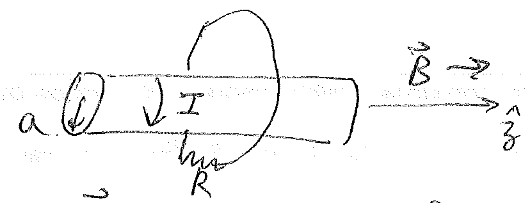
~~7/14~~

7-17



Long solenoid of radius a, N turns per length, i looped by wire of radius R

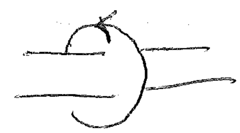
$$\vec{B} = \mu_0 N I \hat{z} \quad \text{inside solenoid}$$

$$\frac{dB}{dt} = \mu_0 N \frac{dI}{dt} \hat{z} \qquad \frac{dI}{dt} = k$$

a)
$$\frac{d\Phi}{dt} = \pi a^2 \mu_0 N k = -\mathcal{E}$$

current flows

$$\frac{\mathcal{E}}{R} = I = \frac{\pi a^2 \mu_0 N k}{R}$$



clockwise since $\mathcal{E} < 0$

b) Suppose solenoid taken out + reversed
what total charge Q passes through the resistor R?

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

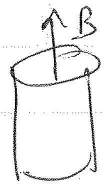
$$I = \frac{\mathcal{E}}{R} = \frac{-d\Phi}{R\,dt}$$

total charge is $Q = \int_0^T dt\, I = \frac{-1}{R} \int_0^T \frac{d\Phi}{dt} = \frac{-1}{R}\left[\Phi_f - \Phi_i\right]$

$$Q = \frac{-\pi a^2}{R}\left[-\mu_0 N I - \mu_0 N I\right] = \frac{2\mu_0 N I\, \pi a^2}{R}$$

7.20)

self inductance: $\vec{B} = \mu_0 N I \hat{z}$ B field inside solenoid



$\underbrace{N \cdot l}_{\#\ \text{turns}}$ l = length of solenoid

$$\Phi = \mu_0 I N \cdot N \cdot l \ \pi R^2$$
$$= L I \Rightarrow \boxed{L = \mu_0 N^2 l \ \pi R^2}$$

Another way to do Above: $W_{mag} = \frac{1}{2} L I^2$

$\cancel{\Phi = L I = \frac{\mu_0 N_1^2}{l_1} \pi R^2 I}$

$$= \frac{1}{2\mu_0} \int d^3r \, B^2$$
$$= \frac{1}{2\mu_0} \pi R^2 l \, (\mu_0 N I)^2$$

$\cancel{\mathcal{E}_1 = \frac{\mu_0 N_1^2 \pi R^2}{l_1} \frac{dI_1}{dt}}$

$$\Rightarrow L = \pi R^2 l \, \mu_0 N^2$$

same as above

~~7.21~~

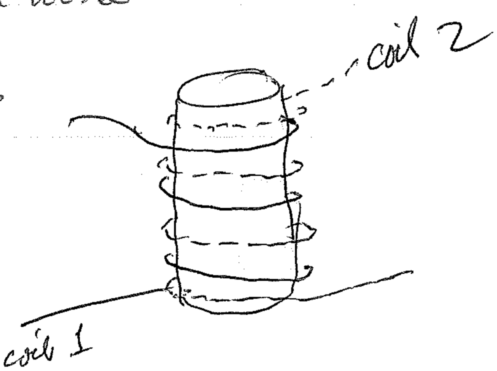
7.57

flux through each turn

$$\mathcal{E}_1 = -\frac{d\Phi}{dt} N_1 \leftarrow \#\ \text{turns}$$

$$\mathcal{E}_2 = -\frac{d\Phi}{dt} N_2$$

$$\Rightarrow \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$$



same flux through both coils

If voltage in coil 1 is $\mathcal{E}_1$, then by this method we can enduce a voltage $\mathcal{E}_2$ in coil 2

This is the principle on which a transformer works

Maxwell's Equations

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho \qquad\qquad \vec{\nabla}\cdot\vec{B}=0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Ampere's law for magnetostatics was

