PHY 218 **Final Exam** **Spring 2019**

• Please write and sign on the inside cover of your blue book the academic honesty pledge, "I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

• For each question, please put a box around your final answer, and cross out any work you do not wish me to look at. If you include sentences explaining your steps you are more likely to receive partial credit if you make a mistake.

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1) [35 points total]

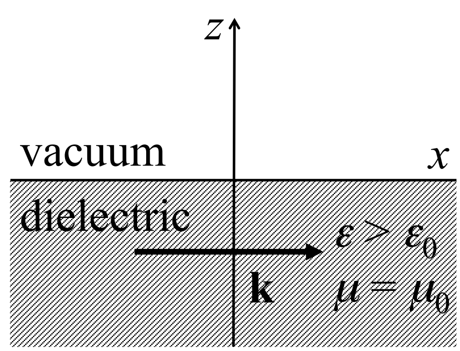
Consider a semi-infinite dielectric ("medium b") with a real positive permittivity $\epsilon > \epsilon_0$ and $\mu = \mu_0$. The surface of the dielectric is the xy plane at $z = 0$ and the dielectric fills all of space below this plane ($z < 0$), while a vacuum ("medium a") fills all of space above this plane ($z > 0$). A plane polarized simple harmonic electromagnetic wave of frequency ω is traveling inside the dielectric in the $\hat{\mathbf{x}}$ direction, as sketched below. We can describe the electric field of this wave as,

$$\mathbf{E}_b(\mathbf{r},t) = \mathrm{Re}\left[\mathbf{E}_b e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right] \quad \text{with} \quad \mathbf{k} = k_x\hat{\mathbf{x}}.$$

The electric field in the vacuum is,

$$\mathbf{E}_a(\mathbf{r},t) = \mathrm{Re}\left[\mathbf{E}_a e^{i(\mathbf{k}'\cdot\mathbf{r}-\omega' t)}\right] \quad \text{with} \quad \mathbf{k}' = k'_x\hat{\mathbf{x}}. + k'_y\hat{\mathbf{y}} + k'_z\hat{\mathbf{z}}.$$

Note, $\mathbf{k}'$ is not necessarily solely in the $\hat{\mathbf{x}}$ direction.



a) [5 points] Write down the equation that determines the relation between $\mathbf{k}$ and ω in the dielectric. Write down the equation that determines the relation between $\mathbf{k}'$ and ω' in the vacuum.

b) [5 points] Write down the boundary conditions at $z = 0$, that determine how the electromagnetic fields in medium a and medium b are related at their interface.

c) [10 points] Using the boundary conditions of part (b), give the relations between the frequencies ω and ω', and the wavevectors $\mathbf{k}$ and $\mathbf{k}'$, of the electromagnetic fields in the two different media.

d) [15 points] Show that the electromagnetic field in the vacuum decays exponentially as one moves in the $\hat{\mathbf{z}}$ direction away from the $z = 0$ interface (Hint: show that k'_z is an imaginary number).

2) [35 points total]

Consider an isolated neutral atom where the nucleus of the atom is held fixed in place at the origin of some coordinate system. A plane, linearly polarized, electromagnetic wave of angular frequency ω acts on the atom, exerting a force on the electron cloud and causing it to oscillate. If $\mathbf{r}(t)$ denotes the center of mass of the electron cloud with respect to the nucleus, the electron cloud moves under the influence of three forces: (i) the force from the electromagnetic wave, (ii) a restoring force $-m\omega_0^2\mathbf{r}$ pulling the electron back to the nucleus, and (iii) a dissipative force $-m\gamma\mathbf{v}$. Here m is the mass of the electron cloud, $-q$ its charge, and $\mathbf{v} = \dot{\mathbf{r}}$ the velocity of the electron cloud's center of mass. You may approximate the electron cloud as a fixed rigid object moving in a classical non-relativistic way.

a) [15 pts] What is the total work done on the electron cloud by the electromagnetic wave in one period of osciallation?

b) [15 pts] What is the total energy radiated away from the atom in electromagnetic radiation in one period of oscillation?

c) [5 pts] If one wishes to interpret the dissipative force $-m\gamma\mathbf{v}$ as arising solely from the radiated energy, what should be the value of the parameter γ? Do you see anything funny about your answer? Is there a physical interpretation?

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3) [30 points total]

For both parts below, consider the inertial frame of reference of the "laboratory" $\mathcal{K}$, and another inertial frame of reference $\mathcal{K}'$ that moves with velocity $\mathbf{v} = v\hat{\mathbf{z}}$ as seen by an observer in $\mathcal{K}$.

a) [15 points] Consider an infinite straight line of charge oriented along the $\hat{z}$ axis, with charge per unit length λ that is at rest in frame $\mathcal{K}$. What are the fields $\mathbf{E}$ and $\mathbf{B}$ that are seen in frame $\mathcal{K}$? What are the fields $\mathbf{E}'$ and $\mathbf{B}'$ that are seen in frame $\mathcal{K}'$?

b) [15 points] Consider an infinite flat slab of finite thickness d, that is oriented in the xy plane centered at height $z = 0$ There is a uniform charge density ρ filling the slab. What are the fields $\mathbf{E}$ and $\mathbf{B}$ that are seen in frame $\mathcal{K}$? What are the fields $\mathbf{E}'$ and $\mathbf{B}'$ that are seen in frame $\mathcal{K}'$? Be sure to give the fields above the slab ($z > d/2$), the fields below the slab ($z < -d/2$), and the fields inside the slab ($-d/2 < z < d/2$).