Consider the 4-acceleration

$$\alpha_\mu = \frac{du_\mu}{ds} = \gamma \frac{d}{dt} u_\mu \qquad \text{since } ds = dt/\gamma$$

use $u_\mu = (\gamma \vec{v}, i\gamma c)$

$$\vec{\alpha} = \gamma \frac{d}{dt}(\gamma \vec{v}) = \gamma^2 \vec{a} + \gamma \vec{v} \frac{d\gamma}{dt}$$

$$\alpha_4 = \gamma i c \frac{d\gamma}{dt}$$

we need $\dfrac{d\gamma}{dt} = \dfrac{d}{dt}\left(\dfrac{1}{\sqrt{1 - v^2/c^2}}\right) = \dfrac{+\frac{\vec{v}}{c^2} \cdot \frac{d\vec{v}}{dt}}{(1 - v^2/c^2)^{3/2}}$

$$= + \frac{\vec{v} \cdot \vec{a}}{c^2} \gamma^3$$

so

$$\vec{\alpha} = \gamma^2 \vec{a} + \gamma^4 \left(\frac{\vec{v} \cdot \vec{a}}{c^2}\right) \vec{v}$$

$$\alpha_4 = \gamma^4 i \left(\frac{\vec{v} \cdot \vec{a}}{c}\right)$$

$$\alpha_\mu = \gamma^4 \left(\left(\frac{\vec{v} \cdot \vec{a}}{c^2}\right) \vec{v} + \frac{\vec{a}}{\gamma^2}, \; i\left(\frac{\vec{v} \cdot \vec{a}}{c}\right) \right)$$

in frame $\overset{\circ}{K}$, $\overset{\circ}{\vec{v}} = 0$ and $\overset{\circ}{\gamma} = 1$, so

$$\overset{\circ}{\alpha}_\mu = (\overset{\circ}{\vec{a}}, 0) \quad \text{and so} \quad \overset{\circ}{a}{}^2 = \alpha_\mu^2 \quad \text{Lorentz invariant scalar}$$

So now we can write the relativistic Larmor formula

$$P = \frac{1}{4\pi\varepsilon_0} \frac{2}{3} \frac{q}{c^3} \overset{\circ}{a}{}^2 = \frac{1}{4\pi\varepsilon_0} \frac{2}{3} \frac{q}{c^3} \alpha_\mu^2 \quad \text{in any frame } K$$

In a general frame K,

$$\alpha_\mu^2 = |\vec{\alpha}|^2 + \alpha_4^2$$

$$= \gamma^8 \left[\left(\frac{\vec{v}\cdot\vec{a}}{c^2}\right)^2 v^2 + \frac{a^2}{\gamma^4} + 2\left(\frac{\vec{v}\cdot\vec{a}}{c^2}\right)\frac{(\vec{v}\cdot\vec{a})}{\gamma^2} - \left(\frac{\vec{v}\cdot\vec{a}}{c}\right)^2 \right]$$

$$= \gamma^8 \left[-\left(\frac{\vec{v}\cdot\vec{a}}{c}\right)^2 \left(1 - \frac{v^2}{c^2}\right) + 2\left(\frac{\vec{v}\cdot\vec{a}}{c}\right)^2 \frac{1}{\gamma^2} + \frac{a^2}{\gamma^4} \right]$$

$$= \gamma^8 \left[\left(\frac{\vec{v}\cdot\vec{a}}{c}\right)^2 \left(\frac{2}{\gamma^2} - \frac{1}{\gamma^2}\right) + \frac{a^2}{\gamma^4} \right]$$

$$= \gamma^8 \left[\frac{a^2}{\gamma^4} + \left(\frac{\vec{v}\cdot\vec{a}}{c}\right)^2 \frac{1}{\gamma^2} \right]$$

$$\alpha_\mu^2 = \gamma^4 \left[a^2 + \gamma^2 \left(\frac{\vec{v}\cdot\vec{a}}{c}\right)^2 \right]$$

Note: as $v \to 0$, $\gamma \to 1$ and we get $\alpha_\mu^2 = a^2$ as we must.