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

but $\underset{\substack{\parallel \\ 0 \\ \text{by general theorem} \\ \text{of vector calculus}}}{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B})} = \mu_0\vec{\nabla}\cdot\vec{J} = \underset{\substack{\neq \\ 0}}{-\mu_0\frac{\partial\rho}{\partial t}}$ by charge conservation

unless have electrostatics + magnetostatics

$\Rightarrow$ Ampere's law can't be valid outside static situations

To fix: write $-\mu_0\frac{\partial\rho}{\partial t} = -\mu_0\varepsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E}) = \vec{\nabla}\cdot\left(-\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$

correction is

So $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\cdot\left(\mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right) = 0$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \underbrace{\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}}_{\text{("displacement current")}}$$

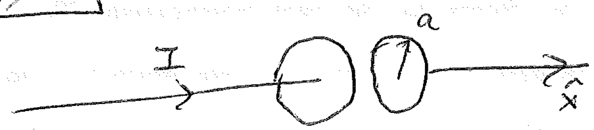
$\leftarrow$ Maxwell's correction to Ampere's Law

$$\oint_P \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\varepsilon_0\int_S\left(\frac{\partial\vec{E}}{\partial t}\right)\cdot d\vec{a}$$

so $\frac{\partial\vec{E}}{\partial t}$ is a source of $\vec{B}$, just like $\frac{\partial\vec{B}}{\partial t}$ is a source of $\vec{E}$

7.35

7.31



$$\nabla \times B = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

$$\oint d\vec{l} \cdot \vec{B} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \int d\vec{a} \cdot \frac{\partial \vec{E}}{\partial t}$$

find B between plates for $r < a$.

charge on plates $Q = It$ on left plate, $-It$ on right plate

$\Rightarrow$ E between plates is $\vec{E} = \frac{\sigma}{\epsilon_0} \hat{x} = \frac{Q}{\pi a^2 \epsilon_0} \hat{x}$

$$\vec{E} = \frac{It}{\epsilon_0 \pi a^2} \hat{x}, \quad \frac{\partial \vec{E}}{\partial t} = \frac{I}{\epsilon_0 \pi a^2} \hat{x}$$

in region between plates, symmetry $\Rightarrow \vec{B} = B(r)\hat{\phi}$

Take loop of radius r centered about wire, inbetween plates

$$\oint d\vec{l} \cdot \vec{B} = 2\pi r B(r) = \mu_0 \underbrace{I_{encl}}_{=0} + \mu_0 \epsilon_0 \int d\vec{a} \cdot \frac{I}{\epsilon_0 \pi a^2} \hat{x}$$

area of loop $= \pi r^2$

$$= \mu_0 \epsilon_0 \frac{\pi r^2 I}{\epsilon_0 \pi a^2}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0 r}{2\pi a^2} I \hat{\phi}$$

just like we had a wire with uniform current density $\vec{J} = \frac{I}{\pi a^2} \hat{x}$

for $r > a$, if ignore "edge" effects from non uniformity of $\vec{E}$ at edges of plates

$$\oint d\vec{\ell}\cdot\vec{B} = 2\pi r\, B(r) = \mu_0 \underbrace{I_{encl}}_{0} + \cancel{\mu_0\epsilon_0 \int d\vec{a}\cdot\frac{I}{\epsilon_0 \pi a^2}\hat{x}}$$

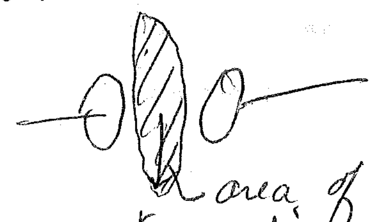
$$+\, \mu_0\epsilon_0 (\pi a^2)\frac{I}{\epsilon_0 \pi a^2}$$

since only area between plates has $\frac{\partial E}{\partial t} \neq 0$

$$= \mu_0 I$$

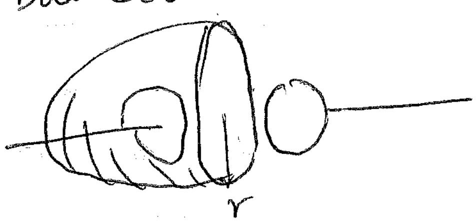
$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$ just like around wire with current I

to do above, took area of loop as



area of integration

but could also take any area bounded by curve,



now $\int d\vec{a}\cdot\frac{\partial\vec{E}}{\partial t} = 0$ on this area

but $I_{encl} = I$

So $B = \frac{\mu_0 I \hat{\phi}}{2\pi r}$ as before

Energy + Momentum Conservation (7.5)

