Having found $\vec{E}$ and $\vec{B}$ we can now compute the power radiated by the accelerating charge. We had

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\,\frac{qR}{[\vec{R}\cdot(c\hat{R}-\vec{v})]^3}\left\{(c\hat{R}-\vec{v})(c^2-v^2)+\vec{R}\times\left[(c\hat{R}-\vec{v})\times\vec{a}\right]\right\}$$

$$\vec{B} = \frac{\hat{R}}{c}\times\vec{E}$$

R is distance from charge's position at the retarded time to the observer's position

Consider first the simplest case where we are in the instantaneous rest frame $\mathring{K}$ as the charge, where $\mathring{\vec{v}}=0$. Then we have

$$\mathring{\vec{E}} = \frac{1}{4\pi\epsilon_0}\,\frac{q\mathring{R}}{c^3\mathring{R}^3}\left\{c^3\mathring{\hat{R}} + c\mathring{R}\times(\mathring{\hat{R}}\times\mathring{\vec{a}})\right\}$$

circles above quantities indicates frame $\mathring{K}$

$$= \frac{q}{4\pi\epsilon_0}\,\frac{1}{\mathring{R}^2}\left\{\mathring{\hat{R}} + \frac{1}{c^2}\mathring{\vec{R}}\times(\mathring{\hat{R}}\times\mathring{\vec{a}})\right\}$$

this just gives the static Coulomb field $\sim 1/\mathring{R}^2$

this gives the radiated fields $\sim 1/\mathring{R}$

Let's consider only the radiation part,

$$\mathring{\vec{E}}^{\,rad} = \frac{q}{4\pi\epsilon_0 c^2}\,\frac{1}{\mathring{R}}\,\mathring{\hat{R}}\times(\mathring{\hat{R}}\times\mathring{\vec{a}}) = \frac{\mu_0}{4\pi}\,\frac{q}{\mathring{R}}\,\mathring{\hat{R}}\times(\mathring{\hat{R}}\times\mathring{\vec{a}}) \qquad \text{using } \mu_0\epsilon_0 = 1/c^2$$

$$\mathring{\vec{B}}^{\,rad} = \frac{\mathring{\hat{R}}}{c}\times\mathring{\vec{E}}^{\,rad} = \frac{\mu_0}{4\pi c}\,\frac{q}{\mathring{R}}\,\underset{A}{\mathring{\hat{R}}}\times\Big(\underset{B}{\mathring{\hat{R}}}\times(\underset{C}{\mathring{\hat{R}}\times\mathring{\vec{a}}})\Big) \qquad \text{use } \vec{A}\times(\vec{B}\times\vec{C}) \text{ rule}$$

$$= \frac{\mu_0}{4\pi c}\,\frac{q}{\mathring{R}}\Big[\mathring{\hat{R}}\,\underbrace{(\mathring{\hat{R}}\cdot(\mathring{\hat{R}}\times\mathring{\vec{a}}))}_{=0} - (\mathring{\hat{R}}\times\mathring{\vec{a}})\underbrace{(\mathring{\hat{R}}\cdot\mathring{\hat{R}})}_{=1}\Big\}$$

$$= -\frac{\mu_0}{4\pi c}\,\frac{q}{\mathring{R}}(\mathring{\hat{R}}\times\mathring{\vec{a}})$$

$$\mathring{\vec{S}} = \frac{1}{\mu_0}\,\mathring{\vec{E}}^{\,rad}\times\mathring{\vec{B}}^{\,rad}$$

$$= \frac{-1}{\mu_0}\left(\frac{\mu_0 q}{4\pi}\right)\left(\frac{\mu_0 q}{4\pi c}\right)\frac{1}{\mathring{R}^2}\left\{\left[\mathring{\hat{R}}\times(\mathring{\hat{R}}\times\mathring{\vec{a}})\right]\times\left[\mathring{\hat{R}}\times\mathring{\vec{a}}\right]\right\}$$

$$\mathring{\hat{R}} \times (\mathring{\hat{R}} \times \mathring{\vec{a}}) = \mathring{\hat{R}}(\mathring{\hat{R}} \cdot \mathring{\vec{a}}) - \mathring{\vec{a}}(\mathring{\hat{R}} \cdot \mathring{\hat{R}}) = \mathring{\hat{R}}(\mathring{\hat{R}} \cdot \mathring{\vec{a}}) - \mathring{\vec{a}}$$

so

$$[\mathring{\hat{R}} \times (\mathring{\hat{R}} \times \mathring{\vec{a}})] \times [\mathring{\hat{R}} \times \mathring{\vec{a}}] = (\mathring{\hat{R}} \cdot \mathring{\vec{a}})\, \mathring{\hat{R}} \times (\mathring{\hat{R}} \times \mathring{\vec{a}}) - \mathring{\vec{a}} \times (\mathring{\hat{R}} \times \mathring{\vec{a}})$$