So power radiated is

$$P = \frac{1}{4\pi\varepsilon_0} \frac{2}{3} \frac{q}{c^3} \gamma^4 \left[a^2 + \gamma^2 \left(\frac{\vec{v}\cdot\vec{a}}{c}\right)^2 \right]$$

Examples:

① For a charge accelerating in linear motion (such as in a linear particle accelerator such as SLAC)

$\vec{v}\cdot\vec{a} = va$ since $\vec{v}$ and $\vec{a}$ are colinear

$$P = \frac{1}{4\pi\varepsilon_0} \frac{2}{3} \frac{q}{c^3} \gamma^4 \left[a^2 + \gamma^2 \frac{v^2 a^2}{c^2} \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{2}{3} \frac{q}{c^3} \gamma^4 a^2 \left[1 + \gamma^2 \frac{v^2}{c^2} \right]$$

$$1+\gamma^2\frac{v^2}{c^2} = 1+\frac{v^2/c^2}{1-v^2/c^2} = \frac{1}{1-v^2/c^2} = \gamma^2$$

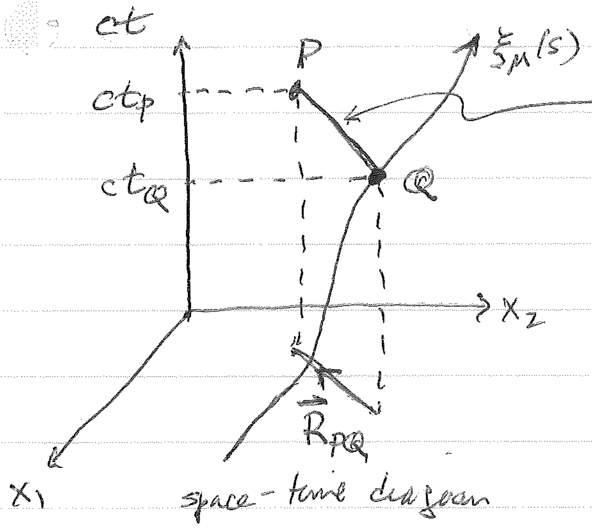
$$P = \frac{1}{4\pi\varepsilon_0}\frac{2}{3}\frac{q}{c^3}a^2\gamma^6$$ relativistic result increased by factor γ^6 compared to non-relativistic result

② For a charge accelerating in circular motion (such as in a synchrotron)

$\vec{v}\cdot\vec{a} = 0$ since $\vec{v}\perp\vec{a}$

$$P = \frac{1}{4\pi\varepsilon_0}\frac{2}{3}\frac{q}{c^3}a^2\gamma^4$$ relativistic result increased by factor γ^4 compared to non-relativistic result

Lienard - Wiechert Potentials in Covariant form



space-time diagram in inertial frame K

light cone of point Q. A pulse of light leaving Q will arrive at P

$\xi_\mu(s)$ is the trajectory of the charged particle 4-position as function of particles proper time s

$$c\Delta t = c(t_P - t_Q) = |\vec{r}_P - \vec{r}_Q| \equiv |\vec{R}_{PQ}|$$

the 4-potential at point P is due to the charge at the earlier point Q. In frame K, $\vec{R}_{PQ}$ is the spatial vector from Q to P, Δt is the time difference between Q and P.

L-W potentials

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi} \frac{q\vec{v}(t')}{|\vec{r}-\vec{r}_0(t')|} \frac{1}{1-\frac{1}{c}\hat{n}(t')\cdot\vec{v}(t')}$$

t' is the retarded time

$$V(\vec{r},t) = \frac{q}{4\pi\varepsilon_0} \frac{1}{|\vec{r}-\vec{r}_0(t')|} \frac{1}{1-\frac{1}{c}\hat{n}(t')\cdot\vec{v}(t')}$$

$$\hat{n}(t') = \frac{\vec{r}-\vec{r}_0(t')}{|\vec{r}-\vec{r}_0(t')|}$$

if we want the potentials at point P, then $(t', \vec{r}_0(t'))$ refers to point Q. So we can rewrite the above as

$$\vec{A}(P) = \frac{\mu_0}{4\pi} \frac{q\vec{v}_Q}{R_{PQ} - \vec{R}_{PQ}\cdot\frac{\vec{v}_Q}{c}}$$

$$\vec{r}-\vec{r}_0(t') = \vec{R}_{PQ}$$

$$V(P) = \frac{\mu_0 c^2}{4\pi} \frac{q}{R_{PQ} - \vec{R}_{PQ}\cdot\frac{\vec{v}_Q}{c}}$$

