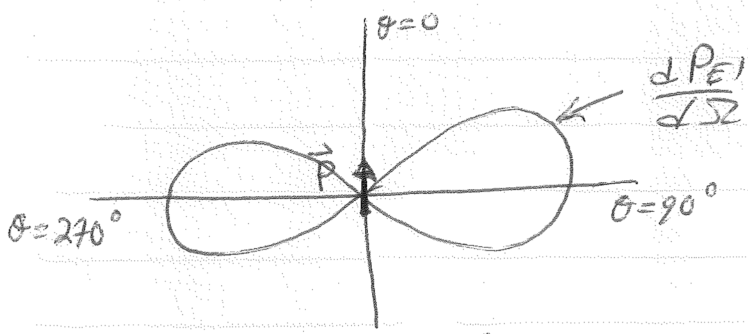
$$\frac{dP_{E1}}{d\Omega} = \hat{r} \cdot \langle \vec{S}_{E1} \rangle r^2 = \frac{ck^4 p^2}{2(4\pi)^2 \varepsilon_0} \sin^2\theta \sim \omega^4 \sin^2\theta$$

$\omega = ck$



polar plot

most of power is directed outwards into the plane $\perp$ to $\vec{p}$, ie at angles θ peaked about $90°$

For energy conservation to hold, it must be true that all the higher order terms, that go as higher powers of $\frac{1}{r}$ (ie $\frac{1}{r^2}$, $\frac{1}{r^3}$, etc ...), must vanish when integrated over surface of sphere compute the time averaged energy flux — otherwise energy would be dissappearing as the wave propagated outwards.

Total power radiated is

$$P_{E1} = \int \frac{dP_{E1}}{d\Omega} d\Omega = \frac{ck^4p^2}{2(4\pi)^2\varepsilon_0} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \sin^2\theta$$

$$= \frac{ck^4p^2}{32\pi^2\varepsilon_0} 2\pi \underbrace{\int_0^{\pi} d\theta \sin\theta (1-\cos^2\theta)}_{\left[-\cos\theta + \frac{\cos^3\theta}{3}\right]_0^{\pi} = \frac{4}{3}}$$

$$P_{E1} = \frac{ck^4p^2}{4\pi\varepsilon_0 \cdot 3} = \boxed{\frac{p^2\omega^4}{4\pi\varepsilon_0 \, 3c^3} = P_{E1}} \sim \omega^4$$

<u>Note:</u> when we wrote for $\vec{E}_{E1} = \frac{k^2}{4\pi\epsilon_0}\frac{e^{ikr}}{r}\hat{r}\times(\vec{p}\times\hat{r})$

$$\mathrm{Re}\left[\vec{E}_{E1}(\vec{r},\omega)e^{-i\omega t}\right] = \frac{k^2}{4\pi\epsilon_0}\frac{\cos(kr-\omega t)}{r}\hat{r}\times(\vec{p}\times\hat{r})$$

we implicitly assumed that the amplitude of the oscillating electric dipole moment $\vec{p}(\omega)$ was a real vector. But that is not necessarily always the case!

For $\vec{p}(\omega) \equiv \vec{p}_1$ real, the time dependent dipole moment is

$$\vec{p}(t) = \mathrm{Re}\left[\vec{p}_1 e^{-i\omega t}\right] = \vec{p}_1\cos\omega t$$

points always in same direction with oscillating magnitude.

But suppose $\vec{p}(\omega) = \vec{p}_1 + i\vec{p}_2$. Then

$$\vec{p}(t) = \mathrm{Re}\left[(\vec{p}_1 + i\vec{p}_2)e^{-i\omega t}\right]$$
$$= \vec{p}_1\cos\omega t + \vec{p}_2\sin\omega t$$

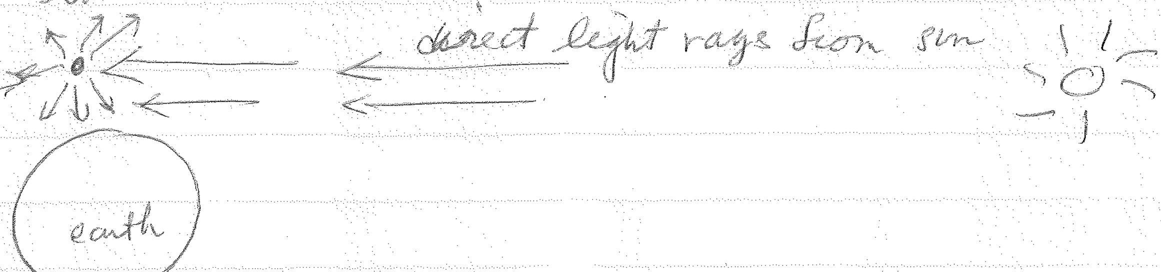
Now <u>direction</u> of $\vec{p}(t)$ is rotating!

if $\vec{p}_1 \perp \vec{p}_2$ then the tip of $\vec{p}(t)$ sweeps out an ellipse! An example of such a $\vec{p}$ would be a charge moving in an elliptical orbit

