Transport Properties of the Sommerfeld Model

We have seen that we must use quantum Fermi-Dirac statistics to describe the thermodynamic behavior of conduction electrons.

What about the equation motion? Semi-classical arguments can be made to show that in many cases it remains OK to use the classical Drude equation of motion. We need to be able to construct electron wave packets which are localized on the desired spatial length scales.

We saw that the typical electron energy is set by the Fermi energy ε_F. Hence the typical momentum is $p \sim p_F$. If we make a wavepacket with $\Delta p \ll p_F$ then spread in spatial position is

$$\Delta x \sim \frac{\hbar}{\Delta p} \gg \frac{1}{k_F} \sim r_s$$

So the electron cannot be localized to atomic length scales, but can be localized on the length scales of macroscopically varying electric, magnetic fields or temperature gradients ~ 1000 Å. So OK for motion in EM waves in visible spectrum but ~~not~~ for X-rays ($\lambda_{xray} \sim \mathring{A}$).

We also need $\Delta x \ll l$ the mean free path. Classical motion may fail when $l \sim 10\mathring{A}$.

$l \simeq v\tau$ where $\tau \sim 10^{-14}$ sec and
$v \sim v_F \sim 10^8$ cm/sec

so $l \sim 10^8 \cdot 10^{-14}$ cm $= 10^{-6}$ cm $= 100\mathring{A}$

so looks OK.

Using Drude Equation motion + Fermi Dirac Statistics we then have

① dc and ac electric conductivity same as for classical Drude model, since no thermo was involved

② For thermal conductivity we had

$$K = \frac{1}{3} v^2 \tau c_v$$

Now we should use $v \sim v_F$, $v_F^2 = \frac{2\varepsilon_F}{m}$

and $c_v = \frac{\pi^2}{3}\left(\frac{k_BT}{\varepsilon_F}\right) n k_B$

$$\Rightarrow K = \frac{1}{3}\frac{2\varepsilon_F}{m}\frac{\pi^2}{2}\left(\frac{k_BT}{\varepsilon_F}\right) n k_B \tau$$

$$= \frac{\pi^2}{3m}\tau n k_B^2 T$$

and Wiedemann – Franz coefficient is

$$\frac{K}{\sigma T} = \frac{\frac{\pi^2}{3m}\tau n k_B^2}{\frac{ne^2\tau}{m}} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 = 2.44\times10^{-8}\ \text{watt-ohm}/{}^\circ K^2$$

excellent agreement with experiment

③ Thermopower

$$Q = -\frac{c_v}{3ne} = -\frac{\pi^2}{6}\frac{k_B}{e}\left(\frac{k_B T}{\varepsilon_F}\right)$$

$$= -1.42\left(\frac{k_B T}{\varepsilon_F}\right)\times10^{-4}\ \text{volt}/{}^\circ K$$

more reasonable result than classical value

Magnetic properties of Free Electron Gas

In the presence of an applied magnetic field $\vec{H}$, the electron gas will develop a net magnetization via two effects

1) the intrinsic spins of the electrons anti-align with $\vec{H}$ ⇒ magnetic moments align with $\vec{H}$ ⇒ paramagnetic effect
   Pauli Paramagnetism

2) the electrons move in closed orbits ⇒ circulating currents ⇒ magnetic moments anti aligned with $\vec{H}$ ⇒ diamagnetic effect.
   Landau diamagnetism

We consider first Pauli Paramagnetism
(A+M Chpt 31

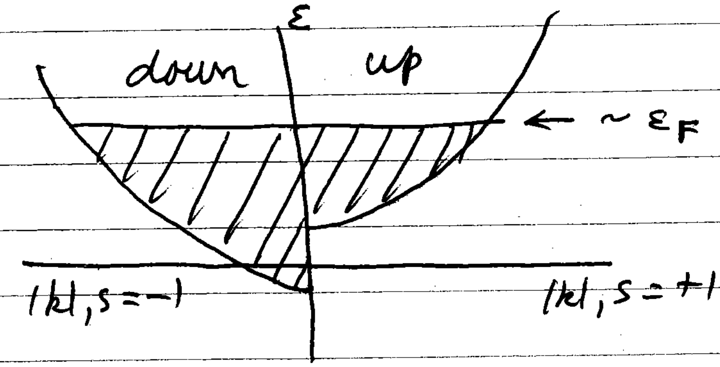
An electron with intrinsic spin $\vec{s}$ $(s_z = \pm 1)$ has intrinsic magnetic moment $\vec{\mu} = -\mu_0 \vec{s}$
where $\mu_0 = \frac{e\hbar}{2mc}$ is the Bohr magneton

the interaction energy of the spin with the applied magnetic field is

$$\delta\mathcal{H} = -\vec{\mu}\cdot\vec{H} = \mu_0 \vec{s}\cdot\vec{H}$$

state, as the energy will be lowered by having up electrons at $\varepsilon_F + \mu_0 H$, $s = +1$ convert into down electrons and go into the empty states at $\varepsilon_F - \mu_0 H$, $s = -1$.