$$\mu_0\varepsilon_0 = \frac{1}{c^2}$$

re-write the denominator in a covariant form

$$R_{PQ} - \vec{R}_{PQ} \cdot \frac{\vec{v}_Q}{c} = \frac{1}{c}\left(cR_{PQ} - \vec{R}_{PQ} \cdot \vec{v} \right)$$

use $R_{PQ} = |\vec{R}_{PQ}| = c\Delta t$

$$R_{PQ} - \vec{R}_{PQ} \cdot \frac{\vec{v}_Q}{c} = \frac{1}{c}\left(c^2 \Delta t - \vec{R}_{PQ} \cdot \vec{v}_Q \right)$$

If x_μ is the 4-position of point P, and ξ_μ is the 4-position of the charge at point Q, then the 4-displacement between the two is

$$\mathcal{R}_\mu \equiv x_\mu - \xi_\mu = (\vec{R}_{PQ}, ic\Delta t)$$

the 4-velocity of the charge at point Q is

$$u_\mu = (\gamma \vec{v}_Q, i\gamma c)$$

we then have $\quad \mathcal{R}_\mu u_\mu = \gamma \vec{R}_{PQ} \cdot \vec{v}_Q - \gamma c^2 \Delta t$

So $\quad R_{PQ} - \vec{R}_{PQ} \cdot \frac{\vec{v}_Q}{c} = -\frac{1}{c\gamma} \mathcal{R}_\mu u_\mu$

the 4-potential is $A_\mu = \left(\vec{A}, \frac{iV}{c} \right)$ so

$$\frac{iV}{c} = \frac{i}{c} \frac{\mu_0 c^2}{4\pi} \frac{q}{\mathcal{R}_\mu u_\mu \left(-\frac{1}{\gamma c}\right)} = -i \frac{\mu_0 c^2 \gamma}{4\pi} \frac{q}{\mathcal{R}_\mu u_\mu}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{q \vec{v}_Q}{\mathcal{R}_\mu u_\mu \left(-\frac{1}{\gamma c}\right)} = -\frac{\mu_0 c \gamma}{4\pi} \frac{q \vec{v}_Q}{\mathcal{R}_\mu u_\mu}$$

So $A_\nu = \dfrac{-\mu_0 c}{4\pi} \dfrac{q}{R_\mu u_\mu} \gamma(\vec{v}_Q, ic)$ $\qquad u_\mu = (\gamma \vec{v}_Q, i\gamma c)$

$$\boxed{A_\nu = -\frac{\mu_0 c}{4\pi} \frac{q\, u_\nu}{R_\mu u_\mu}}$$ covariant form for the Lienard-Wiechert 4-potential

here $u_\mu = \dfrac{d\xi_\mu}{ds}$ is the 4-velocity of the charge at point Q.

The retarded time is determined by the condition

$R_\mu^2 = (x_\mu - \xi_\mu)^2 = 0$ holds since P is on the light cone of Q

Define the Lorentz invariant scalar

$$D \equiv -R_\mu u_\mu$$

$$A_\mu(P) = \frac{\mu_0 c}{4\pi} \frac{q\, u_\mu(s)}{D}$$

x_μ is the position of point P. $u_\mu(s)$ is 4-velocity of charge at point Q.

Now we will find the fields, $F_{\mu\nu} = \dfrac{\partial A_\nu}{\partial x_\mu} - \dfrac{\partial A_\mu}{\partial x_\nu}$

When we do differentiation with respect to x_μ, we must also take care of the fact s, which locates point Q, also depends on the value of x_μ through the relation

$$R_\lambda^2 = (x_\lambda - \xi_\lambda(s))^2 = 0$$

$$\frac{\partial}{\partial x_\mu}(R_\lambda R_\lambda) = 0 \Rightarrow R_\lambda \frac{\partial R_\lambda}{\partial x_\mu} = R_\lambda\left(\delta_{\mu\lambda} - \frac{\partial \xi_\lambda}{\partial s}\frac{\partial s}{\partial x_\mu}\right) = 0$$

$$\Rightarrow R_\mu = R_\lambda u_\lambda \frac{\partial s}{\partial x_\mu} \Rightarrow \frac{\partial s}{\partial x_\mu} = \frac{R_\mu}{R_\lambda u_\lambda} = \boxed{-\frac{R_\mu}{D} = \frac{\partial s}{\partial x_\mu}}$$