We say in electrostatics $W_{elec} = \frac{\varepsilon_0}{2}\int d^3r\, E^2$

magnetostatics $W_{mag} = \frac{1}{2\mu_0}\int d^3r\, B^2$

Now treat for full electrodynamic situation

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \frac{\partial\vec{E}}{\partial t}\mu_0\varepsilon_0 \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$\mathcal{E}$ emf
"
voltage drop

power dissipated in current carrying wire is VI ← total current

$V = EL$ — electric field in wire

$I = Aj$ — cross sectional area

$\Rightarrow$ power dissipated is $EjLA = Ej\,\text{vol}$

in general, if we expect energy to be conserved, then, power dissipated $= \frac{d}{dt}$(mechanical or chemical ~~work done~~ energy ~~[illegible]~~ of particles)

$$\frac{dW_{mech}}{dt} = \int_{vol} d^3r\, \vec{E}\cdot\vec{j}$$

= kinetic energy + potential energy of the charges

also can get this: work done by electromagnetic fields to move charge $d\vec{r}$ is

$$W = \vec{F}\cdot d\vec{r} = (q\vec{E} + q\vec{v}\times\vec{B})\cdot d\vec{r}$$

work per unit time is

$$\frac{dW}{dt} = \vec{F}\cdot\frac{d\vec{r}}{dt} = \vec{F}\cdot\vec{v} = q\vec{E}\cdot\vec{v} + \underbrace{q(\vec{v}\times\vec{B})\cdot\vec{v}}_{=0}$$

work per unit time done on all charges is

$$\frac{dW}{dt} = \int d^3r\; n(\vec{r})\, q\,\vec{v}\cdot\vec{E} = \int d^3r\, \vec{j}\cdot\vec{E}$$

$n(\vec{r})$: density of charges

$$\frac{dW_m}{dt} = \int d^3r\, \vec{J}\cdot\vec{E}$$

Ampere's Law $\vec{J} = \dfrac{\vec{\nabla}\times\vec{B}}{\mu_0} - \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

$$\frac{dW_m}{dt} = \int d^3r \left[\underbrace{\frac{\vec{E}\cdot(\vec{\nabla}\times\vec{B})}{\mu_0}}_{\substack{\text{integrate}\\ \text{by parts}}} - \underbrace{\varepsilon_0 E\cdot\frac{\partial\vec{E}}{\partial t}}_{= \frac{1}{2}\frac{\partial E^2}{\partial t}} \right]$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{B})$$

$$\frac{dW_m}{dt} = \int d^3r \left[\frac{1}{\mu_0}\vec{B}\cdot(\vec{\nabla}\times\vec{E}) - \frac{1}{\mu_0}\vec{\nabla}\cdot(\vec{E}\times\vec{B}) - \varepsilon_0 \frac{1}{2}\frac{\partial E^2}{\partial t}\right]$$

use $\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$ Faraday

use $\vec{B}\cdot\dfrac{\partial\vec{B}}{\partial t} = \dfrac{1}{2}\dfrac{\partial B^2}{\partial t}$

$$= \int d^3r \left[\left(\frac{-1}{2}\right)\left(\frac{\partial B^2}{\mu_0\partial t} + \varepsilon_0\frac{\partial E^2}{\partial t}\right) - \frac{1}{\mu_0}\vec{\nabla}\cdot(\vec{E}\times\vec{B})\right]$$

$$\frac{dW_m}{dt} = -\frac{d}{dt}\int_{Vol} d^3r \left(\frac{1}{2\mu_0}B^2 + \frac{\varepsilon_0}{2}E^2\right) - \frac{1}{\mu_0}\oint_{Surface} d\vec{a}\cdot(\vec{E}\times\vec{B})$$

define $W_{EB} = \displaystyle\int_{Vol} d^3r \left(\frac{1}{2\mu_0}B^2 + \frac{\varepsilon_0}{2}E^2\right)$ electro-magneto energy in volume V

$\vec{S} \equiv \dfrac{\vec{E}\times\vec{B}}{\mu_0}$ "Poynting vector"

= energy density current