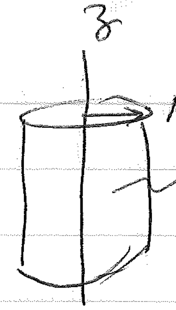


By symmetry $\vec{E}(\vec{r}) = E(r)\,\hat{r}$ where r is the cylindrical radial coordinate

For a concentric cylindrical Gaussian surface of radius r and length L

$$\oint_S d\vec{a}\cdot\vec{E} = 2\pi r L\, E(r) = \frac{Q_{encl}}{\varepsilon_0} = \begin{cases} 0 & r<R \\ \dfrac{2\pi R L \sigma}{\varepsilon_0} & r>R \end{cases}$$

$$\vec{E}(r) = \begin{cases} 0 & r<R \\ \dfrac{R}{\varepsilon_0 r}\sigma\hat{r} & r>R \end{cases}$$

outward normal

So $\vec{E}_{above} - \vec{E}_{below} = \dfrac{R}{\varepsilon_0 R}\sigma\hat{r} - 0 = \dfrac{\sigma}{\varepsilon_0}\hat{r} = \dfrac{\sigma}{\varepsilon_0}\hat{n}$

So Griffiths Eqn (2.33) is satisfied

We can compute $V(r) = -\int_{r_0}^{r} d\vec{l}\cdot\vec{E}$ Take reference

~~point to be~~ $\cancel{r_0 = \frac{R}{10}\hat{x}}$

$$\cancel{V(r) = -\int_0^r dr'\,\hat{r}\cdot E(r')\hat{r}}$$

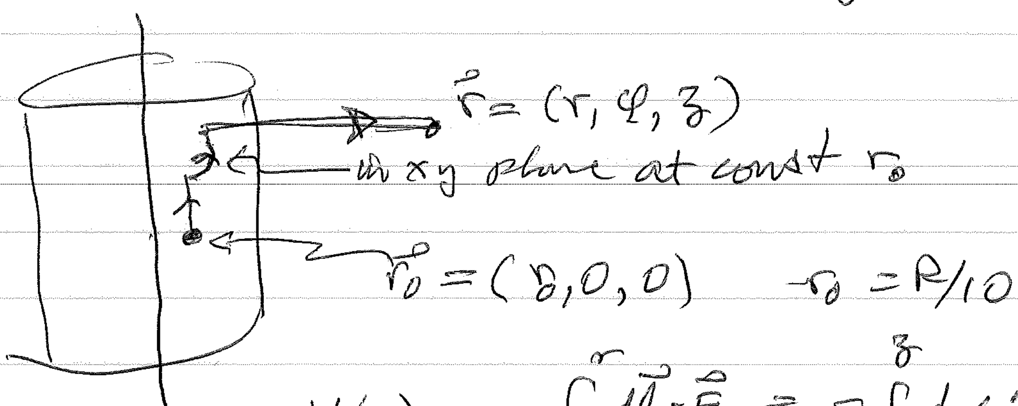
$$\cancel{= -\int_0^r dr' E(r') = 0 \quad r<R}$$

$$\cancel{= -\int_0^r dr' \frac{R\sigma}{\varepsilon_0 r} = \frac{R\sigma}{\varepsilon_0}}$$

reference point to be inside cylinder, but not $\vec{r}_0 = 0$ to avoid a singularity.

So we could take $\vec{r}_0 = \frac{R}{10}\hat{x}$ for example.

To integrate to $\vec{r}$, first integrate $-\int_{r_0} d\vec{l}\cdot\vec{E}$ along $\hat{z}$ to the desired height – this contributes nothing since $\vec{E}\cdot\hat{z} = 0$.
Then integrate along φ to the desired ~~value~~ polar angle – this contributes nothing since $\vec{E}\cdot\hat{\varphi} = 0$



$$V(r) = -\int_{r_0}^{r} d\vec{l}\cdot\vec{E} = -\int_{z=0}^{z} dz'\,\hat{z}\cdot\vec{E} - \int_{\varphi=0}^{\varphi} d\varphi'\,r_0\,\hat{\varphi}\cdot\vec{E}$$

$$- \int_{r_0}^{r} dr\,\hat{r}\cdot\vec{E} = 0 + 0 - \int_{r_0}^{r} dr\,E(r)$$

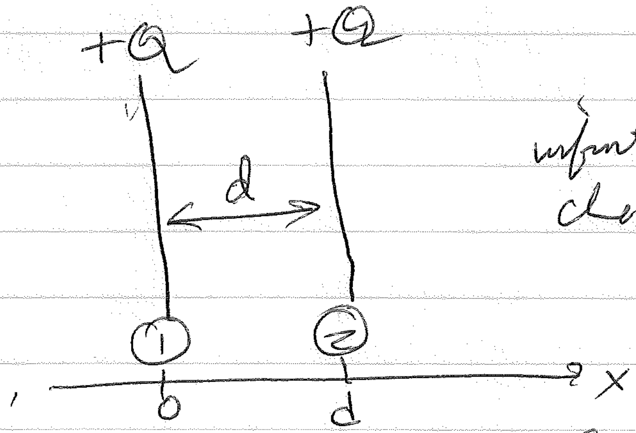
$$V(r) = -\int_{r_0}^{r} dr\,E(r) = 0 \qquad r_0 < r < R$$