Now $$\frac{\partial A_\nu}{\partial x_\mu} = \frac{\mu_0 c}{4\pi} q \frac{\partial}{\partial x_\mu}\left(\frac{u_\nu}{D}\right) = \frac{\mu_0 c q}{4\pi}\left\{\frac{1}{D}\frac{\partial u_\nu}{\partial s}\frac{\partial s}{\partial x_\mu} - \frac{u_\nu}{D^2}\frac{\partial D}{\partial x_\mu}\right\}$$

where $$\frac{\partial D}{\partial x_\mu} = -\frac{\partial}{\partial x_\mu}(R_\lambda u_\lambda) = -R_\lambda \frac{\partial u_\lambda}{\partial s}\frac{\partial s}{\partial x_\mu} - u_\lambda \frac{\partial}{\partial x_\mu}\underbrace{(x_\lambda - \xi_\lambda)}_{R_\lambda}$$

$$= -R_\lambda \dot{u}_\lambda\left(-\frac{R_\mu}{D}\right) - u_\lambda\left(\delta_{\lambda\mu} - \frac{\partial \xi_\lambda}{\partial s}\frac{\partial s}{\partial x_\mu}\right)$$

$$= \frac{R_\mu R_\lambda \dot{u}_\lambda}{D} - u_\mu + \underbrace{u_\lambda u_\lambda}_{-c^2}\left(-\frac{R_\mu}{D}\right)$$

$$\frac{\partial D}{\partial x_\mu} = -u_\mu + \frac{R_\mu c^2}{D}\left(1 + \frac{1}{c^2}\dot{u}_\lambda R_\lambda\right)$$

plug back in

$$\frac{\partial A_\nu}{\partial x_\mu} = \frac{\mu_0 c}{4\pi} q\left\{\frac{1}{D}\dot{u}_\nu\left(-\frac{R_\mu}{D}\right) - \frac{u_\nu}{D^2}\left[-u_\mu + \frac{R_\mu c^2}{D}\left(1 + \frac{\dot{u}_\lambda R_\lambda}{c^2}\right)\right]\right\}$$

$$= \frac{\mu_0 c}{4\pi} q\left\{-\frac{R_\mu \dot{u}_\nu}{D^2} + \frac{u_\nu u_\mu}{D^2} - \frac{u_\nu R_\mu c^2}{D^3}\left(1 + \frac{\dot{u}_\lambda R_\lambda}{c^2}\right)\right\}$$

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu} = \frac{\mu_0 c q}{4\pi D^2}\left\{\dot{u}_\mu R_\nu - \dot{u}_\nu R_\mu + \frac{c^2}{D}\left([u_\mu R_\nu]\left[1 + \frac{\dot{u}_\lambda R_\lambda}{c^2}\right] - [u_\nu R_\mu]\left[1 + \frac{\dot{u}_\lambda R_\lambda}{c^2}\right]\right)\right\}$$

$$\boxed{F_{\mu\nu} = \frac{\mu_0 c}{4\pi}\frac{q}{D^2}\left\{\dot{u}_\mu R_\nu - \dot{u}_\nu R_\mu + \frac{c^2}{D}(u_\mu R_\nu - u_\nu R_\mu)\left(1 + \frac{\dot{u}_\lambda R_\lambda}{c^2}\right)\right\}}$$

the terms proportional to u_μ are the "velocity terms"
the terms proportional to $\dot{u}_\mu$ are the "acceleration terms"

$$F_{\mu\nu} = \frac{\mu_0 c}{4\pi}\frac{q}{D^2}\left\{ \dot{u}_\mu R_\nu - \dot{u}_\nu R_\mu + \frac{c^2}{D}(u_\mu R_\nu - u_\nu R_\mu)\left(1 + \frac{\dot{u}_\lambda R_\lambda}{c^2}\right)\right\}$$

we want to show that this gives the same $\vec{E}$ as in Griffiths

$$F_{4i} = \frac{iE_i}{c} \Rightarrow E_i = -icF_{4i}$$

the pieces we need are:

