PHY 251/420 **Midterm Exam** **Spring 2006**

Each part below is worth 20 points.

Consider a two-dimensional gas of free spin$-1/2$ electrons confined to the xy plane (i.e. assume motion in the $\hat{z}$ direction is quantized to its lowest energy level with $k_z = 0$). The number of electrons per unit area is n, and initially, there is no magnetic field applied to the system.

a) The density of states $g(\epsilon)$ is defined to be the number of single electron states per unit energy per unit area. Compute $g(\epsilon)$.

b) Find the Fermi energy, ϵ_F, of the system as a function of the electron density n.

c) Find the total energy per unit area, E/A, in the ground state.

d) Now assume that a uniform magnetic field, $\mathbf{H} = H\hat{z}$, is applied to the system. Ignore the interaction between $\mathbf{H}$ and the intrinsic electron spin. If n is such that the lowest p Landau levels are completely filled, and the $(p+1)^{st}$ Landau level is a fraction λ filled, find the total energy per unit area, $E(H)/A$.

e) Compute the change in energy per unit area due to the applied magnetic field, $\Delta E/A \equiv [E(H) - E(0)]/A$, and make a sketch of $\Delta E/A$ as a function of density n.