The ground state will instead look like



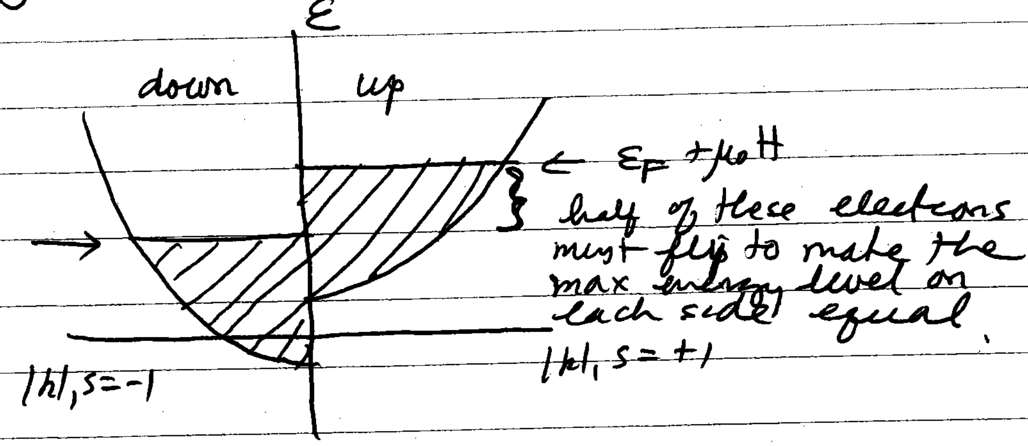
(shaded areas are filled electron states

and we thus see that there will be more down than up electrons in the ground state. Since $s = -1$ electrons have magnetic moment $-\mu_0 s$, the system has a net <u>positive</u> magnetization M aligned with H.

To see how big is M we need to compute the number of up electrons that flip into down electrons when H is turned on.

We will assume that H is small enough that $\mu_0 H \ll \varepsilon_F$. When this is so, we can ignore the fact that the density of states has a slight variation with energy ε over the range $\varepsilon_F + \mu_0 H$ to $\varepsilon_F - \mu_0 H$ and assume it to be roughly constant $g(\varepsilon_F)$.

The number of up electrons that flip is then easily computed from the following sketch



The number of up electrons that must flip is therefore

$$g_+(\varepsilon_F)\,\Delta\varepsilon$$

where $g_+(\varepsilon_F) = \frac{1}{2} g(\varepsilon_F)$ is the density of states of up electrons at ε_F, which is half the total density of states at ε_F, and $\Delta\varepsilon = \frac{1}{2}[\mu_0 H - (-\mu_0 H_0)] = \mu_0 H$ is the energy interval that must flip. The number that flip is therefore

$$\Delta n = \frac{1}{2} g(\varepsilon_F)\, \mu_0 H.$$

In the new ground state, the number of down electrons is now $n_0 + \Delta n$, and the number of up electrons is $n_0 - \Delta n$, where $n_0 = \frac{1}{2} n$ is the ~~tot~~ number when $H = 0$.

So the net magnetization is now (at $T=0$)

$$M = \mu_0 (n_- - n_+)$$
$$= \mu_0 (n_0 + \Delta n - (n_0 - \Delta n))$$
$$= 2\mu_0 \Delta n$$
$$= g(\varepsilon_F)\, \mu_0^2 H$$

and the <u>Pauli paramagnetic susceptibility</u> is

$$\boxed{\chi_P = \frac{\partial M}{\partial H} = g(\varepsilon_F)\,\mu_0^2}$$

proportional to density of states at Fermi energy

For the <u>free electron gas</u> we had

$$g(\varepsilon_F) = \frac{3}{2}\frac{n}{\varepsilon_F}$$

$$\Rightarrow \boxed{\chi_P = \frac{3}{2}\frac{n}{\varepsilon_F}\mu_0^2}$$

$n \sim k_F^3$, $\varepsilon_F \sim k_F^2$, so $\chi_P \sim k_F \sim \frac{1}{(r_s/a_0)}$

$$\chi_P = \frac{2.59}{(r_s/a_0)} \times 10^{-6}$$

corrections to above result at finite T of order $\left(\frac{T}{T_F}\right)^2$ so above is very good ~~that~~ at all $T \ll T_F$ and so good at room temperature.

Compare to experiment

metal	r_s/a_0	χ_p^{theory}	χ_p^{ext}	$\times 10^{-6}$
Li	3.25	0.80	2.0	
Na	3.93	0.66	1.1	
K	4.86	0.53	0.8	
Rb	5.20	0.50	0.8	
Cs	5.62	0.46	0.8	

turns out that the discrepancy between theory and expt is mainly due to having neglected electron-electron interactions!

Note that χ_p above is very different from what one gets with classical statistics.

Classically $$M \sim \left[\frac{(+1)e^{-\mu_0 H/k_BT} + (-1)e^{+\mu_0 H/k_BT}}{e^{-\mu_0 H/k_BT} + e^{+\mu_0 H/k_BT}}\right](-\mu_0)\, n$$

$$= \left[\frac{e^{\mu_0 H/k_BT} - e^{-\mu_0 H/k_BT}}{e^{\mu_0 H/k_BT} + e^{\mu_0 H/k_BT}}\right]\mu_0\, n$$

$$\sim \frac{\mu_0^2 H n}{k_BT} \quad \text{when } \mu_0 H \ll k_BT$$

$$\chi_{classical} = \frac{dM}{dH} \sim \frac{\mu_0^2 n}{k_BT} \qquad \text{Curie law} \sim \frac{1}{T}$$

so $$\frac{\chi_p}{\chi_{classical}} \sim \left(\frac{T}{T_F}\right) \ll 1$$