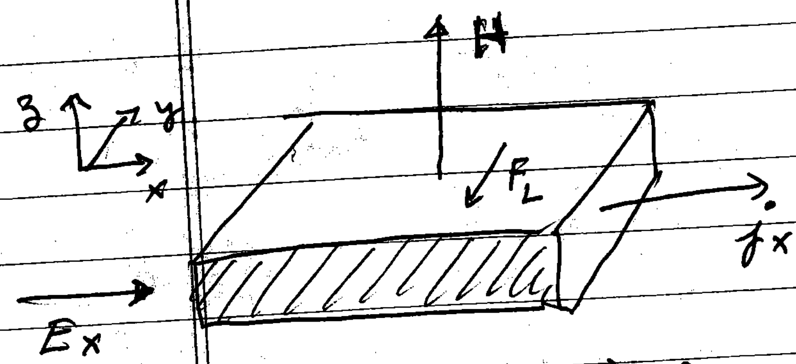
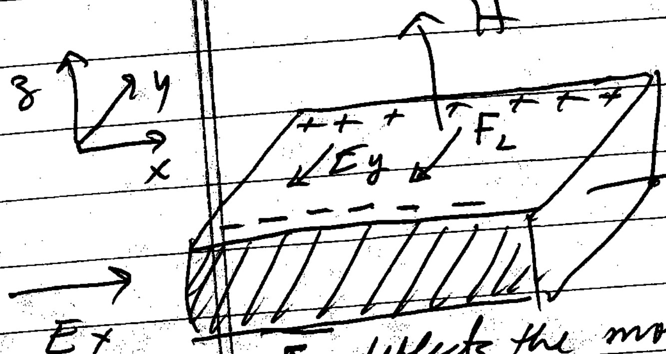
Hall effect (1879) - determines the sign of the charges that carry the electric current in a metal

electron motion in combined static electric and magnetic fields



Electric field E_x applied in $\hat{x}$ direction produces flowing electric current j_x in $\hat{x}$ direction. Magnetic field H in $\hat{z}$ direction exerts Lorentz force $\frac{\vec{j}}{c}\times\vec{H}$ on the ~~charges~~ moving charges carrying the current $\vec{j}$. For $\vec{j}$ in $\hat{x}$ direction and $\hat{H}$ in $\hat{z}$ direction, the Lorentz force $\vec{F}_L$ is in the $-\hat{y}$ direction. $\vec{F}_L$ deflects the charge carriers to the side wall of the wire (the shaded wall in the figure) where they build up and create a surface charge density. The surface charge density produces an electric field E_y in $\hat{y}$ direction. For a steady state situation, the force from E_y will exactly cancel out the Lorentz force F_L. If W is the width of the wire, then measuring the "Hall voltage" $V_y = E_y W$ allows one to determine the sign of the charges that carry the electric current.

If current is carried by negative charges $-q$, then



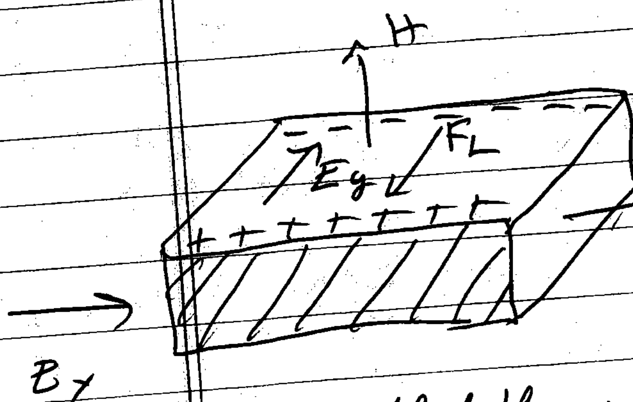
$j_x = -q n v_x \Rightarrow v_x < 0$

$\leftarrow v_x$

F_L deflects the mobile negative charges carrying the current, and negative charges build up on shaded surface (neutrality of system $\Rightarrow$ absence of negative charge, i.e. positive charge, builds up on opposite surface)

The electric field E_y is in $-\hat{y}$ direction and Hall voltage is <u>negative</u>

If current is carried by positive charges $+q$, then



$j_x = q n v_x, \quad v_x > 0$

$v_x \rightarrow$

F_L deflects the mobile positive charges carrying the current and positive charge builds up on the shaded surface

The electric field E_y is in the $+\hat{y}$ direction and the Hall voltage is <u>positive</u>.

For most (but not all) metals one finds a negative Hall voltage. This established that it was negatively charged electrons that carry the electric current in most metals.

Quantities to measure

Hall coefficient $R_H \equiv \dfrac{E_y}{j_x H}$

since we expect force from E_y to exactly balance out Lorentz force F_L in steady state, we expect R_H to be independent of H

magnetoresistance $\rho(H) \equiv \dfrac{E_x}{j_x}$

We can compute both R_H and ρ using the Drude model.

$$\frac{d\vec{p}}{dt} = -e\left(\vec{E} + \frac{\vec{p}}{mc} \times \vec{H}\right) - \frac{\vec{p}}{\tau} = 0 \text{ in steady state}$$

for x and y components

$$0 = -eE_x - \frac{eH}{mc} p_y - \frac{p_x}{\tau}$$

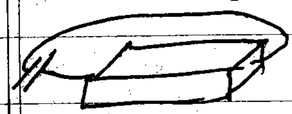
$$0 = -eE_y + \frac{eH}{mc} p_x - \frac{p_y}{\tau}$$

$\omega_c \equiv \dfrac{eH}{mc}$ cyclotron frequency = angular frequency of a charged particle in circular motion in uniform H