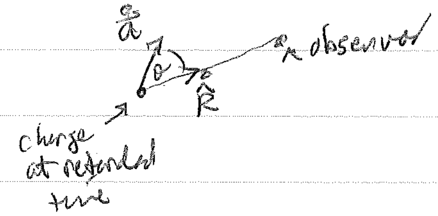
$$= (\mathring{\hat{R}} \cdot \mathring{\vec{a}})[\mathring{\hat{R}}(\mathring{\hat{R}} \cdot \mathring{\vec{a}}) - \mathring{\vec{a}}] - \mathring{\hat{R}}\, \mathring{a}^2 + \mathring{\vec{a}}(\mathring{\hat{R}} \cdot \mathring{\vec{a}})$$

$$= \mathring{\hat{R}}(\mathring{\hat{R}} \cdot \mathring{\vec{a}})^2 - \mathring{\vec{a}}(\mathring{\hat{R}} \cdot \mathring{\vec{a}}) - \mathring{\hat{R}}\, \mathring{a}^2 + \mathring{\vec{a}}(\mathring{\hat{R}} \cdot \mathring{\vec{a}})$$

$$= -\mathring{\hat{R}}\left(\mathring{a}^2 - (\mathring{\hat{R}} \cdot \mathring{\vec{a}})^2\right)$$

let θ be the angle between $\mathring{\vec{a}}$ and $\mathring{\hat{R}}$. Then $(\mathring{\hat{R}} \cdot \mathring{\vec{a}})^2 = \mathring{a}^2 \cos^2\theta$

$$= -\mathring{\hat{R}}\, \mathring{a}^2 (1 - \cos^2\theta) = -\mathring{\hat{R}}\, \mathring{a}^2 \sin^2\theta$$



So

$$\mathring{\vec{S}} = \frac{\mu_0}{(4\pi)^2 c} \frac{q^2}{\mathring{R}^2} \mathring{a}^2 \sin^2\theta\, \mathring{\hat{R}}$$

$$\frac{d\mathring{P}}{d\Omega} = \mathring{R}^2\, \mathring{\vec{S}} \cdot \mathring{\hat{R}} = \frac{\mu_0}{(4\pi)^2 c} q^2 \mathring{a}^2 \sin^2\theta$$

same as we found earlier from electric dipole approx

Total radiated power in frame $\mathring{K}$ is

$$\mathring{P} = \int d\Omega \frac{d\mathring{P}}{d\Omega} = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \frac{\mu_0}{(4\pi)^2 c} q^2 \mathring{a}^2 \sin^2\theta$$

$$= \frac{\mu_0}{(4\pi)^2 c} q^2 \mathring{a}^2\, 2\pi \underbrace{\int_0^{\pi} d\theta \sin^3\theta}_{4/3}$$

$$= \frac{\mu_0}{(4\pi)^2 c} q^2 \mathring{a}^2\, 2\pi \left(\frac{4}{3}\right) = \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{q^2 \mathring{a}^2}{c^3}$$

using $\mu_0 = \frac{1}{\epsilon_0 c^2}$

exactly the same as Larmor's formula!

So Larmor's formula, which we derived originally from the electric dipole approx, holds <u>exactly</u> in the instantaneous rest frame of the charge where $\vec{v}=0$

This is not surprising since we saw that all the higher moments in our multipole expansion for radiation, i.e. the magnetic dipole, the electric quadrupole, etc, were all of order $\left(\frac{v}{c}\right)^n \times$ (electric dipole term) and so vanish as $v \to 0$.