So if $\vec{p}(\omega)$ is complex we need to be more careful in our calculation of $\vec{S}$

Why the sky is blue - Lord Rayleigh

When look at sky, you are seeing the indirect light of the sun, which is the light emitted by the atoms and ~~molec~~ molecules of the atmosphere as they osculate + so radiate, due to the electric field of the direct light from the sun



molecule, ~~atom~~ in atmosphere oscilates in $\vec{E}$ field of direct rays, and then emitts radiated light with power (can view this as a scattering of the direct rays)

$$P \sim \omega^4 p^2$$

polarizability

p is dipole moment of ~~atom~~ molecule in atmosphere $p = \alpha E$

electric field of direct rays

$$\alpha \sim \frac{e^2}{m} \frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

for molecules in atmosphere, N_2, etc, ω_0 is typically a freq higher than the visible spectrum. Therefore for light in visible spectrum, $\alpha \sim \frac{e^2}{m\omega_0}$ indep of ω

Power emitted $\sim \omega^4$ largest at higher freq.

Since light from sun is "white light" it has components of all freqs. ~~For~~ From the above, we see that this ~~indirect scattered~~ light is most scattered at the higher freqs, due to the ω^4 dependence of

Scattered power in electric dipole approx
$\Rightarrow$ indirect light is strongest in the blue (large ω) part of the visible spectrum. $\Rightarrow$ sky is blue!

When we look at sunrise or sunset however, we are looking at the direct rays of the sun. Since these rays are scattered most in the blue, the direct rays are strongest in the red (small ω) part of the spectrum $\Rightarrow$ sunset & sunrise are red!

Magnetic Dipole Radiation - in the Radiation Zone Approx

$$\vec{A}_{M1} = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} ik\, \hat{r} \times \vec{m}$$

$$\vec{B}_{M1} = \vec{\nabla} \times \vec{A}_{M1} = \frac{\mu_0}{4\pi} ik\, \vec{\nabla} \times \left(e^{ikr} \frac{\hat{r} \times \vec{m}}{r} \right)$$

Exactly the same form as when we computed $\vec{E}_{E1}$ from B_{E1} except $\vec{p} \to \vec{m}$

use $\vec{\nabla} \times (f\vec{g}) = \vec{\nabla} f \times \vec{g} + f\, \vec{\nabla} \times \vec{g}$

$$\vec{B}_{M1} = \frac{\mu_0}{4\pi} ik \left[\underbrace{(\vec{\nabla} e^{ikr}) \times \left(\frac{\hat{r} \times \vec{m}}{r} \right)}_{ike^{ikr}\, \hat{r} \times \left(\frac{\hat{r} \times \vec{m}}{r} \right)} + \underbrace{e^{ikr} \vec{\nabla} \times \left(\frac{\hat{r} \times \vec{m}}{r} \right)}_{\sim O\left(\frac{1}{r^2}\right) \text{ so ignore in RZ approx}} \right]$$

$$\vec{B}_{M1} = -\frac{\mu_0}{4\pi} k^2 \frac{e^{ikr}}{r}\, \hat{r} \times (\hat{r} \times \vec{m})$$

$$\vec{E}_{M1} = \frac{i}{\omega \mu_0 \varepsilon_0} \vec{\nabla} \times \vec{B}_{M1}$$ from Amperes law with $\vec{J} = 0$

$$= \frac{-i\mu_0 k^2}{4\pi \omega \mu_0 \varepsilon_0} \vec{\nabla} \times \left(\frac{e^{ikr}}{r}\, \hat{r} \times (\hat{r} \times \vec{m}) \right)$$

$$= \frac{-ik^2}{4\pi\omega\varepsilon_0} \left[(\underbrace{\vec{\nabla} e^{ikr}}_{ik\hat{r}e^{ikr}}) \times \left(\frac{\hat{r} \times (\hat{r} \times \vec{m})}{r} \right) + \underbrace{e^{ikr} \vec{\nabla} \times \left(\frac{\hat{r} \times (\hat{r} \times \vec{m})}{r} \right)}_{\sim O\left(\frac{1}{r^2}\right) \text{ so ignore in RZ approx}} \right]$$

$$= \frac{k^3}{4\pi\omega\varepsilon_0} \frac{e^{ikr}}{r}\, \hat{r} \times (\hat{r} \times (\hat{r} \times \vec{m}))$$ use $\omega = ck$

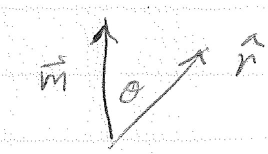
use triple product rule

$$\hat{r} \times (\hat{r} \times (\hat{r} \times \vec{m})) = \hat{r}(\hat{r} \cdot (\hat{r} \times \vec{m})) - (\hat{r} \times \vec{m})(\hat{r} \cdot \hat{r})$$
$$= 0 - \hat{r} \times \vec{m}$$

$$\vec{E}_{M1} = \frac{-k^2}{4\pi\varepsilon_0 c} \frac{e^{ikr}}{r}\, \hat{r} \times \vec{m}$$

$$E_{M1} = \frac{k^2}{4\pi\varepsilon_0 c} \frac{e^{ikr}}{r} \left[-\hat{r} \times \vec{m} \right]$$

$$\boxed{\begin{aligned} \vec{E}_{M1} &= \frac{k^2}{4\pi\varepsilon_0 c} \frac{e^{ikr}}{r} \left[\vec{m} \times \hat{r} \right] \\ \vec{B}_{M1} &= \frac{\mu_0 k^2}{4\pi} \frac{e^{ikr}}{r} \left[\hat{r} \times (\vec{m} \times \hat{r}) \right] \end{aligned}}$$