(1) $eE_x = -\omega_c p_y - \frac{p_x}{\tau}$

(2) $eE_y = \omega_c p_x - \frac{p_y}{\tau}$

In steady state, current flows only in $\hat{x}$ direction.



v_x

No current flows out the side walls of the wire $\Rightarrow p_y = 0$

with $p_y = 0$,

(1) $\Rightarrow \quad p_x = -eE_x\tau$

$$j_x = -ne v_x = -\frac{ne p_x}{m} = \frac{ne^2\tau}{m} E_x$$

$$\boxed{\frac{E_x}{j_x} = \frac{m}{ne^2\tau} = \rho}$$

magnetoresistance $\rho(H) = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$

same as ordinary d.c. resistivity ρ when $H=0$

In Drude model, $\rho(H)$ is independent of H!

agreed with expt measurements by Drude.

More modern expts however do find ρ can vary with H.

(2) $\Rightarrow E_y = \frac{\omega_c}{e} p_x = -\omega_c \tau E_x$

Hall coefficient $\quad R_H = \frac{E_y}{j_x H} = \frac{\left(\frac{\omega_c}{e} p_x\right)}{\left(-\frac{ne p_x}{m}\right) H} = \frac{-\omega_c m}{ne^2 H}$

use $\omega_c = \frac{eH}{mc} \Rightarrow R_H = -\frac{\left(\frac{eH}{mc}\right) m}{ne^2 H} = -\frac{1}{nec}$

$$\boxed{R_H = -\frac{1}{nec}}$$ Hall coefficient independent of H

But also, R_H is independent of our phenomenological parameter τ, the relaxation time.

R_H is something we can directly test against experiment since it only depends on the electron density n, which can be easily calculated.

In practice R_H is found to depend on H and T and also on sample preparation. But at low T, high H ($\sim 10^4$ gauss) very pure samples, R_H is found to approach a constant value, often very close to the Drude value

	metal	valence	$-1/R_H nec$ (=1 for Drude)	
alkalis	Li	1	0.8	Drude prediction very good for alkali metals which have single s shell electron as valence electron
	Na	1	1.2	
	K	1	1.1	
	Rb	1	1.0	
	Cs	1	0.9	
	Cu	1	1.5	
	Ag	1	1.3	
	Au	1	1.5	
	Be	2	−0.2	sign is negative! ⇒ current is carried by objects with positive sign
	Mg	2	−0.4	
	In	3	−0.3	
	Al	3	−0.3	

ac electric conductivity

$$\vec{E}(t) = Re\left[\vec{E}_\omega e^{-i\omega t}\right]$$ harmonic oscillating electric field

$$\frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} - e\vec{E}(t)$$ Drude eqn of motion

assume solution is also harmonic oscillation

$$\vec{p}(t) = Re\left[\vec{p}_\omega e^{-i\omega t}\right]$$

$$\Rightarrow \quad -i\omega\vec{p}_\omega = -\frac{\vec{p}_\omega}{\tau} - e\vec{E}_\omega$$

$$\left(\frac{1}{\tau} - i\omega\right)\vec{p}_\omega = -e\vec{E}_\omega$$

$$\vec{p}_\omega = \frac{-e}{\frac{1}{\tau} - i\omega}\vec{E}_\omega = \frac{-e\tau}{1-i\omega\tau}\vec{E}_\omega$$

current $\vec{j}(t) = Re\left[\vec{j}_\omega e^{-i\omega t}\right]$

$$\vec{j} = -en\vec{v} = -en\frac{\vec{p}}{m}$$

$$\vec{j}_\omega = -en\frac{\vec{p}_\omega}{m}$$

$$= \frac{ne^2\tau}{m(1-i\omega\tau)}\vec{E}_\omega$$

a-c conductivity

$$\vec{j}_\omega = \sigma(\omega)\vec{E}_\omega$$

$$\Rightarrow \quad \sigma(\omega) = \frac{ne^2\tau}{m(1-i\omega\tau)} = \frac{\sigma_{dc}}{1-i\omega\tau}$$

where $\sigma_{dc} = \frac{ne^2\tau}{m}$ is d.c. Drude conductivity

as $\omega \to 0$, $\sigma(\omega) \to \sigma_{dc}$

as $\omega \to \infty$, $\sigma(\omega) \to \frac{ne^2\tau}{-i\omega\tau m} = \frac{i n e^2}{m\omega}$ indep of τ

for $\omega\tau \gg 1$, ie $\omega \gg \frac{1}{\tau}$, oscillation is fast compared to collision rate, so $\sigma(\omega)$ becomes independent of τ.

Electromagnetic wave propagation in a metal

approx 1) In CGS units, for a propagating electromagnetic plane wave $|\vec{E}| = |\vec{H}|$.
so for the forces the EM wave exerts on a conduction electron

$$\frac{|\vec{F}_{mag}|}{|\vec{F}_{elec}|} = \frac{-e\,|\frac{\vec{v}}{c}\times\vec{H}|}{-e\,|\vec{E}|} \sim \frac{v}{c} \ll 1$$

so we will ignore the force that the $\vec{H}$ component of the wave exerts on the electron

approx 2) when wavelength λ of EM wave is much larger than mean free path l of collisions, $\lambda \gg l$, the electric field that an electron sees over the time between collisions is roughly ~~consta~~ uniform in space. Good for waves in visible spectrum where $\lambda \sim 5000\,\text{Å}$, $l \sim 10\,\text{Å}$.

(1) + (2) $\Rightarrow$ we can use the above computed a.c. conductivity $\sigma(\omega)$ to find the relation