We can now consider the general case where $\vec{v} \neq 0$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{qR}{[\vec{R}\cdot(c\hat{R}-\vec{v})]^3} \left\{ (c\hat{R}-\vec{v})(c^2-v^2) + \vec{R}\times[(c\hat{R}-\vec{v})\times\vec{a}] \right\}$$

this term gives the velocity field $\sim 1/R^2$ — this term gives the radiated field $\sim 1/R$

we keep only the radiation part

$$\vec{E}^{rad} = \frac{1}{4\pi\varepsilon_0} \frac{qR^2}{R^3c^3[1-\frac{\vec{v}}{c}\cdot\hat{R}]^3} \left\{ \hat{R}\times[(c\hat{R}-\vec{v})\times\vec{a}] \right\}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{c^2 K^3 R} \hat{R}\times[(\hat{R}-\frac{\vec{v}}{c})\times\vec{a}] \qquad \text{where } K \equiv 1-\frac{\vec{v}}{c}\cdot\hat{R}$$

$$\vec{B}^{rad} = \frac{\hat{R}}{c}\times\vec{E}^{rad}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}^{rad}\times\vec{B}^{rad} = \frac{1}{\mu_0 c}\vec{E}^{rad}\times(\hat{R}\times\vec{E}^{rad})$$

use triple product rule

$$= \frac{1}{\mu_0 c}\left[|\vec{E}^{rad}|^2\hat{R} - \vec{E}^{rad}(\hat{R}\cdot\vec{E}^{rad})\right] \qquad \text{but } \hat{R}\cdot\vec{E}^{rad} = 0$$

$$= \frac{1}{\mu_0 c}|\vec{E}^{rad}|^2\hat{R}$$

$$\vec{S} = \frac{1}{\mu_0 c}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{q^2}{c^4 K^6 R^2} \left|\hat{R}\times[(\hat{R}-\frac{\vec{v}}{c})\times\vec{a}]\right|^2 \hat{R}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi}\frac{q^2}{c^3K^6R^2}\left|\hat{R}\times[(\hat{R}-\frac{\vec{v}}{c})\times\vec{a}]\right|^2\hat{R} \qquad \text{using } \frac{1}{\mu_0\varepsilon_0} = c^2$$

in general this is a messy expression!

$$\frac{dP}{d\Omega} = R^2\langle\vec{S}\rangle\cdot\hat{R} = \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi}\frac{q^2}{c^3K^6}\left|\hat{R}\times[(\hat{R}-\frac{\vec{v}}{c})\times\vec{a}]\right|^2$$

Consider the special case of linear motion where $\vec{v} \parallel \vec{a}$

Then $\hat{R} \times [(\hat{R} - \frac{\vec{v}}{c}) \times \vec{a}] = \hat{R} \times (\hat{R} \times \vec{a})$ since $\vec{v} \times \vec{a} = 0$

$= \hat{R}(\hat{R} \cdot \vec{a}) - \vec{a}$

$$
\begin{aligned}
|\hat{R} \times (\hat{R} \times \vec{a})|^2 &= |\hat{R}(\hat{R} \cdot \vec{a}) - \vec{a}|^2 \\
&= (\hat{R} \cdot \vec{a})^2 + a^2 - 2(\hat{R} \cdot \vec{a})^2 \\
&= a^2 - (\hat{R} \cdot \vec{a})^2
\end{aligned}
$$

let θ be the angle between $\hat{R}$ and $\vec{v}$, which is also the angle between $\hat{R}$ and $\vec{a}$ since $\vec{v} \parallel \vec{a}$

$$
|\hat{R} \times (\hat{R} \times \vec{a})|^2 = a^2 - a^2 \cos^2\theta = a^2 \sin^2\theta
$$

$$
K = 1 - \frac{\vec{v}}{c} \cdot \hat{R} = 1 - \frac{v}{c}\cos\theta
$$

$$
\frac{dP}{d\Omega} = \frac{1}{4\pi\varepsilon_0} \frac{1}{4\pi} \frac{q^2 a^2 \sin^2\theta}{c^3 (1 - \frac{v}{c}\cos\theta)^6}
$$

when $v/c \ll 1$ the denominator $\sim 1 - 6\frac{v}{c}\cos\theta$ gives a very small correction to what we had from Larmor's nonrelativistic formula

But now consider $\frac{v}{c} \approx 1$ very relativistic case

For θ small we have $\cos\theta \simeq 1 - \frac{\theta^2}{2}$, $\sin\theta \approx \theta$, the above becomes

$$
\frac{dP}{d\Omega} = \frac{1}{4\pi\varepsilon_0} \frac{1}{4\pi} \frac{q^2 a^2}{c^3} \frac{\theta^2}{(1 - \frac{v}{c} + \frac{v}{c}\frac{\theta^2}{2})^6}
$$

at small θ

To estimate the behavior approx

$$\frac{1}{2}\left(1-\frac{v^2}{c^2}\right) = \frac{1}{2}\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right) \simeq \frac{1}{2}\left(1-\frac{v}{c}\right)(2) \quad \text{when } \frac{v}{c} \approx 1$$
$$\simeq 1-\frac{v}{c}$$