$$= -\int_{r_0}^{r} dr\,\frac{R\sigma}{\varepsilon_0 r} = -\frac{R\sigma}{\varepsilon_0}\ln(r/r_0) \qquad R < r$$

So $\frac{\partial V_{below}}{\partial n} = \frac{\partial V_{below}}{\partial r} = 0$, $\frac{\partial V_{above}}{\partial n} = \frac{\partial V_{above}}{\partial r} = -\frac{R\sigma}{\varepsilon_0}\frac{1}{r}\Big|_{r=R}$

So $-\frac{\partial V_{above}}{\partial n} + \frac{\partial V_{below}}{\partial n} = \frac{R\sigma}{\varepsilon_0 R} = \frac{\sigma}{\varepsilon_0}$ agrees with (2-36)

2.41)



infinite parallel planes area A
charge Q

$\sigma = \frac{Q}{A}$

Field from plane ① is $\begin{cases} \frac{\sigma}{2\epsilon_0}\hat{x} & x > 0 \\ -\frac{\sigma}{2\epsilon_0}\hat{x} & x < 0 \end{cases}$

Field from plane ② is $\begin{cases} \frac{\sigma}{2\epsilon_0}\hat{x} & x > 0 \\ -\frac{\sigma}{2\epsilon_0}\hat{x} & x < d \end{cases}$

The total field is then $\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} \frac{-\sigma}{\epsilon_0}\hat{x} & x < 0 \\ 0 & 0 < x < d \\ \frac{+\sigma}{\epsilon_0}\hat{x} & x > d \end{cases}$

force per unit area on plane ② is

$$\vec{f}_2 = \frac{1}{2}\sigma\left[\vec{E}_{above} + \vec{E}_{below}\right] = \frac{1}{2}\sigma\left[\left(\frac{\sigma}{\epsilon_0}\right)\hat{x} + 0\right]$$

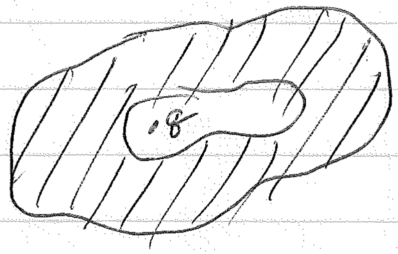
$= \frac{1}{2}\frac{\sigma^2}{\epsilon_0}\hat{x}$ pushes in $+\hat{x}$ direction

force per unit area on plane ① is

$$\vec{f}_1 = \frac{1}{2}\sigma\left[\vec{E}_{above} + \vec{E}_{below}\right] = \frac{1}{2}\sigma\left[-\frac{\sigma}{\epsilon_0}\hat{x} + 0\right]$$

$= \frac{-1}{2}\frac{\sigma^2}{\epsilon_0}\hat{x}$ pushes in $-\hat{x}$ direction

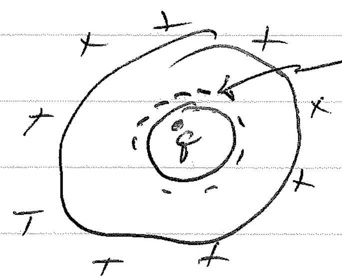
2.40

a) 

is the force on a charge q inside an arbitrary shaped cavity inside an arbitrary shaped neutral conductor necessarily zero?

No: ~~the force~~ Say q is positive. It will induce a negative surface charge on the surface of the cavity and equal but opposite total surface charge on outside of conductor. Force on q is due to the induced charges.

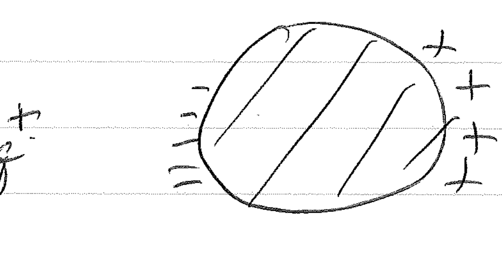
If q is close to the wall of the cavity it will induce charge on the wall near it that will attract q to the wall



more (−) charge induced close to q, so net force on q attracts it to the wall.

b) Is the force between a point charge q outside a neutral conductor necessarily attractive?

