PHY 218 **Final Exam** **Spring 2015**

Please put a box around your final answer, and cross out any work you do not wish me to look at. Note that not all problems are worth the same number of points!

1) [25 points total]

Give a brief and to the point answer to each of the following. You may cite appropriate equations when it helps to explain a point, but no calculations should be necessary.

a) [5 pts] In a conductor or a non-conducting dielectric, the permittivity ε is in general not a constant, but is rather a frequency dependent function $\varepsilon(\omega)$ with complex values. Give two consequences of the fact that $\varepsilon(\omega)$ varies with frequency ω. Give two consequences of the fact that $\varepsilon(\omega) = \varepsilon_1(\omega) + i\varepsilon_2(\omega)$ may have an imaginary component.

b) [5 pts] Describe one main difference between the propagation of a plane polarized simple-harmonic (i.e. single frequency) electromagnetic wave in a conductor vs in a non-conducting dielectric.

c) [5 pts] Describe one physical phenomenon associated with the plasma frequency of a conductor.

d) [5 pts] What is Snell's law? In what case is Snell's law not valid?

e) [5 pts] What happens at Brewster's angle?

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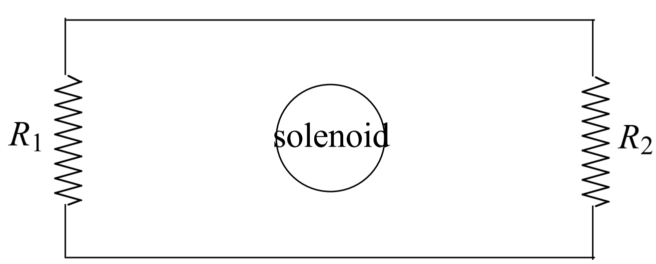
2) [15 points total]

The current in a long solenoid is increasing linearly with time, so that the magnetic flux through the solenoid is proportional to time: $\Phi(t) = \alpha t$. Suppose the solenoid is placed in the center of a loop with two resistors R_1 and R_2 as shown in the figure below, with the flux pointing out of the page.

a) [5 pts] In which direction will the induced current in the loop flow (you must explain the reason for your answer to get credit)?

b) [10 pts] What will be the voltage drop across each resistor?



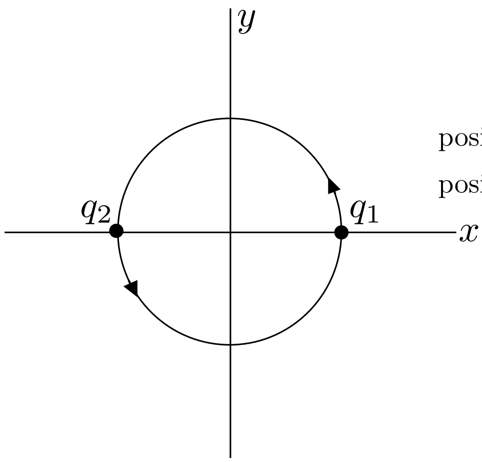
3) [30 points total]

For a time dependent charge distribution with an oscillating electric dipole moment $\mathbf{p}(t) = \mathrm{Re}[\mathbf{p}_\omega \mathrm{e}^{-i\omega t}]$, the radiated fields (i.e. in the radiation zone) in the electric dipole approximation are given by,

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\left[\frac{k^2}{4\pi\epsilon_0}\frac{\mathrm{e}^{i(kr-\omega t)}}{r}\hat{\mathbf{r}} \times (\mathbf{p}_\omega \times \hat{\mathbf{r}})\right], \quad \mathbf{B}(\mathbf{r},t) = \mathrm{Re}\left[\frac{-c\mu_0 k^2}{4\pi}\frac{\mathrm{e}^{i(kr-\omega t)}}{r}\mathbf{p}_\omega \times \hat{\mathbf{r}}\right],$$

where $k = \omega/c$.

Consider two charges q_1 and q_2 which are moving in a circular orbit of radius d in the xy plane. The charges are located on opposite sides of the orbit, and the rotate with angular frequency ω, as in the sketch below.



position of q_1 is: $\mathbf{r}_1 = d\cos(\omega t)\hat{\mathbf{x}} + d\sin(\omega t)\hat{\mathbf{y}}$

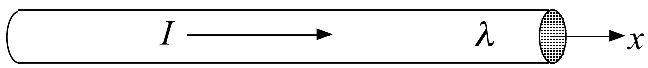
position of q_2 is: $\mathbf{r}_2 = -d\cos(\omega t)\hat{\mathbf{x}} - d\sin(\omega t)\hat{\mathbf{y}}$

a) [15 pts] If $q_1 = -q_2 = q$, find the radiated $\mathbf{E}$ and $\mathbf{B}$ fields in the electric dipole approximation. Express your answers as *real valued* functions of space and time. Find the time averaged Poynting vector $\langle\mathbf{S}\rangle$as a function of space. Make a polar plot of the angular distribution of the radiated power $dP/d\Omega$.

b) [15 pts] What happens if one now has the case $q_q = q_2 = q$? What term in the multipole expansion is responsible for the radiation? What is the frequency of the radiation? (You do not need to explicitly calculate $\mathbf{E}$, $\mathbf{B}$, or $\mathbf{S}$ for this part, just argue convincingly for your conclusion.)

4) [30 points total]

Consider an infinitely long straight cylindrical wire of radius R, centered about the x axis. Suppose that in an inertial frame K, there is a total current I flowing down the wire in the x direction and the wire has a net charge per unit length λ. Assume the current I and the charge λ are uniformly distributed over the cross-sectional area of the wire.



a) [10 pts] We know that the 4-current $j_\mu = (\mathbf{j}, ic\rho)$ is a 4-vector, where $\mathbf{j}$ is the current density and ρ is the charge density. For the geometry described above, argue why $I_\mu = (I\hat{\mathbf{x}}, ic\lambda)$ also transforms like a 4-vector *with respect to Lorentz transformations directed along the x axis.* Does I_μ also transform like a 4-vector with respect to Lorentz transformations in directions perpendicular to the x axis (you must explain your answer, not just say "yes" or "no")?

b) [10 pts] If I and λ are the current and charge per length seen in frame K, what are the current I' and charge per length λ' as seen in a frame K' that moves with velocity $v\hat{\mathbf{x}}$ with respect to frame K? In what situation is it possible to transform to such a frame K' in which the current $I' = 0$? In what situation is it possible to transform to a frame K' in which the charge per length $\lambda' = 0$?

c) [10 pts] Compute the electric and magnetic fields $\mathbf{E}'$ and $\mathbf{B}'$ in frame K' by applying a Lorentz transformation directly to the fields $\mathbf{E}$ and $\mathbf{B}$ in frame K (of course, you must first compute the fields in frame K in terms of the given current and charge per length, I and λ). Now compute the fields $\mathbf{E}'$ and $\mathbf{B}'$ in frame K' directly by solving the appropriate electrostatic and magnetostatic problems with the charge per length λ' and current I' that you found in part (b). Show that these two approaches give the same result for the fields in K'. For this part you may assume that the radius of the wire is vanishingly small.

Note: When K' moves with velocity $v\hat{\mathbf{x}}$ with respect to K, the Lorentz transformation law for fields is:

$$E'_x = E_x, \qquad B'_x = B_x$$

$$E'_y = \gamma(E_y - vB_z), \qquad B'_y = \gamma(B_y + \frac{v}{c^2}E_z)$$

$$E'_z = \gamma(E_z + vB_y), \qquad B'_z = \gamma(B_z - \frac{v}{c^2}E_y)$$