So $1-\frac{v}{c} \sim \frac{1}{2\gamma^2}$

$$1-\frac{v}{c}+\underbrace{\frac{v}{c}}_{\approx 1}\frac{\theta^2}{2} \simeq \frac{1}{2\gamma^2}+\frac{\theta^2}{2} \simeq \frac{1}{2\gamma^2}\left(1+\gamma^2\theta^2\right)$$

So

$$\frac{dP}{d\Omega} = \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi}\frac{q^2a^2}{c^3}\frac{\theta^2}{\left[\frac{1}{2\gamma^2}\left(1+\gamma^2\theta^2\right)\right]^6}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi}\frac{q^2a^2}{c^3}\,2^6\,\gamma^{10}\frac{(\gamma\theta)^2}{\left[1+(\gamma\theta)^2\right]^6}$$

this vanishes at $\theta=0$, but the maximum will be at

$$0 = \frac{d}{d\theta}\left\{\frac{(\gamma\theta)^2}{\left[1+(\gamma\theta)^2\right]^6}\right\} = \frac{\left[1+(\gamma\theta)^2\right]^6 2\gamma^2\theta - (\gamma\theta)^2 6\left(1+(\gamma\theta)^2\right)^5 2\gamma^2\theta}{\left(1+(\gamma\theta)^2\right)^{12}}$$

$$\left[1+(\gamma\theta)^2\right] - 6(\gamma\theta)^2 = 0$$

$$1-5(\gamma\theta)^2 = 0 \qquad \gamma\theta = \frac{1}{\sqrt{5}}$$

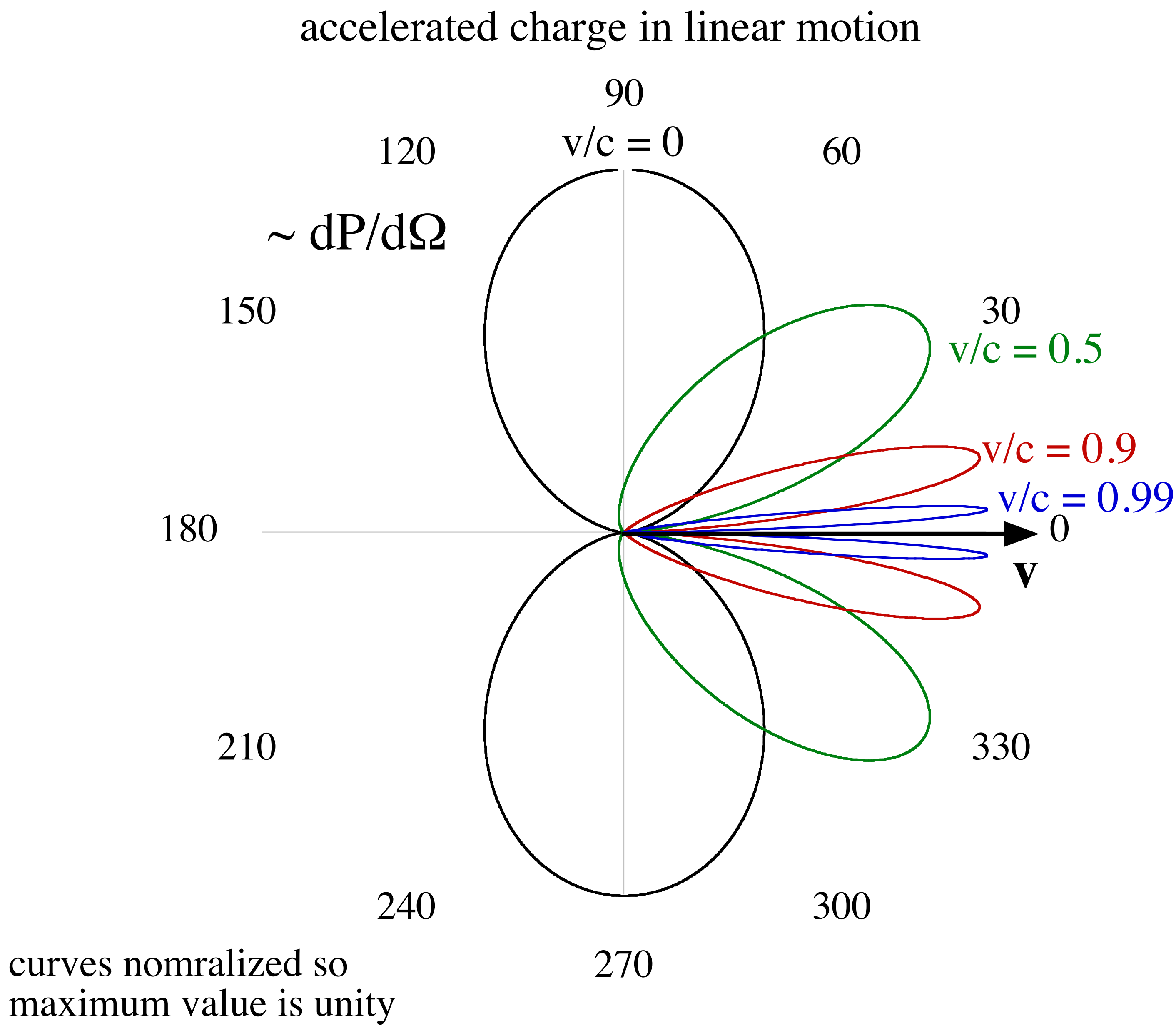
$$\theta_{max} = \frac{1}{\sqrt{5}\,\gamma}$$

