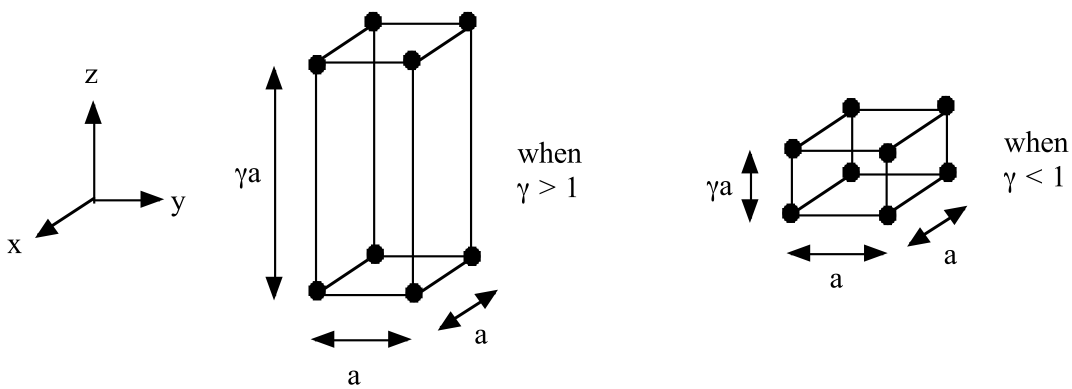
PHY 251/420 **Final Exam** **Fall 2006**

This is a **closed book** exam. You are on your honor not to consult any books, notes, classmates, friends, etc. while working on this exam, and you may take no more than three hours to complete it. You must return your exam to the box outside my office, B&L 455B, by noon on Monday, December 18. Note: problem 1 is worth 60 points, while problems 2 and 3 are worth 20 points each.

1) [60 points total]

Consider a crystal where the ions sit at the sites of a Bravais lattice as shown below. The primitive vectors are, $\mathbf{a}_1 = a\hat{x}$, $\mathbf{a}_2 = a\hat{y}$, $\mathbf{a}_3 = \gamma a\hat{z}$, where γ is a real positive number. Each ion contributes one conduction electron.



a) What are the primitive vectors of the reciprocal lattice? What range of wavevectors $\mathbf{k}$ are in the first Brillouin Zone? Sketch the first Brillouin Zone. [15 points]

b) Compute the radius k_F of the free electron Fermi sphere. *Hint*: It might help to recall that the first Brillouin Zone contains N allowed (i.e. consistent with Born-von Karman boundary conditions) wavevectors, where N is the number of sites of the Bravais lattice. [15 points]

c) Assuming the periodic potential from the ions is very weak (but not zero), for what range of γ will the crystal be well approximated by the free electron Sommerfeld model? [15 points]

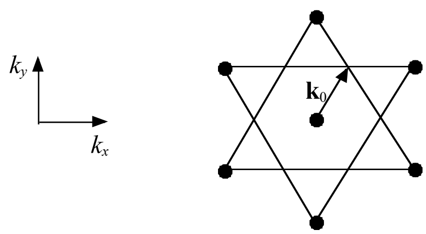
d) If $\gamma = 1.5$, what does the Fermi surface look like in the repeated zone scheme (the repeated zone scheme is the same as the periodic zone scheme)? Sketch it qualitatively. Suppose one does a Hall effect experiment with a strong magnetic field $\mathbf{H}$. For which of the following three allignments of $\mathbf{H}$, if any, do you expect the Hall coefficient to agree with the Drude prediction: $\mathbf{H} = H\hat{x}$, $\mathbf{H} = H\hat{y}$, $\mathbf{H} = H\hat{z}$. Explain your conclusions. If $\mathbf{H} = H\hat{x}$, describe qualitatively how the magneto-resistance varies as the applied $\mathbf{E}$ is rotated in the yz plane. [15 points]

2) [20 points total]

Consider a two dimensional triangular Bravais lattice of lattice constant a. Its reciprocal lattice is also triangular and has basis vectors,

$$\mathbf{b}_1 = K_0 \left(\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y} \right), \qquad \mathbf{b}_2 = K_0 \hat{y}$$

where $K_0 = (4\pi/\sqrt{3}a)$. The origin and the six smallest reciprocal lattice vectors, and their corresponding Bragg planes, are shown below. Consider an electron with crystal momentum $\mathbf{k}_0$, at the boundary of the first Brillouin zone, as shown in the figure below. Assume the weak potential approximation for the ionic potential $U(\mathbf{r})$.



a) What are the leading (i.e. those of $O(1)$) Fourier components that appear in the Bloch wavefunction for the electron at $\mathbf{k}_0$? [10 points]

b) Write a matix equation from which one could determine the energy of this electron in the lowest band, and an equation from which one could determine the Fourier components of part (a). You do not need to solve these equations, but you should explain clearly what one in principle would do to determine the desired results. [10 points]

3) [20 points total]

Consider a band with an anisotropic dispersion relation,

$$\epsilon(\mathbf{k}) = \frac{\hbar^2}{2} \left(\frac{k_x^2}{m_x} + \frac{k_y^2}{m_y} + \frac{k_z^2}{m_z} \right) .$$

a) Using the fact that an ellipsoid given by the equation, $(x/a)^2 + (y/b)^2 + (z/c)^2 = 1$, encloses a volume $(4\pi/3)abc$, calculate the density of states $g(\epsilon)$. *Hint*: One can write for the density of states, $g(\epsilon) = dG(\epsilon)/d\epsilon$, where $G(\epsilon)$ is the number of single electron states per unit volume with energy *less than* ϵ. [10 points]

b) We learned that, at sufficiently low temperatures, the contribution to the specific heat at constant volume due to the electrons is given by $c_V = \gamma T$, where γ is a constant independent of temperature. For the anisotropic band structure considered here, explain why γ is proportional to the effective mass, $m^* = (m_x m_y m_z)^{1/3}$. [10 points]