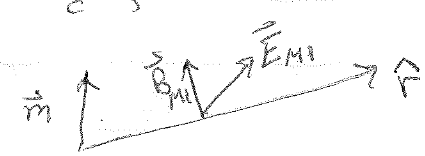


for $\underline{\underline{\vec{m} = m\hat{z}}}$, $\vec{m} \times \hat{r} = m \sin\theta \, \hat{\phi}$

$\hat{r} \times (\vec{m} \times \hat{r}) = m \sin\theta \, (-\hat{\theta})$

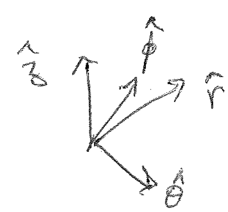
$$\left.\begin{aligned} \vec{E}_{M1} &= \frac{k^2 m}{4\pi\varepsilon_0 c} \frac{e^{ikr}}{r} \sin\theta \, \hat{\phi} \\ \vec{B}_{M1} &= -\frac{\mu_0 k^2 m}{4\pi} \frac{e^{ikr}}{r} \sin\theta \, \hat{\theta} \end{aligned}\right\}$$

Note similarity with $\vec{E}_{E1}$ and $\vec{B}_{E1}$
$\vec{p} \rightarrow \frac{\vec{m}}{c}$, $\vec{E}$ + $\vec{B}$ rotated by $90°$



Poynting vector

$$\cancel{\vec{S}_{M1} = \frac{1}{\mu_0} \vec{E}_{M1}(\vec{r},t) \times \vec{B}_{M1}(\vec{r},t) = \frac{1}{\mu_0} \frac{k^2 m}{}}$$



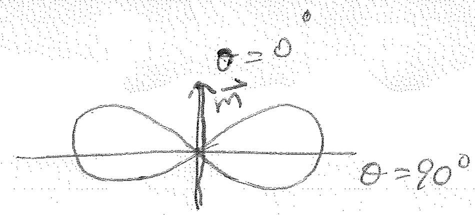
$$\vec{S}_{M1} = \frac{1}{\mu_0} \mathrm{Re}\left\{ \vec{E}_{M1} e^{-i\omega t} \right\} \times \mathrm{Re}\left\{ \vec{B}_{M1} e^{-i\omega t} \right\}$$

$$= \frac{1}{\mu_0} \left(\frac{k^2 m}{4\pi\varepsilon_0 c} \right) \left(-\frac{\mu_0 k^2 m}{4\pi} \right) \frac{\sin^2\theta}{r^2} \cos^2(kr - \omega t) \underbrace{\hat{\phi} \times \hat{\theta}}_{-\hat{r}}$$

$$= \frac{k^4 m^2}{4\pi\varepsilon_0 \, 4\pi c} \frac{\sin^2\theta}{r^2} \cos^2(kr - \omega t) \, \hat{r}$$

time average

$$\langle \vec{S}_{M1} \rangle = \frac{k^4 m^2}{32\pi^2 \varepsilon_0 c} \frac{\sin^2\theta}{r^2} \hat{r}$$



power cross section

$$\frac{dP_{M1}}{d\Omega} = \hat{r}\cdot\langle \vec{S}_{M1}\rangle r^2 = \boxed{\frac{k^4 m^2}{2(4\pi)^2 \varepsilon_0 c}\sin^2\theta = \frac{dP_{M1}}{d\Omega}}$$

same form as $\frac{dP_{E1}}{d\Omega}$ with $p \to \frac{m}{c}$

total power

$$P_{M1} = \int \frac{dP_{M1}}{d\Omega}\, d\Omega = \frac{2\pi\, k^4 m^2}{2(4\pi)^2 \varepsilon_0 c} \underbrace{\int_0^{\pi} d\theta \sin^3\theta}_{4/3}$$

$k^4 = \frac{\omega^4}{c^4}$

$$\boxed{P_{M1} = \frac{\omega^4 m^2}{4\pi\varepsilon_0\, 3c^5}}$$

compare to $P_{E1} = \frac{\omega^4 p^2}{4\pi\varepsilon_0\, 3c^3}$

$$\frac{P_{M1}}{P_{E1}} = \left(\frac{m}{cp}\right)^2 \quad \text{as } m \sim vp \Rightarrow \frac{P_{M1}}{P_{E1}} \sim \left(\frac{v}{c}\right)^2$$

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electric quadrupole radiation - radiation zone approx

$$\vec{A}_{E2} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\, ik\left(\frac{i\omega}{6}\right)\hat{r}\cdot\overleftrightarrow{Q}$$

check

Find fields for homework!