for very relativistic motion with $\frac{v}{c} \approx 1$, thus $\gamma \gg 1$

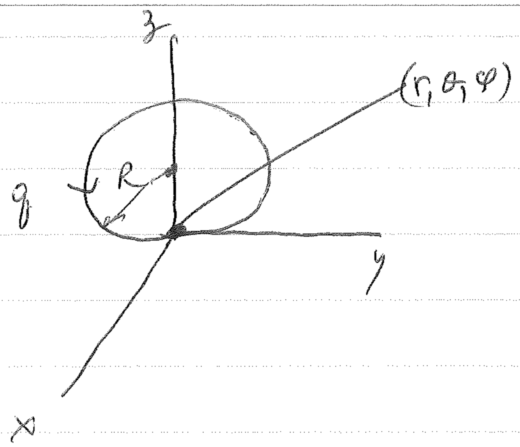
θ_{max} close to zero

radiation is very strongly collimated about θ_{max}

Note the factor γ^{10}!


accelerated charge in linear motion
90
v/c = 0
120
60
~ dP/dΩ
150
30
v/c = 0.5
v/c = 0.9
v/c = 0.99
180
0
v
210
330
240
300
270
curves nomralized so
maximum value is unity

Charged particle in circular motion



charge moving in circular orbit of radius R. orbit in yz plane as shown. What is radiation when orbit is at origin at time $t=0$?

Note: radiation emitted by charge at $t=0$ will reach observer at (r, θ, φ) at time $t_1 = cr$.

$$\frac{dP}{d\Omega} = \frac{1}{4\pi\varepsilon_0} \frac{1}{4\pi} \frac{q^2}{c^3 \kappa^6} \left| \hat{n} \times \left[\left(\hat{n} - \frac{\vec{v}}{c} \right) \times \vec{a} \right] \right|^2$$

where $\kappa = 1 - \frac{\vec{v}}{c} \cdot \hat{n}$

where $\hat{n} = \hat{r}$ is unit vector from charge to observer

(previously we called this $\hat{R}$, but since we want to use R as the radius of the orbit, we go back to our older notation and use $\hat{n}$)

At $t=0$ when charge is at the origin, $\vec{v} = v\,\hat{y}$,

$\vec{a} = \frac{v^2}{R}\hat{z}$, $\quad \hat{n} = \hat{r} = \sin\theta\cos\varphi\,\hat{x} + \sin\theta\sin\varphi\,\hat{y} + \cos\theta\,\hat{z}$

$$\left(\hat{n} - \frac{\vec{v}}{c}\right) \times \vec{a} = \left(\sin\theta\cos\varphi\,\hat{x} + \left(\sin\theta\sin\varphi - \frac{v}{c}\right)\hat{y} + \cos\theta\,\hat{z} \right) \times \frac{v^2}{R}\hat{z}$$