4-velocity $\quad u_\mu = \gamma(\vec{v}, ic)$

4-acceleration $\quad \alpha_\mu = \dot{u}_\mu = \left(\gamma^4\frac{(\vec{v}\cdot\vec{a})}{c^2}\vec{v} + \gamma^2\vec{a},\ i\gamma^4\frac{(\vec{v}\cdot\vec{a})}{c}\right)$

4-difference $\quad R_\mu = (\vec{R}, iR)$ $\quad$ displacement from Q to P

$$D = -R_\lambda u_\lambda = \gamma(cR - \vec{v}\cdot\vec{R})$$

$$\dot{u}_\lambda R_\lambda = \gamma^4\frac{(\vec{v}\cdot\vec{a})}{c^2}(\vec{v}\cdot\vec{R}) + \gamma^2(\vec{a}\cdot\vec{R}) - \gamma^4\frac{(\vec{v}\cdot\vec{a})}{c}R$$

$$= \gamma^4\frac{(\vec{v}\cdot\vec{a})}{c^2}[\vec{v}\cdot\vec{R} - cR] + \gamma^2(\vec{a}\cdot\vec{R})$$

$$E_i = \frac{-i\mu_0 c^2}{4\pi}\frac{q}{D^3}\left\{[\dot{u}_4 R_i - \dot{u}_i R_4]D + c^2(u_4 R_i - u_i R_4)\left(1 + \frac{\dot{u}_\lambda R_\lambda}{c^2}\right)\right\}$$

use $\mu_0 c^2 = 1/\varepsilon_0$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{D^3}\left\{\left[\overbrace{\gamma^4\frac{(\vec{v}\cdot\vec{a})}{c}\vec{R}}^{\dot{u}_4 R_i} \overbrace{- \gamma^4\frac{(\vec{v}\cdot\vec{a})}{c^2}R\vec{v} - \gamma^2 R\vec{a}}^{-\dot{u}_i R_4}\right]\overbrace{\gamma(cR - \vec{v}\cdot\vec{R})}^{D}\right.$$

$$\left. + \underbrace{\gamma(c\vec{R} - R\vec{v})}_{u_4 R_i - u_i R_4}\underbrace{\left(c^2 + \gamma^4\frac{(\vec{v}\cdot\vec{a})}{c^2}[\vec{v}\cdot\vec{R} - cR] + \gamma^2(\vec{a}\cdot\vec{R})\right)}_{c^2 + \dot{u}_\lambda R_\lambda}\right\}$$

multiply through

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{D^3}\left\{\gamma^5\frac{(\vec{v}\cdot\vec{a})}{c^2}(c\vec{R} - R\vec{v})(cR - \vec{v}\cdot\vec{R}) - \gamma^3(cR - \vec{v}\cdot\vec{R})R\vec{a}\right.$$

$$\left. + \gamma^5\frac{(\vec{v}\cdot\vec{a})}{c^2}(\vec{v}\cdot\vec{R} - cR)(c\vec{R} - R\vec{v}) + \gamma^3(c\vec{R} - R\vec{v})(\vec{a}\cdot\vec{R}) + \gamma c^2(c\vec{R} - R\vec{v})\right\}$$

The γ^5 terms cancel

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{D^3}\left\{\gamma c^2(c\vec{R}-R\vec{v}) + \gamma^3\left[(c\vec{R}-R\vec{v})(\vec{a}\cdot\vec{R}) - \vec{a}\,(cR-\vec{v}\cdot\vec{R})R\right]\right\}$$

rewrite last term

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{D^3}\left\{\gamma c^2(c\vec{R}-R\vec{v}) + \gamma^3\left[(c\vec{R}-R\vec{v})(\vec{a}\cdot\vec{R}) - \vec{a}\,(c\vec{R}-R\vec{v})\cdot\vec{R}\right]\right\}$$

simplify the γ^3 term by using the triple product rule

$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$ with $\vec{A}\equiv\vec{R}$, $\vec{B} = c\vec{R}-R\vec{v}$, $\vec{C}=\vec{a}$

substitute for D

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{\gamma^3(cR-\vec{v}\cdot\vec{R})^3}\left\{\gamma c^2(c\vec{R}-R\vec{v}) + \gamma^3\vec{R}\times\left([c\vec{R}-R\vec{v}]\times\vec{a}\right)\right\}$$

