$$\frac{dW_m}{dt} = -\frac{d W_{EB}}{dt} - \oint d\vec{a}\cdot\vec{S}$$

(kinetic energy of charges)
increase in mechanical energy = energy lost from $\vec{E} + \vec{B}$ fields − energy from E + B fields flowing out of volume through surface

write $u_{EB} = \frac{\varepsilon_0}{2}E^2 + \frac{B^2}{2\mu_0}$ energy density of electromagnetic fields

u_M = mechanical energy density

$$\frac{d}{dt}\int_V d^3r\, u_m + \frac{d}{dt}\int_V d^3r\, u_{EB} = -\oint_S \vec{S}\cdot d\vec{a} = -\int_V d^3r\, \vec{\nabla}\cdot\vec{S}$$

$$\frac{\partial}{\partial t}\left(u_M + u_{EB}\right) = -\vec{\nabla}\cdot\vec{S}$$

law of local conservation of energy for e-m fields

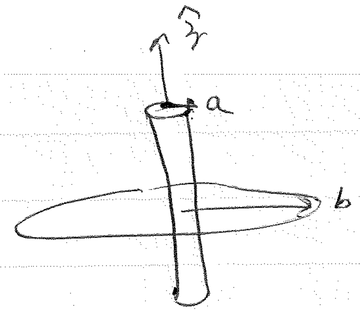
(same form as charge conservation $\frac{\partial \rho}{\partial t} = -\vec{\nabla}\cdot\vec{j}$

$\vec{S}$ is flux of energy carried by $\vec{E} + \vec{B}$ fields

$\oint_S \vec{S}\cdot d\vec{a}$ is energy per unit time carried by $\vec{E} + \vec{B}$ fields through surface S

8-13

~~7-6~~



a) Field in solenoid is $\vec{B} = \mu_0 N I_s \hat{z}$, $\vec{B}=0$ outside from solenoid

Flux through ring is $\Phi = \oint \vec{B}\cdot d\vec{a} = B\pi a^2 = \mu_0 \pi a^2 N I_s$

emf around ring (going in $\hat{\phi}$ direction)

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\mu_0 \pi a^2 N \frac{dI_s}{dt}$$

current in ring is $\vec{I}_r = \frac{\mathcal{E}}{R}\hat{\phi} = -\frac{\mu_0 \pi a^2 N}{R}\frac{dI_s}{dt}\hat{\phi}$

b) power delivered to ring is $I_r^2 R = \left(\mu_0 \pi a^2 N \frac{dI_s}{dt}\right)^2 \frac{1}{R} \equiv P$

must come from solenoid.

Show that power ~~emd~~ carried away from solenoid

by $\vec{E} + \vec{B}$ fields is just P above.

energy flux is $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$. Calculate $\vec{S}$ on outside surface of solenoid.

integral form of Faraday's law

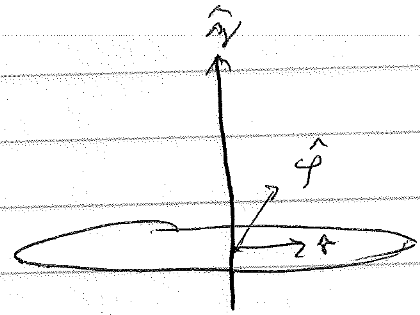
$\vec{E}$ is produced by the $\frac{\partial \vec{B}}{\partial t}$. Since $\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int d\vec{a}\cdot\vec{B}$

evaluate on path of radius $r=a$ just outside solenoid

By symmetry, $\vec{E} = E(r)\hat{\phi}$

$$\oint \vec{E}\cdot d\vec{l} = 2\pi a E(a) = -\frac{d\Phi}{dt} = -\mu_0 \pi a^2 N \frac{dI_s}{dt}$$

$$\vec{E} = \frac{-\mu_0 \pi a^2 N}{2\pi a}\frac{dI_s}{dt}\hat{\phi}$$



Biot - Savart Law

$$B(\vec r) = \frac{\mu_0}{4\pi}\int d^3r' \; \frac{\vec J(r') \times (\vec r - \vec r')}{|\vec r - \vec r'|^3}$$

$$B(z)\,\hat z = \frac{\mu_0 I r}{4\pi}\oint \frac{d\vec\ell \times (z\hat z - b\hat r)}{(z^2+b^2)^{3/2}}$$

$\hat\varphi \times \hat z = \hat r$

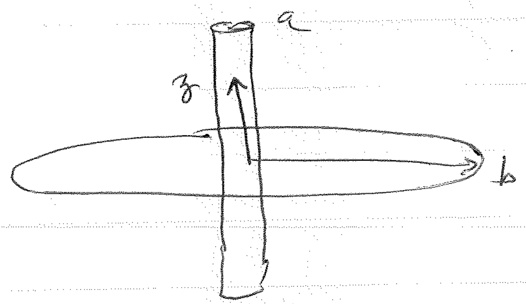
$\hat\varphi \times \hat r = -\hat z$

$$= \frac{\mu_0 I r}{4\pi} \oint_0^{2\pi} d\varphi\, b \; \frac{\hat\varphi \times (z\hat z - b\hat r)}{(z^2+b^2)^{3/2}}$$

$$= \frac{\mu_0 I r}{4\pi}\; \frac{b}{(z^2+b^2)^{3/2}} \int_0^{2\pi} d\varphi \,(\underbrace{z\hat r}_{\text{integrates to zero}} + b\hat z)$$