$$= \frac{v^2}{R}\left[-\sin\theta\cos\varphi\,\hat{y} + \left(\sin\theta\sin\varphi - \frac{v}{c}\right)\hat{x} \right]$$

$$\hat{n} \times \left[\left(\hat{n} - \frac{\vec{v}}{c}\right) \times \vec{a} \right] = \left(\sin\theta\cos\varphi\,\hat{x} + \sin\theta\sin\varphi\,\hat{y} + \cos\theta\,\hat{z} \right)$$

$$\times \frac{v^2}{R}\left(-\sin\theta\cos\varphi\,\hat{y} + \left(\sin\theta\sin\varphi - \frac{v}{c}\right)\hat{x} \right)$$

$$= \frac{v^2}{R}\Big(-\sin^2\theta\cos^2\varphi\,\hat{z} - \sin\theta\sin\varphi\left(\sin\theta\sin\varphi - \frac{v}{c}\right)\hat{z}$$

$$+ \cos\theta\sin\theta\cos\varphi\,\hat{x} + \cos\theta\left(\sin\theta\sin\varphi - \frac{v}{c}\right)\hat{y}\Big)$$

$$= \frac{v^2}{R}\Big[\left(-\sin^2\theta + \frac{v}{c}\sin\theta\sin\varphi\right)\hat{z}$$

$$+ \cos\theta\sin\theta\cos\varphi\,\hat{x} + \cos\theta\left(\sin\theta\sin\varphi - \frac{v}{c}\right)\hat{y}\Big]$$

$$\left|\hat{n}\times\left[\left(\hat{n}-\frac{\vec{v}}{c}\right)\times\vec{a}\right]\right|^2$$

$$= \frac{v^4}{R^2}\Big[\sin^4\theta + \left(\frac{v}{c}\right)^2\sin^2\theta\sin^2\varphi - 2\left(\frac{v}{c}\right)\sin^3\theta\sin\varphi$$

$$+ \cos^2\theta\sin^2\theta\cos^2\varphi + \cos^2\theta\sin^2\theta\sin^2\varphi$$

$$- 2\left(\frac{v}{c}\right)\cos^2\theta\sin\theta\sin\varphi\Big]$$

$$= \frac{v^4}{R^2}\Big[\sin^4\theta + \sin^2\theta\cos^2\theta\cos^2\varphi + \cos^2\theta\sin^2\theta\sin^2\varphi$$

$$- 2\left(\frac{v}{c}\right)\left(\sin^3\theta\sin\varphi + \sin\theta\cos^2\theta\sin\varphi\right)$$

$$+ \left(\frac{v}{c}\right)^2\left(\sin^2\theta\sin^2\varphi + \cos^2\theta\right)\Big]$$

$$= \frac{v^4}{R^2}\Big[\sin^2\theta - 2\left(\frac{v}{c}\right)\sin\theta\sin\varphi + \left(\frac{v}{c}\right)^2\left(1 - \sin^2\theta\cos^2\varphi\right)\Big]$$

$$\boxed{\frac{dP}{d\Omega} = \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi}\frac{q^2a^2}{c^3}\,\frac{\left[\sin^2\theta - 2\left(\frac{v}{c}\right)\sin\theta\sin\varphi + \left(\frac{v}{c}\right)^2\left(1-\sin^2\theta\cos^2\varphi\right)\right]}{\left[1-\frac{v}{c}\sin\theta\sin\varphi\right]^6}}$$

Note, in general we have both θ and φ dependence to $\frac{dP}{d\Omega}$

Note $\frac{v^4}{R^2} = a^2$

Special cases:

(1) Radiation into the xz plane ($x>0$) - perpendicular to plane of orbit

$\varphi = 0 \Rightarrow \sin\varphi = 0, \cos\varphi = 1$

$$\frac{dP}{d\Omega} = \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi}\frac{q^2a^2}{c^3}\left[\sin^2\theta + \left(\frac{v}{c}\right)^2(1-\sin^2\theta)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi}\frac{q^2a^2}{c^3}\left[\sin^2\theta + \left(\frac{v}{c}\right)^2\cos^2\theta\right]$$

For xz plane with $x<0$, $\varphi = \pi \Rightarrow \sin\varphi = 0$, $\cos\varphi = -1$

$$\frac{dP}{d\Omega} = \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi}\frac{q^2a^2}{c^3}\left[\sin^2\theta + \left(\frac{v}{c}\right)^2(1-\sin^2\theta)\right] \quad \text{same as } x>0$$

