PHY 218 **Midterm Exam** **Spring 2017**

1) [40 points total] You should be able to give a short answer to each part of this question without any detailed calculations needed. [8 pts each part]

The dispersion relation for transverse electromagnetic waves propagating in a dielectric material or a conductor is determined by the permittivity $\epsilon(\omega)$ of the material.

a) Give two physical consequences of the fact that $\epsilon(\omega)$ varies with the wave frequency ω.

b) Give two physical consequences of the fact that $\epsilon(\omega)$ may be a complex valued function.

If ϵ_1 and ϵ_2 are the real and imaginary parts of ϵ respectively, and k_1 and k_2 the real and imaginary parts of the amplitude of the wavevector $|\mathbf{k}|$, then characterize the values of ϵ_1, ϵ_2, k_1 and k_2 if one is in:

c) a region of nearly transparent propagation.

d) a region of strong absorption.

e) a region of total reflection.

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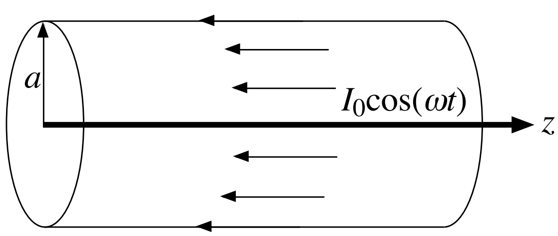
2) [30 points total]

An alternating current $I(t) = I_0 \cos(\omega t)$ flows down a long straight wire, and returns flowing uniformly distributed along the surface of a coaxial conducting tube of radius a, as in the sketch below. Assume that the frequency ω is sufficiently small, that the resulting magnetic field $\mathbf{B}$ can be computed quasistatically, i.e. using the magnetostatic Ampere's law.



a) [8 pts] Find the magnetic field $\mathbf{B}(\mathbf{r}, t)$. Be sure to give an answer for all regions of space.

b) [5 pts] Because $\mathbf{B}(\mathbf{r}, t)$ from part (a) varies in time, there will be an induced electric field $\mathbf{E}(\mathbf{r}, t)$. In what direction is the induced electric field? [*Hint: It might help to think about an analog magnetostatic problem.*]

c) [10 pts] Find the electric field $\mathbf{E}(\mathbf{r}, t)$, assuming that $\mathbf{E} \to 0$ as one goes infinitely far from the wire.

d) [7 pts] Compute the Poynting vector $\mathbf{S}(\mathbf{r}, t)$. In what direction does the electromagnetic energy flow? What is the average $\langle \mathbf{S}(\mathbf{r}) \rangle$, averaged over one period of oscillation of the current?

problem 3 on back side!

3) [30 points total]

Consider a plane polarized electromagnetic wave in the vacuum described by the vector and scalar potentials

$$\mathbf{A}(\mathbf{r},t) = \mathbf{A}_0 \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \quad \text{and} \quad V(\mathbf{r},t) = V_0 \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

where the orientation of $\mathbf{A}_0$ is arbitrary and $\omega = c|\mathbf{k}|$.

a) [10 pts] Using Maxwell's equations, find the relationship that must hold between the amplitudes $\mathbf{A}_0$ and V_0.

b) [10 pts] Using the principle of gauge invariance, show that one can transform to a new but physically equivalent vector potential $\mathbf{A}'(\mathbf{r},t)$ which is transversely polarized, i.e. $\mathbf{A}'_0 \cdot \mathbf{k} = 0$.

c) [10 pts] What is the scalar potential $V'(\mathbf{r},t)$ in the gauge of part (b)?