PHY 415 **FINAL EXAM** Dec. 19, 2003

Cross out work you do not wish me to look out, and circle your final answer where appropriate.

---

1) [25 points] Consider a plane polarized electromagnetic wave described by the vector and scalar potentials

$$\mathbf{A}(\mathbf{r},t) = \mathbf{A}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \quad \text{and} \quad \phi(\mathbf{r},t) = \phi_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

where the orientation of the vector $\mathbf{A}_0$ is arbitrary.

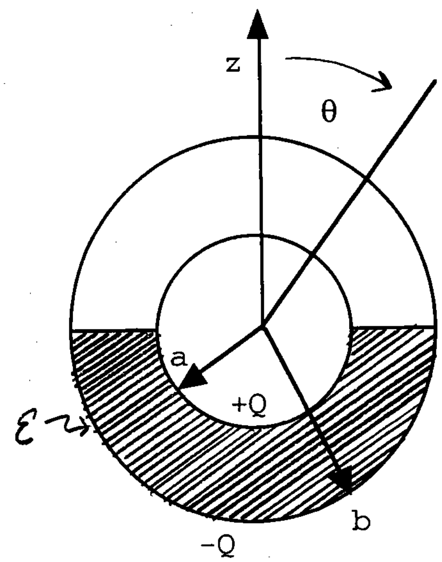
a) Using Maxwell's equations, find the relationship that must hold between $\mathbf{A}_0$ and ϕ_0.

b) Using the principle of gauge invarience, show that one can transform to a new but physically equivalent vector potential which is transversely polarized, $\mathbf{A}_0 \cdot \mathbf{k} = 0$.

c) What is the scalar potential in the gauge of part (b)?

---

2) [25 points] Two concentric conducting spheres of inner and outer radii a and b, carry total free charges +Q and -Q, respectively. The empty space between the spheres is half filled, as shown below, by a dielectric material with dielectric constant ε.



a) Find the electric field everywhere between the spheres.

b) Calculate the total surface charge density $\sigma_{tot}(\theta)$ on the surface of the inner sphere at r=a.

c) Calculate the induced bound surface charge density $\sigma_b(\theta)$ on the surface of the dielectric at r=a.

3) [25 points] The electric and magnetic fields of a plane electromagnetic wave traveling along the z axis, in a dissipative dielectric media, can be written as:

$$\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}\left\{\mathbf{E}_\omega e^{-k_2 z} e^{i(k_1 z - \omega t)}\right\},$$

$$\mathbf{B}(\mathbf{r}, t) = \mathrm{Re}\left\{\frac{c\,|\,k\,|}{\omega}(\hat{z} \times \mathbf{E}_\omega)\, e^{-k_2 z} e^{i(k_1 z - \omega t + \phi)}\right\}$$

where k_1 and k_2 are the real and imaginary parts of the wave vector, $|k| = \sqrt{k_1^2 + k_2^2}$, and $\tan(\phi) = k_2/k_1$.

For a linearly polarized wave, where the amplitude $\mathbf{E}_\omega$ is a real vector, **E** and **B** are orthogonal, ie. $\mathbf{E}\cdot\mathbf{B} = 0$. However for a general elliptically polarized wave, this is no longer true!

For a general elliptically polarized wave with

$\mathbf{E}_\omega = E\cos\theta\,\hat{x} + E\sin\theta\, e^{i\chi}\,\hat{y}$ (θ and χ arbitrary parameters)

a) Compute the value of $\mathbf{E}\cdot\mathbf{B}$.

b) Does $\mathbf{E}\cdot\mathbf{B}$ vary with time or spatial position?

c) Under what general conditions will $\mathbf{E}\cdot\mathbf{B} = 0$?

---

4) [25 points] Consider the radiation emitted by a circular wire loop of radius R, when the current flowing in the loop is given by

$$I(\phi, t) = \mathrm{Re}\left\{I_o \cos(n\phi)\, e^{-i\omega t}\right\}$$

where the loop is centered at the origin in the xy plane, and ϕ is just the usual spherical polar angle. The frequency ω is such that $R\omega \ll c$. Show that:

a) if n = 0, there is magnetic dipole radiation, but no electric dipole radiation.

b) if n = 1, there is electric dipole radiation, but no magnetic dipole radiation.

c) if n = 2, there is neither electric dipole, nor magnetic dipole radiation. What do you think happens in this case? You must explain why.