PHY 218 **Midterm Exam** **Spring 2016**

1) [40 points total] You should be able to give a short answer to each part of this question without any detailed calculations needed. [10 pts each part]

a) A loop #1 lies in the xy plane at $z = 0$, and carries a current flowing in the clockwise direction. Another loop #2 lies in the xy plane at height $z = d$ directly above loop #1. If the current in loop #1 is gradually increased, in which direction will the induced current in loop #2 flow?

b) Ampere's Law in magnetostatics is $\boldsymbol{\nabla}\times\mathbf{B} = \mu_0\mathbf{j}$. Explain why this equation can no longer hold true once one considers non-static situations. Don't just write the corrected equation, explain why this one must be wrong.

c) What is meant by the electromagnetic mass of a charged particle?

d) What is the Coulomb gauge? In the Coulomb gauge, the expression for the scalar potential V in terms of the charge density ρ remains exactly the same as in electrostatics,

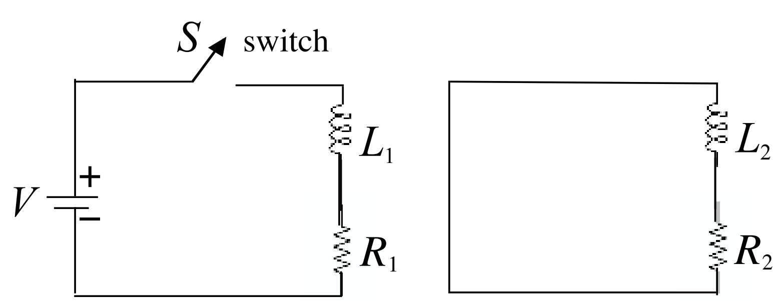
$$V(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\int_{-\infty}^{\infty} d^3r' \frac{\rho(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}$$

i.e. the potential at position $\mathbf{r}$ at time t is determined by the charge far away at position $\mathbf{r}'$ at exactly the *same* time t. Explain why this does not necessarily violate the physical principle that it should take some finite amount of time for information from a source at one position $\mathbf{r}'$ to be felt at another position $\mathbf{r}$ a finite distance away.

---

2) [30 points total]

Two circuits with self inductances L_1 and L_2 and resistances R_1 and R_2 are located near each other as in the sketch below. The mutual inductance of the two circuits is M. Circuit #1 has a battery of voltage V and a switch S. Initially, the switch S is open, but then at time $t = 0$ it is closed and a current in circuit #1 increases from zero, eventually reaching a steady constant value. Find the total charge Q that circulates around circuit #2 during this process. In what direction does the charge Q flow?



---

problem 3 on back side!

3) [30 points total]

Consider an electromagnetic wave traveling in the vacuum, where the electric field in complex exponential form is given by,

$$\mathbf{E}(\mathbf{r}, t) = E_0(\hat{\mathbf{x}} + i\hat{\mathbf{y}})\mathrm{e}^{i(kz-\omega t)}$$

where $\omega = ck$.

a) [8 pts] Find the corresponding magnetic field $\mathbf{B}(\mathbf{r}, t)$ for this wave and write your result in complex exponential form.

b) [4 pts] Take the real parts of the above complex exponential forms to write the real valued electric and magnetic fields.

c) [6 pts] Compute the instantaneous electromagnetic field energy density $u_{\mathrm{EB}}(\mathbf{r}, t)$ of this wave.

d) [6 pts] Compute the instantaneous energy current $\mathbf{S}(\mathbf{r}, t)$ of this wave.

e) [6 pts] Compute all components of the instantaneous Maxwell stress tensor $\mathbf{T}_{ij}(\mathbf{r}, t)$ of this wave and interpret your results in terms of the force per unit area that the wave will exert on an absorbing surface oriented with normal in the $+\hat{\mathbf{x}}$, $+\hat{\mathbf{y}}$ and $+\hat{\mathbf{z}}$ directions.