Momentum Conservation

want similar conservation law for mechanical & electromagnetic momentum

$$\frac{\partial}{\partial t}\left(p_{mi} + p_{EBi} \right) = \vec{\nabla}\cdot\vec{T}_i \qquad i = x, y, z$$

p_{mi} = i^{th} component of mechanical momentum density

p_{EBi} = i^{th} component of electromagnetic momentum density

$\vec{T}_i$ = $-$flux density (or "current") of i^{th} component of momentum density

Since $\vec{T}_i$ is a vector with 3 components, and there are three such vectors, for $i = x, y, z$, we will see that these 3 vectors form the components of a 3×3 tensor (ie matrix)

mechanical momentum density given by Newtons Law $\Big\}$ $\dfrac{\partial \vec{p}_m}{\partial t} = \vec{f} = \rho\vec{E} + \vec{j}\times\vec{B}$

force density $\vec{f} = \rho\vec{E} + \vec{j}\times\vec{B} = \varepsilon_0(\vec{\nabla}\cdot\vec{E})\vec{E} + \left(\frac{1}{\mu_0}\vec{\nabla}\times\vec{B} - \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)\times\vec{B}$

Now apply vector algebra + Maxwell's equ (see text sec 8.2.2) to manipulate into the form

$$\vec{f} = \varepsilon_0\left[(\vec{\nabla}\cdot\vec{E})\vec{E} + (\vec{E}\cdot\vec{\nabla})\vec{E}\right] + \frac{1}{\mu_0}\left[(\vec{\nabla}\cdot\vec{B})\vec{B} + (\vec{B}\cdot\vec{\nabla})\vec{B}\right]$$
$$-\frac{1}{2}\vec{\nabla}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \varepsilon_0\frac{\partial}{\partial t}(\vec{E}\times\vec{B}) \qquad \cancel{(7.91)}\ (8.15)$$