$$\boxed{B(z)\,\hat z = \frac{\mu_0 I r}{2}\;\frac{b^2}{(z^2+b^2)^{3/2}}}$$

the $\vec{B}$ just outside the solenoid, is the $\vec{B}$ produced by I_r flowing in the ring (I_s in solenoid produces no $\vec{B}$ outside $r \geq a$)



since $b \gg a$, we can approx $\vec{B}$ just outside solenoid, at heigh z, by the value at the center of the ring + height z. See example 6 Chpt 5 for result:

$$\vec{B}(z) = \frac{\mu_0 I_r}{2} \frac{b^2 \hat{z}}{(b^2+z^2)^{3/2}} = \frac{\mu_0}{2}\left(-\frac{\mu_0 \pi a^2 N}{R}\frac{dI_s}{dt}\right)\frac{b^2 \hat{z}}{(b^2+z^2)^{3/2}}$$

$\Rightarrow$ Poynting vector
$$\vec{S} = \frac{1}{\mu_0}\left(-\frac{\mu_0 \pi a^2 N}{2\pi a}\frac{dI_s}{dt}\right)\frac{\mu_0}{2}\left(-\frac{\mu_0 \pi a^2 N}{R}\frac{dI_s}{dt}\right)$$

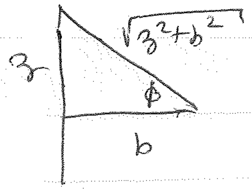
$$\times \frac{b^2}{(b^2+z^2)^{3/2}} \underbrace{(\hat{\phi} \times \hat{z})}_{=\hat{r}}$$

$$\vec{S} = \left(\mu_0 \pi a^2 N \frac{dI_s}{dt}\right)^2 \frac{1}{2\pi a R} \frac{b^2 \hat{r}}{2(b^2+z^2)^{3/2}}$$

total power leaving solenoid is just $\oint_S \vec{S} \cdot d\vec{a}$ where S is outside surface of solenoid

$$\oint \vec{S} \cdot d\vec{a} = 2\pi a \int_{-\infty}^{\infty} dz\, \vec{S}(z) \cdot \hat{r}$$

$$= \left(\mu_0 \pi a^2 N \frac{dI_s}{dt}\right)^2 \frac{1}{R} \frac{b^2}{2} \int_{-\infty}^{\infty} dz \frac{1}{(b^2+z^2)^{3/2}}$$

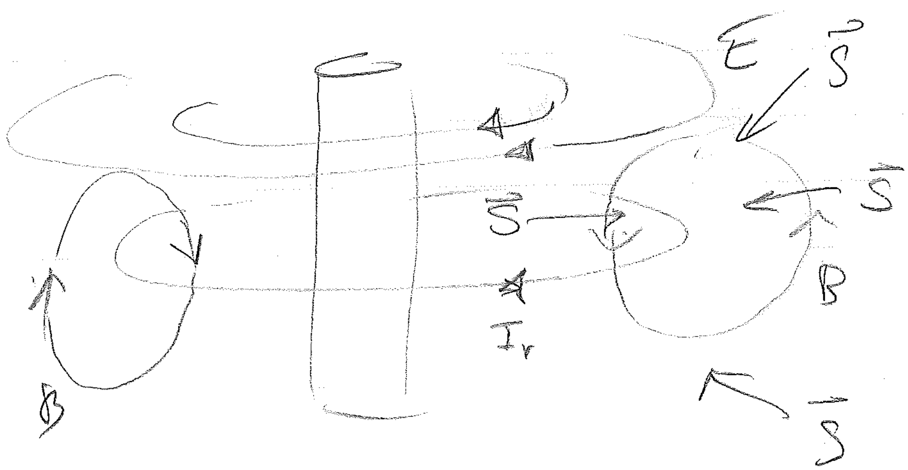


$$b\tan\phi = z \Rightarrow dz = \frac{b}{\cos^2\phi}\, d\phi$$

$$\frac{1}{\sqrt{z^2+b^2}} = \frac{\cos\phi}{b}$$

$$\int_{-\infty}^{\infty} dz\, \frac{1}{(b^2+z^2)^{3/2}} = \int_{-\frac{\pi}{2}}^{\pi/2} d\phi\, \frac{b}{\cos^2\phi}\, \frac{\cos^3\phi}{b^3} = \int_{-\frac{\pi}{2}}^{\pi/2} d\phi\, \frac{\cos\phi}{b^2} = \frac{2}{b^2}$$

$$\Rightarrow \oint \vec{S}\cdot d\vec{a} = \left(\mu_0 \pi a^2 N \frac{dI_s}{dt}\right)^2 \frac{1}{R} = P$$ power absorbed by ring.



$$\vec{E} = \frac{\mathcal{E}}{2\pi a}\,\hat{\phi} \qquad \vec{B} = \frac{\mu_0}{2} I_r \frac{b^2\,\hat{z}}{(b^2+z^2)^{3/2}}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{\mathcal{E} I_r}{2\pi a}\,\frac{1}{2}\,\frac{b^2}{(b^2+z^2)^{3/2}}\,\hat{r}$$

$$\oint \vec{S}\cdot d\vec{a} = \mathcal{E} I_r \int_{-\infty}^{\infty} dz\, \frac{b^2}{2\,(b^2+z^2)^{3/2}} = \mathcal{E} I_r$$