write $\begin{cases} c\vec{R}-R\vec{v} = R(c\hat{R}-\vec{v}) \\ cR-\vec{v}\cdot\vec{R} = \vec{R}\cdot(c\hat{R}-\vec{v}) \end{cases}$ since $\vec{R} = R\hat{R}$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qR}{[\vec{R}\cdot(c\hat{R}-\vec{v})]^3}\left\{\frac{1}{\gamma^2}c^2(c\hat{R}-\vec{v}) + \vec{R}\times\left([c\hat{R}-\vec{v}]\times\vec{a}\right)\right\}$$

use $\frac{c^2}{\gamma^2} = c^2(1-v^2/c^2) = c^2-v^2$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{qR}{[\vec{R}\cdot(c\hat{R}-\vec{v})]^3}\left\{(c\hat{R}-\vec{v})(c^2-v^2) + \vec{R}\times[(c\hat{R}-\vec{v})\times\vec{a}]\right\}}$$

taking $\vec{R} = \vec{r}-\vec{r}_0(t')$, $\hat{R} = \hat{n}(t')$, $\vec{v} = \vec{v}(t')$, $\vec{a} = \vec{a}(t')$, t' the retarded time
then the above is the same as the expression written at end of lecture 21.

If one defines $\vec{w} = c\hat{R}-\vec{v}$, then

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{qR}{(\vec{R}\cdot\vec{w})^3}\left\{\vec{w}(c^2-v^2) + \vec{R}\times(\vec{w}\times\vec{a})\right\}$$

this is the same as Griffiths Eqn (10.72) if we use his notation $\vec{R}\to\vec{\mathscr{r}}$, $\vec{w}\to\vec{u}$

Similarly we can find $\vec{B}$

$B_i = F_{jk}$ with i, j, k a cyclic permutation of $1, 2, 3$

$$B_i = \frac{\mu_0 c}{4\pi}\frac{q}{D^2}\left\{ \mathring{u}_j R_k - \mathring{u}_k R_j + \frac{c^2}{D}(u_j R_k - u_k R_j)\left(1 + \frac{\mathring{u}_\lambda R_\lambda}{c^2}\right)\right\}$$

use $\mathring{u}_j R_k - \mathring{u}_k R_j = (\mathring{\vec{u}} \times \vec{R})_i = -(\vec{R} \times \mathring{\vec{u}})_i$

$u_j R_k - u_k R_j = (\vec{u} \times \vec{R})_i = -(\vec{R} \times \vec{u})_i$

$\vec{R} = R\hat{R}$

$$\vec{B} = \frac{\hat{R}}{c} \times \left[-\frac{\mu_0 c^2}{4\pi}\frac{q}{D^2}\left\{ R\mathring{\vec{u}} + \frac{c^2}{D^2} R\vec{u}\left(1 + \frac{\mathring{u}_\lambda R_\lambda}{c^2}\right)\right\}\right]$$

compare this to

$$\frac{\hat{R}}{c} \times \vec{E} = \frac{\hat{R}}{c} \times \left[\frac{-i\mu_0 c^2}{4\pi}\frac{q}{D^2}\left\{ \mathring{u}_4 \vec{R} - \mathring{\vec{u}} R_4 + \frac{c^2}{D}(u_4 \vec{R} - \vec{u} R_4)\left(1 + \frac{\mathring{u}_\lambda R_\lambda}{c^2}\right)\right\}\right]$$

gives zero as $\hat{R} \times \vec{R} = 0$

use $R_4 = iR$

$$\frac{\hat{R}}{c} \times \vec{E} = \frac{\hat{R}}{c} \times \left[\frac{-i\mu_0 c^2}{4\pi}\frac{q}{D^2}\left\{ -iR\mathring{\vec{u}} + \frac{c^2}{D}(-iR\vec{u})\left(1 + \frac{\mathring{u}_\lambda R_\lambda}{c^2}\right)\right\}\right]$$

$$= \frac{\hat{R}}{c} \times \left[-\frac{\mu_0 c^2}{4\pi}\frac{q}{D^2}\left\{ R\mathring{\vec{u}} + \frac{c^2}{D} R\vec{u}\left(1 + \frac{\mathring{u}_\lambda R_\lambda}{c^2}\right)\right\}\right]$$

$$= \vec{B}$$

so $\boxed{\vec{B} = \frac{\hat{R}}{c} \times \vec{E}}$