(2) in yz plane ($y>0$), $\varphi = \frac{\pi}{2} \Rightarrow \sin\varphi = 1, \cos\varphi = 0$

$$\frac{dP}{d\Omega} = \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi}\frac{q^2a^2}{c^3}\frac{\left[\sin^2\theta - 2\left(\frac{v}{c}\right)\sin\theta + \left(\frac{v}{c}\right)^2\right]}{\left[1-\frac{v}{c}\sin\theta\right]^6}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi}\frac{q^2a^2}{c^3}\frac{(\sin\theta - v/c)^2}{\left[1-\frac{v}{c}\sin\theta\right]^6}$$

yz plane, $y<0$ $\quad \varphi = -\frac{\pi}{2} \Rightarrow \sin\varphi = -1, \cos\varphi = 0$

$$\frac{dP}{d\Omega} = \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi}\frac{q^2a^2}{c^3}\frac{(\sin\theta + v/c)^2}{\left[1+\frac{v}{c}\sin\theta\right]^6}$$

Non relativistic limit $\frac{v}{c} \ll 1$ ignore all terms in v/c

$$\frac{dP}{d\Omega} \simeq \frac{1}{4\pi\epsilon_0} \frac{q^2}{c^3} a^2 \sin^2\theta$$

Same result as earlier non-relativistic Larmor formula

extreme relativistic limit $\frac{v}{c} \simeq 1$ $1 - v/c \equiv \varepsilon$ very small

$$\frac{v}{c} \simeq 1 - \varepsilon$$

in xz plane

$$\frac{dP}{d\Omega} = \frac{1}{4\pi\epsilon_0} \frac{1}{4\pi} \frac{q^2 a^2}{c^3} \left[\sin^2\theta + \underbrace{(1-\varepsilon)^2}_{1-2\varepsilon} \cos^2\theta \right]$$

$$\simeq \frac{1}{4\pi\epsilon_0} \frac{1}{4\pi} \frac{q^2 a^2}{c^3} \left[1 - 2\varepsilon \cos^2\theta \right]$$

becomes rotationally symmetric as $\varepsilon \to 0$

in yz plane, $y<0$ backwards direction

$$\frac{dP}{d\Omega} \simeq \frac{1}{4\pi\epsilon_0} \frac{1}{4\pi} \frac{q^2 a^2}{c^3} \frac{[\sin\theta + 1 - \varepsilon]^2}{[1-(1-\varepsilon)\sin\theta]^6}$$

$$\simeq \frac{1}{4\pi\epsilon_0} \frac{1}{4\pi} \frac{q^2 a^2}{c^3} \frac{1}{[1+\sin\theta]^4}$$

can ignore the ε

in yz plane, $y>0$ forward direction

$$\frac{dP}{d\Omega} \simeq \frac{1}{4\pi\epsilon_0} \frac{1}{4\pi} \frac{q^2 a^2}{c^3} \frac{[\sin\theta - 1 + \varepsilon]^2}{[1-(1-\varepsilon)\sin\theta]^6}$$

need to be careful since as $\theta \to \frac{\pi}{2}$, the denominator $\to \varepsilon$ and $\frac{dP}{d\Omega}$ gets large! so can't just take $\varepsilon \to 0$

$\frac{dP}{d\Omega}(\theta = \frac{\pi}{2}, \varphi = \frac{\pi}{2})$ along $\hat{y}$ axis ie in forward direction

$$= \frac{1}{4\pi\varepsilon_0} \frac{1}{4\pi} \frac{q^2 a^2}{c^3} \frac{[1-1+\varepsilon]^2}{[1-1+\varepsilon]^6}$$

