PHY 218 **Final Exam** **Spring 2016**

Please put a box around your final answer, and cross out any work you do not wish me to look at.

1) [25 points total]

Give a brief and to the point answer to each of the following. You may cite appropriate equations when it helps to explain a point, but no calculations should be necessary. Note, part (a) is worth 10 points, but the other parts are worth only 5 points each.

a) [10 pts] For electromagnetic wave propagation in a dielectric, we discussed three different regions of behavior as a function of frequency ω – (i) nearly transparent propagation, (ii) resonant absorption, (iii) total reflection. Give a one (or at most two) sentence description of the physical behavior of the wave in each region. Then state, from a mathematical point of view, the difference in the the permittivity $\epsilon(\omega)$ and the wavenumber k from region to region. Where along the ω axis does each region lie?

b) [5 pts] Describe one main physical difference between the propagation of a plane polarized simple-harmonic (i.e. single frequency) electromagnetic wave in a conductor vs in a non-conducting dielectric – give a clear physical effect rather than just a mathematical expression.

c) [5 pts] Why was it necessary for us to discuss the dynamical corrections to Maxwell's equations for electro- and magnetostatics in order to derive an expression for the energy stored in a magnetostatic magnetic field $\mathbf{B}$?

d) [5 pts] Why does Larmor's formula imply that the classical picture of the atom as a negatively charged electron orbiting the positively charged nucleus (as a planet orbits the sun) cannot be correct?

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2) [25 points total]

Consider a plane polarized electromagnetic wave in the vacuum described by the vector and scalar potentials

$$\mathbf{A}(\mathbf{r},t) = \mathbf{A}_0 \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \quad \text{and} \quad V(\mathbf{r},t) = V_0 \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

where the orientation of $\mathbf{A}_0$ is arbitrary and $\omega = ck$.

a) [8 pts] Using Maxwell's equations, find the relationship that must hold between the amplitudes $\mathbf{A}_0$ and V_0.

b) [8 pts] Using the principle of gauge invariance, show that one can transform to a new but physically equivalent vector potential $\mathbf{A}'(\mathbf{r},t)$ which is transversely polarized, i.e. $\mathbf{A}'_0 \cdot \mathbf{k} = 0$.

c) [7 pts] What is the scalar potential $V'(\mathbf{r},t)$ in the gauge of part (b)?

3) [25 points total]

Consider the radiation emitted by a thin circular wire loop of radius R, centered about the origin in the xy plane at $z = 0$. The current flowing in the loop is given by

$$I(\varphi, t) = \mathrm{Re}\left[I_0 \cos(n\varphi) \mathrm{e}^{-i\omega t}\right]$$

where φ is the usual azimuthal angle in spherical coordinates. The frequency ω is such that $R\omega \ll c$.

a) [8 pts] If $n = 0$, show that there is magnetic dipole radiation but no electric dipole radiation.

b) [8 pts] If $n = 1$, show that there is electric dipole radiation but no magnetic dipole radiation.

c) [7 pts] If $n = 2$, show that there is neither electric dipole nor magnetic dipole radiation. What happens in this case? You must explain your answer, not just give a guess.

Hint: Think about what the charge in the loop is doing. If you argue convincingly, you don't need to do a calculation.

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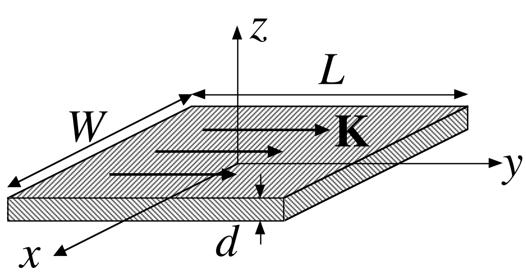
4) [25 points total]

Consider a flat conducting sheet lying in the xy-plane at $z = 0$. The sheet is electrically neutral (no net charge) and carrying a steady uniform (magnetostatic) sheet current $\mathbf{K} = K\hat{\mathbf{y}}$ in the y-direction. It may help conceptually to think of the sheet as having length L, width W, and small thickness d, as in the sketch below, with a uniform current density $\mathbf{j} = j\hat{\mathbf{y}}$ flowing through the cross-sectional area dW, so that $K = jd$. But we are interested in the limits that L and W get infinitely large, while d becomes infinitesmally small.



a) [5 pts] Find the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ for the above configuration.

b) [5 pts] Consider the above configuration as the inertial rest frame $\mathcal{K}$ of the sheet. Consider now making a transformation to an inertial frame $\mathcal{K}'$ that moves with velocity $\mathbf{v} = v\hat{\mathbf{y}}$ as seen by $\mathcal{K}$. What are the electric and magnetic fields $\mathbf{E}'$ and $\mathbf{B}'$ that are seen in frame $\mathcal{K}'$?

c) [5 pts] If you did part (b) correctly, you will have found that $\mathbf{E}' \neq 0$. Show explicitly that this $\mathbf{E}'$ results from the presence of a charge density on the sheet, as seen in $\mathcal{K}'$. Explicitly find this charge density by making a Lorentz transformation of the 4-current from $\mathcal{K}$ to $\mathcal{K}'$.

d) [5 pts] Where does this charge come from, given that the sheet is neutral in frame $\mathcal{K}$? Give a physical answer in terms of simple concepts from relativity, like time dilation or FitzGerald contraction (rather than from the machinery of 4-vectors and transformation matrices). You only have to give a good explanation, you do not have to do a calculation.

e) [5 pts] Is there a frame $\mathcal{K}'$ in which $\mathbf{B}' = 0$? You must explain your answer.

Some helpful formulae for Problem 4

For an inertial frame $\mathcal{K}'$ moving with velocity $\mathbf{v} = v\hat{\mathbf{x}}$ as seen by the inertial frame $\mathcal{K}$, the Lorentz transformation of a 4-vector from $\mathcal{K}$ to $\mathcal{K}'$ is given by the matrix $a_{\mu,\nu}$, with $\mu, \nu = 1, 2, 3, 4$. As we discussed in lecture, this matrix is

$$a_{\mu\nu} = \begin{pmatrix} \gamma & 0 & 0 & i(v/c)\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i(v/c)\gamma & 0 & 0 & \gamma \end{pmatrix}$$

The corresponding transformation of the electric and magnetic fields is,

$$E_1' = E_1 \qquad B_1' = B_1$$

$$E_2' = \gamma(E_2 - vB_3) \qquad B_2' = \gamma(B_2 + (v/c^2)E_3)$$

$$E_3' = \gamma(E_3 + vB_2) \qquad B_3' = \gamma(B_3 - (v/c^2)E_2)$$