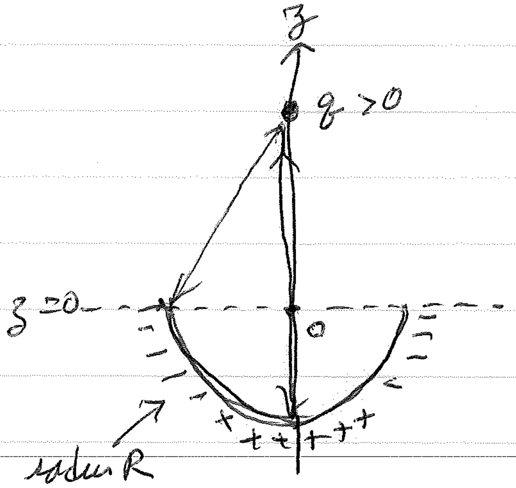
In general the force is <u>usually</u> attractive



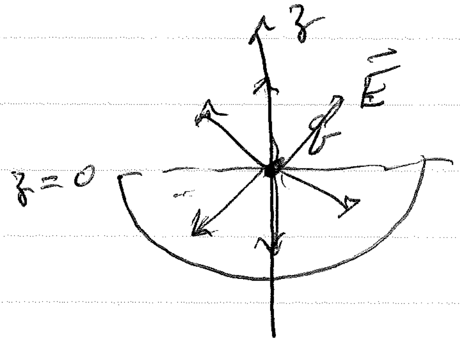
as induced charge of opposite sign is induced on surface of conductor closest to q and so attracts it.

But we can construct special situations where this is not true and the force can be repulsive!

Example: a point charge q in front of an inverted neutral hemispherical shell



When q at height z is far away, $z \gg R$, the pattern of induced charge will look as illustrated. Since the distance from q to the induced (−) charge is smaller than the distance from q to the induced (+) charge, there will be a net attractive force.



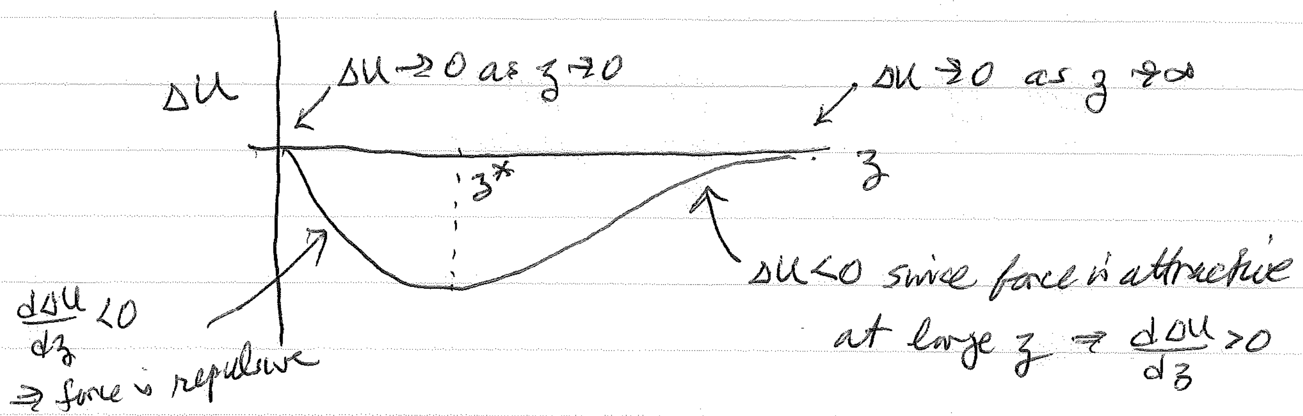
But when q is at $z=0$, there is no attractive force $\vec{f}$ because there is no induced charge! This is because the $\vec{E}$ field from q by itself already satisfies all the necessary boundary conditions on the conducting surface, i.e. $\vec{E} \perp$ surface

Similarly, when $z \to \infty$ there are no induced charges on the shell. So the electrostatic energy U must be the same when $z=\infty$ as when $z=0$

$$U(z=0) = U(z=\infty)$$

Denote $\Delta U(z) = U(z) - U(z=\infty)$, so $\Delta U(0) = \Delta U(\infty) = 0$

$\Delta U(z)$ must therefore look ~~like~~ qualitatively like



Since the force is attractive at large z, the electrostatic energy must decrease as z decreases at large z. But since $\Delta U(0) = \Delta U(\infty)$ should be a continuous function, it is therefore necessary that $\Delta U(z)$ starts to increase as z decreases below some z^*. Therefore at sufficiently small $z < z^*$, ΔU will decrease as z increases $\Rightarrow$ force is repulsive. So z^* marks a crossover: for $z > z^*$ the force between q and the shell is attractive, for $z < z^*$ the force between q and the shell is repulsive.

One has to do a calculation to determine the value of z^*

see https://arxiv.org/pdf/1007.217v1.pdf