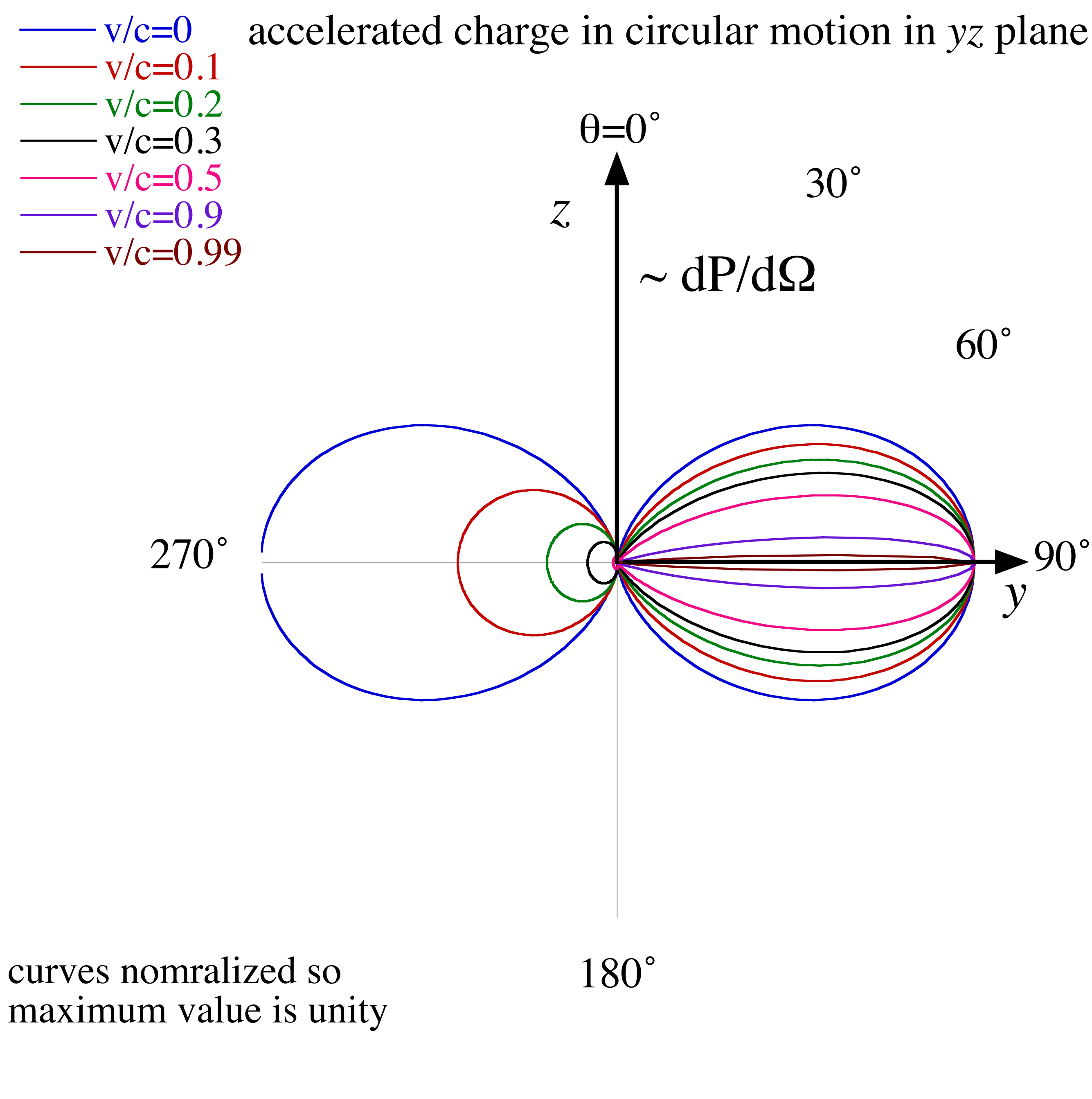
$$= \frac{1}{4\pi\varepsilon_0} \frac{1}{4\pi} \frac{q^2 a^2}{c^3} \frac{1}{\varepsilon^4} = \frac{1}{4\pi\varepsilon_0} \frac{1}{4\pi} \frac{q^2 a^2}{c^3} \frac{1}{(1-v/c)^4}$$

as $\frac{v}{c} \to 1$ becomes very strongly peaked about $\hat{y}$ axis

See polar plot next page for $\frac{dP}{d\Omega}(\theta)$ at $\varphi = \frac{\pi}{2}$ in yz plane at various v/c.

We see that in the relativistic case, the radiation gets strongly focused in the forward direction – very different from the non-relativistic limit.

Radiation from charged particles in synchrotrons give very high energy, very focused EM beams, for probing materials – "synchrotron radiation" source


accelerated charge in circular motion in yz plane
v/c=0
v/c=0.1
v/c=0.2
v/c=0.3
v/c=0.5
v/c=0.9
v/c=0.99
θ=0°
30°
60°
90°
180°
270°
z
y
~ dP/dΩ
curves nomralized so
maximum value is unity